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CORE DENSITY TOPOLOGIES

We shall investigate some topologies connected with the core topology on the plane.

We recall the notion of the core topology (cf. [2]).

A set $A \subset \mathbb{R}^2$ is open in the core topology ($\mathcal{T}$-core) if for each point $x \in A$ and for each $y \in \mathbb{R}^2$ there exists $\varepsilon > 0$ such that $x + ty \in A$ for $t \in (-\varepsilon, \varepsilon)$, it means that for each point $x \in A$ a set A contains an open segment centered at the x in every direction.

There are two kinds of natural modification of this definition — changing the number of directions on which we want to have a neighbourhood in a topology on the line or changing this topology. We obtain following definitions.

A set A is open in a core-almost everywhere topology ($\mathcal{T}$-core-a.e.) if for each point $x \in A$ the set A contains an open segment centered at x on almost each direction, it means that for each point $x \in A$ there exists a linear set H with full linear Lebesgue measure such that for all $\Theta \in H$ a set $p_\Theta \cap A$ contains an open segment centered at x, where p_Θ denotes a line running through the point x and forming an angle Θ with the $0x$-axis.

A set A is open in an approximate-core topology ($\mathcal{T}$-ap-core) if for each $x \in A$, for each line p_Θ passing through the point x there exists a linearly measurable set B contained in $p_\Theta \cap A$ such that the x is a point of the linear density of the set B and $x \in B$ (cf. [4]).

A set A is open in a Hashimoto-core topology ($\mathcal{T}$-Hash-core) if for each $x \in A$, for each $\Theta \in \langle 0, \pi)$ a set $p_\Theta \cap A$ contains a neighbourhood of the point x in the Hashimoto topology on the line, it means a set of the form $G - N$, where G is open in Euclidean topology and N is a null-set, and $x \in G - N$ (cf. [1]).

A set A is open in a Hashimoto-core-almost everywhere topology ($\mathcal{T}$-Hash-core-a.e.) if for each $x \in A$ a set A contains a neighbourhood of the point x in the Hashimoto topology on the line on almost each direction.

A set A is open in an approximate-core-almost everywhere topology ($\mathcal{T}$-ap-core-a.e.) if for each $x \in A$ a set A contains a neighbourhood of the point x in the density topology on the line on almost each direction.

Families of open sets defined above really form topologies.

To compare them with density topologies on the plane we add one more assumption that a set to be open should be Lebesgue measurable.

We can analogically make definitions of topologies conected with the ideal of meager sets: $\mathcal{T}$-I-ap-core, $\mathcal{T}$-core-I-a.e., $\mathcal{T}$-I-Hash-core, $\mathcal{T}$-I-Hash-core-I-a.e. and $\mathcal{T}$-I-ap-core-I-a.e.

In these definitions we require open sets to have the Baire property on the plane, we replace the density topology on the line by the I-density topology, the Hashimoto topology by the I-Hashimoto topology which consists of sets of the form $G - P$, where G is open in Euclidean topology and P is a meager set, in topologies "I-a.e." we require existence of a proper neighbourhood on directions from a residual set (cf. [3], [5]).

We come back to the density-core topologies. The chart below shows the relationships between the following classes.

$\mathcal{T}$-Hash-core	$\subset$	$\mathcal{T}$-Hash-core-a.e.	$\subset$	$\mathcal{T}$-ap-core-a.e.	$\subset$	d_2
$\mathcal{O}$	$\subset$	$\mathcal{T}$-core	$\subset$	$\mathcal{T}$-Hash-core	$\subset$	$\mathcal{T}$-ap-core

We denote here the product of the density topologies by $d \times d$, an ordinary density topology on the plane by d_2 and a strong density topology on the plane by d_2^s, the Euclidean topology by $\mathcal{O}$, the Hashimoto topology by $\mathcal{O}^*$.

All the inclusions are proper.

We receive a similar diagram for the core topologies connected with the ideal of meager sets. Only results of comparing with the ordinary I-density topology are a little bit different.

References

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