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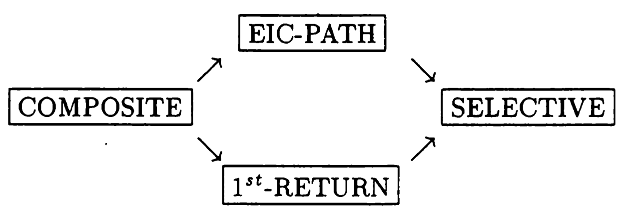
STRAINING FOR THE HARMONY AMIDST A CACOPHONY OF DERIVATIVES

In 1984 A. M. Bruckner, R. J. O'Malley, and B. S. Thomson [1] introduced path differentiation and showed it to be a valid synthesizing framework to encompass most of the fruitful notions of generalized differentiation of functions of a real variable which were under study at that time. Included among these types of differentiation were approximate, Peano, and selective differentiation. In that paper the authors introduced several intersection properties for the underlying path systems as a means of studying the resulting path derivatives and path differentiable functions. (Among these were the internal intersection condition (I.I.C.), the external intersection condition (E.I.C.), and the one-sided external intersection condition (1-sided E.I.C.).) Subsequent to [1], the concept of composite differentiation was introduced by O'Malley and C. E. Weil in [6]. In [7] O'Malley showed that this type of differentiation can also be described in terms of path differentiation with the underlying path system satisfying a type of intersection condition different from those investigated in [1]. Recently, O'Malley [8] introduced the intriguing notion of first return differentiation. The purpose of the present talk is to observe where composite and first return differentiation fit in to the path differentiation scheme of generalized differentiation based on the intersection properties introduced in [1] and to shed additional light on the relationships between these various types of path differentiation. Specifically, a schematic outline of the situation is illustrated in the following diagram.

(1)



Each node of diagram (1) is to be interpreted as a statement about an ordered pair (F, f) of real valued functions defined on $[0,1]$. The COMPOSITE node is intended to represent the statement "F has f as a bilateral derived function and is compositely differentiable to f on $[0,1]$." We take the E.I.C.–PATH node to represent the statement "F is E.I.C. path differentiable to f." Node 1^{st}–RETURN represents the statement "F is first return differentiable to f on $[0,1]$." Finally, the SELECTIVE node represents the conjunction of the statement "f is a bilateral Baire 1 derivate function of F" and any of the following equivalent statements: a) "F is selectively differentiable to f on $[0,1]$," b) "F is I.I.C.–path differentiable to f on $[0,1]$," c) "F is one-sided E.I.C.–path differentiable to f on $[0,1]$," and d) "Every perfect set $M \subseteq [0,1]$ contains a dense G_δ set K such that F restricted to M is differentiable to f at each point of K." The equivalence of a) – d) under the assumption that f is a Baire 1 bilateral derivate function of F was established in [4], where d) was called Condition $\mathcal{B}$. The validity of diagram (1), including examples to show that no additional arrows can be drawn between nodes, is presented in [2].

References

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