Jack Ceder, Department of Mathematics, University of California, Santa Barbara, CA 93106

A Report on Globs

Agronsky [1], showed that each F_σ bilaterally c-dense-in-itself subset of R can be expressed in the form $\cup\{A(\alpha) : \alpha \in [1, \infty)\}$ where each $A(\alpha)$ is closed and whenever $\alpha < \beta$ each point of $A(\alpha)$ is a bilateral c-limit point of $A(\beta)$. (Any set expressible in this form will be called a linear glob.) Then according to Agronsky the function defined by

$$f(x) = \begin{cases} \dfrac{1}{\inf\{\alpha : x \in A(\alpha)\}} & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases}$$

becomes a Darboux, upper semi-continuous function such that $A = f^{-1}(0,\infty))$. Therefore, the following are true

(1) Each bilaterally c-dense-in-itself F_σ is a linear glob;

(2) a set is a linear glob if and only if it is the inverse image of a non-void open set under a Darboux Baire 1 (semi-continuous function.

Can these results be extended to two dimensions? In order to do so we must first extend the notions of a bilateral c-limit point and a Darboux function to two dimensions.

A point z is called a panoramic c-limit point of a planar set A if each non-degenerated closed triangle containing z also contains

$2^{\aleph_0}$ points of A. We write $A \subseteq p\ B$ if $A \subseteq B$ and each point of A is a panoramic c-limit point of B. A family of closed sets $\{F(\alpha) : \alpha \in [1,\infty)\}$ is called a <u>hierarchy</u> if $F(\alpha) \subseteq p\ F(\beta)$ whenever $\alpha < \beta$. The union of a hierarchy is called a <u>glob</u>. A function $f : R^2 \to R$ is said to be <u>Darboux</u> if the image of each non-degenerate closed triangle is an interval.

It is shown in [2] that in contrast to the one dimensional case, that a panoramically c-dense-in-itself F_σ set is not necessarily a glob. Thus, (1) does not extend to our two dimensional setting. As far as (2) is concerned, it is true that each glob is the inverse image of an open set under a Darboux semi-continuous function; however, the converse remains an unsolved problem.

Our unsuccessful attempt to extend (1) and (2) satisfactorily then leads to studying globs in detail. We obtain some interesting results, one of which, (e) below, is of independent interest. Many open questions remain. The main additional results can be summarized as follows:

(a) <u>Globness is not a topological invariant</u>;

(b) <u>a panoramically c-dense-in-itself</u> F_σ <u>which is "locally" a is a glob</u>;

(c) <u>a product of two linear globs each dense in (0,1) is not necessarily a glob</u> (it is if one set is open in the density topology).

(d) <u>Any planar set of positive Lebesgue measure contains a glob</u> [3].

(e) if A is a null subset (resp. of first category) of the unit square I^2, then there exists a non-void perfect subset P of I and a subset Q of I which has full measure (resp. is residual) such that $P \times Q$ misses A.

It is unknown whether a residual subset of the plane contains a glob.

[1] Agronsky, S., Characterizations of certain subclasses of the Baire class 1, Py.D. dissertation, Dept. of Mathematics, University of California, Santa Barbara, 1974.

[2] Ceder, J. On Globs to appear in Acta Math. Hungar.

[3] Mustafa I, Ph.D. dissertation UCSB, in preparation.