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PARAMETRIC DERIVATIVES ARE IN BAIRE CLASS ONE

In recent years much effort by many different researchers has gone into proving that particular generalized derivatives are in Baire class one. (For example, see Filipczak [F], Larson [L2], Tolstoff [T] and Preiss [P].) The purpose of this note is to present two theorems concerning the Baire class of approximate parametric derivates, from which it can be concluded that many common generalized derivatives are in Baire class one. These theorems are proved in [L1].

A function h will be called a parameter function iff it satisfies:

(a) h is measurable;

(b) if G is any measurable set, then $h(G) = \{h(x): x \in G\}$ is measurable; and

(c) if A_n is any sequence of Borel sets such that $|A_n| \to 0$ as $n \to \infty$, then $|h^{-1}(A_n)| \to 0$.

To define the class of generalized derivatives with which we are concerned, let $h_1, \ldots, h_n$ be parameter functions and $a_i \in R$, $i = 1, \ldots, n$. A generalized difference quotient is defined as

$$Q(x,t) = [\Sigma_{i=1}^{n} a_i f(x + h_i(t))]/g(t)$$

where f and g are arbitrary functions. (Note that $Q(x,t)$ may not be defined for some values of x and t.) Using this difference quotient, we define an approximate parametric derivative of f at x to be

$$f^*(x) = \text{app} \lim_{t \to 0} Q(x,t).$$

The usual combinations of upper, lower, left and right parametric derivatives are defined analogously. For example, the right generalized approximate parametric derivative is

$$f^{*+}(x) = \text{app} \lim_{t \to 0^+} Q(x,t)$$

and the lower left approximate parametric derivate is

$$\underline{f}^{*-}(x) = \text{app lim inf}_{t \to 0^-} Q(x,t).$$

Appropriate choices of a_i, h_i, g and n allow $Q(x,t)$ to be the difference quotient of many different generalized derivatives.

Using the definitions given above, the following theorems can be proved.

<u>THEOREM 1.</u> If f is a measurable function such that f^{*+} exists everywhere, finite or infinite, then f^{*+} is in Baire class one.

<u>THEOREM 2.</u> If f is a measurable function, then its extreme approximate parametric derivates are in Baire class three.

The simplest nontrivial example of a difference "quotient" which satisfies the definitions given above is $Q_0(x,t) = f(x + t)$. The various limits of $Q_0(x,t)$ yield forms of continuity. As an easy example from Theorem 1, we state the following corollary.

<u>COROLLARY 3.</u> If f is a function which is right approximately continuous at every point, then f is in Baire class one.

Because it is well-known that an approximately continuous function is in Baire class one, we suspect that Corollary 3 is known, but we cannot find a reference for it.

Another example of a difference quotient which satisfies the conditions stated above is that of the symmetric derivative, $[f(x + t) - f(x - t)]/2t$. An application of Theorem 1 shows that the approximate symmetric derivative of a measurable function is in Baire class one. This was proved in Larson [La1]. In fact, using roughly the same method, the following corollaries can be proved.

<u>COROLLARY 4.</u> Let f be a measurable function which has an approximate Riemann derivative of order n, $f^{(n)}$, at every point. If n is odd, then $f^{(n)}$ is in Baire class one and if n is even, then f is in Baire class one.

Theorem 1 does not immediately imply anything about f when n is odd, or about $f^{(n)}$ when n is even.

<u>COROLLARY 5.</u> Any measurable approximately symmetric function is in Baire class one.

C. E. Weil [W] proved that if f is a Darboux-Baire one function which has a nonnegative second symmetric derivative at every point, then f is convex. Corollary 4 allows a partial strengthening of this result.

COROLLARY 6. If f is a measurable function with the Darboux property such that $f^{(2)}$ exists and is nonnegative everywhere, then f is convex.

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