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ON NON-BAIRE SETS

We are giving some theorems in which the existence of non-Baire sets in category bases has been established. The definitions of category base, Baire set, non-Baire set, meager and abundant set, region can be found in the book of J.Morgan II (see [3]). By (X, S) we will denote a categogry base with the familly $B(S)$ of Baire sets and the family $M(S)$ of meager sets.

Theorem A (see [1] and [2]). *Let (X, S) be a category base such that the following conditions are satisfied:*

1. $M_0 \subset M(S)$ *where* $M_0 = \{A \subset X : card(A) < card(X)\}$,
2. $M(S)$ *has a base of cardinality not greater than* $card(X)$.

Then each abundant subset of X contains a non-Baire set.

A subfamily $S' \subset S$ is π-base of a category base (X, S), if each region $A \in S$ contains a subregion $B \in S'$. A set $A \in X$ is not exhausted if, for each meager set P in category base (X, S), we have $card(A - P) = card(X)$.

Theorem B. *Let (X, S) be a category base satisfying the following conditions:*

1. *there exists a π-base S' such that $card(S') \leq card(X)$ and each member of S' is not exhausted,*
2. $M(S)$ *has a base of cardinality not greater than* X.

Then each abundant set $B \subset X$ contains a non-Baire set.

Theorem C (see [3]). *If (X, S) is a point meager category base satisfying c.c.c., then each abundant set of cardinality ω_1 contains a non-Baire set.*

Problem 1. Let (X, S) be a point meager category base such that the family $M(S)$ has a base of cardinality not greater than $card(X)$. Does every abundant set of cardinality ω_1 contain a non-Baire set?

Theorem D. *Let (X, S) be a category base satisfying the following conditions:*

1. *for an arbitrary cardinal $\alpha \leq card(X)$, the ideal $M(S)$ is α -additive,*
2. $M(S)$ *has a base of cardinality not greater than* $card(X)$.

If X is not meager set, then for an arbitrary family $\{X_t : t \in T\}$ of meager sets being a partition of X, there exists a set $T' \subset T$ such that $\cup\{X_t : t \in T'\}$ is a non-Baire set.

Shilling has recently introduced an interesting example of a category base $(2^\omega, C^*_M)$ in Cantor set exploring the concept of Mycielski ideal I_M (see [4]).

Problem 2. Does every abundant set in category base $(2^\omega, C^*_M)$ contain a non-Baire set?

Problem 3. Does the equality $Borel(2^\omega)\triangle M(C^*_M) = B(C^*_M)$ hold?

REFERENCES

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