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On Functions With Non-Negative Divided Differences

SUMMARY

Let $V_n(F) \equiv V_n(F;x_0,x_1,\dots,x_n)$ be the nth divided difference of F with respect to the (n+1) points $x_0,x_1,\dots,x_n$ on an interval [a,b].

If the inequality $V_n(F) \geq 0$ for all choices of points $x_0,x_1,\dots,x_n$ in [a,b] then F is said to be n-convex on [a,b].

It is shown that if F(x) is n-convex on [a,b], if $F^{(r)}(x)$ exists and is continuous on [a,b], $0 \leq r \leq n-2$ and if $F_{(n-1),+}(a)$ is finite, then

$$F(x) = G(x) + \sum_{k=0}^{n-1} F_{(k),+}(a) \frac{(x-a)^k}{k!}, \quad x \in [a,b],$$

where G(x) is monotonic increasing on [a,b], and $F_{(k),+}(a)$ is the one-sided Peano derivative of F at a.

The result has applications to approximation theory.

(A more comprehensive presentation of the above results will appear in the Proceedings of the Oberwolfach Conference on General Inequalities (1981).)