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Michael J. Evans, Department of Mathematics, Washington and Lee University, Lexington, VA 24450, email:mjevans@@wlu.edu

FIRST RETURN DIFFERENTIATION

Composite differentiation was introduced by O'Malley and Weil in [6], and first return differentiation was introduced by O'Malley in [5]. A notion seemingly stronger than first return differentiation, called universal first return differentiation, was introduced by Darji, Evans, and O'Malley in [1], where it was shown that if a function $F : [0,1] \to \mathbb{R}$ is compositely differentiable to f on $[0,1]$, where for each $x \in [0,1]$ $f(x)$ is a bilateral derived number of F, then F is universally first return differentiable to f on $[0,1]$. Since approximate derivatives, Peano derivatives, and approximate Peano derivatives are all known to be composite derivatives (See [4], [2], and [3], respectively.), it follows that all of these derivatives are universal first return derivatives. My first question is whether the same is true for the $\mathcal{I}$-approximate derivatives introduced by Wilczyński in [7].

Question 1 *If $F : [0,1] \to \mathbb{R}$ has f as its $\mathcal{I}$-approximate derivative on $[0,1]$, is F compositely differentiable to f on $[0,1]$?*

My second question asks whether first return differentiation and universal first return differentiation are really different.

Question 2 *If $F : [0,1] \to \mathbb{R}$ is first return differentiable to f on $[0,1]$, is F universally first return differentiable to f on $[0,1]$?*

References

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