Real Analysis Exchange
Vol. 20(2), 1994/5, pp. 405–405

Tomasz Natkaniec*, Mathematical Institute WSP, Chodkiewicza 30, 85–064 Bydgoszcz, Poland, email: wspb11@pltumk11.bitnet

FIXED POINTS AND ITERATIONS OF DARBOUX FUNCTIONS

(1) If $f : \mathbb{R} \to \mathbb{R}$ is weakly connected (i.e., for every interval I, $f|I$ can be separated by no continuous function $h : I \to \mathbb{R}$) and for every $x \in \mathbb{R}$ there is an n_x with $f^{n_x}(x) = 1$, then $f(1) = 1$.

(2) There exists a Darboux function $g : \mathbb{R} \to \mathbb{R}$ such that $g(1) \neq 1$ and for every $x \in \mathbb{R}$ there is an n_x with $g^{n_x}(x) = 1$.

(3) Assume that $f : \mathbb{R} \to \mathbb{R}$ is a Darboux function and for every $x \in \mathbb{R}$ an n_x is chosen with $f^{n_x}(x) = 1$. If the set of all n_x is bounded above, then $f(1) = 1$.

(4) Assume that f is a Darboux function and for each $x \in \mathbb{R}$ there exists an $n_x \in \mathbb{N}$ with $f^{n_x}(x) = x$. Then either f is the identity on $\mathbb{R}$, or else f is continuous and decreasing with $f = f^{-1}$.

References

[1] T. Natkaniec, *On iteration of Darboux functions*, Real Anal. Exchange, to appear.

Key Words: Darboux function, weakly connected function, quasi-continuous function, Lusin condition (N), strong measure zero set, fixed point
Mathematical Reviews subject classification: Primary 26A18. Secondary 26A15, 47H10
*Supported by KBN Research Grant 1992-94, 2 1144 91 01.