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THE APPROXIMATE CONTINUITY OF L_{D} SMOOTH FUNCTIONS

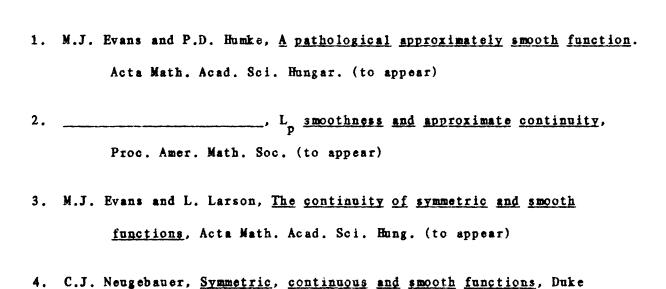
A real valued function f defined on the real line R is said to be smooth at a point xelk if

(*)
$$\lim_{t\to 0} \frac{\Delta^2 f(x,t)}{t} = 0$$

where $\Delta^2 f(x,t) = f(x+h)+f(x-h)-2f(x)$. If, in place of the usual limit in (*), we use the approximate limit, then f is said to be approximately smooth at the point xelk. Similarly, a measurable function f is said to be L_n $(1 \le p \le n)$ smooth at xeR if (*) holds with the limit taken in the L sense. The function f is called smooth or approximately smooth or L smooth if it is so at each xelk. The continuity properties of the associated classes of smooth functions have been studied quite extensively and many of these investigations have focused on identifying the set of those points at which a given function is discontinuous. In specific, Neugebauer showed that if f is measurable and smooth, then \mathbb{R} -C(f) is a nowhere dense countable set [C(f) = the continuity]points of f]. Subsequently, Evans and Larson showed that for measurable smooth functions, IR-C(f) is characterized as clairseme (or scattered). In each of the approximately and L_{p} smooth cases, Neugebauer showed that $\mathbf{R}\text{-}\mathbf{C}(\mathbf{f})$ can have large measure but that for approximately smooth f, IR-AC(f) has measure zero and for L smooth f, \mathbb{R} -L $\mathbb{C}(f)$ has measure zero. Here, $\mathbb{AC}(f)$ denotes the points of approximate continuity of f and $L_p^{C(f)}$ denotes the $L_p^{continuity}$ points of f.As Neugebauer mentions, a natural question is whether the nowhere dense and measure zero set R-L C(f) must be countable for an L smooth function f. An

associated question is whether the set R-AC(f) must be countable for an approximately smooth function f. In this lecture, the orator presents a general construction technique which shows that in either case the answer is negative. In specific, an appropriately (L_p) or approximately smooth function is constructed such that R-AC(f) is uncountable and as L_p C(f) \subset AC(f) the result(s) follow.

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