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Differentiability of Peano type functions - multidimensional case

Let us introduce some notions. Let $f = (f_1,\ldots,f_m) : R^n \to R^m$. We say that f is:

k-differentiable if for each $x \in R^n$ there exists a sequence $1 \le i_1 < \ldots < i_k \le m$ such that the function $(f_{i_1},\ldots,f_{i_k})$ is differentiable at x;

k-measurable if there exists a sequence $1 \le i_1 < \ldots < i_k \le m$ such that the function $(f_{i_1},\ldots,f_{i_k})$ is a Lebesgue measurable mapping from R^n to R^k.

In [1] the following theorem has been proved:

Theorem. For arbitrary natural numbers $m,n > 0$ and $k \ge 0$ the following sentences are equivalent:

(1) $2^{\aleph_0} \le \aleph_{n-k}$

(2) there exists a function $f : R^n \to R^{n+m}$ onto R^{n+m} which is n-differentiable and k-measurable.

(1) is known to be independent of the set theory ZFC (see [4]).

Some other related questions concerning k-differentiability of functions $f : R^n \to R^{n+m}$ and modifications of (2) were also considere in [1]. The results in [1] generalize theorems of [2] and [3].

References

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