**Real Analysis Exchange**
Vol. 20(1), 1994/95, pp. 26–27

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# SMOOTHING $\Lambda$-SEQUENCES

In a recent investigation concerning bounded $\Lambda$-variation as a gap Tauberian condition, a question about $\Lambda BV$ spaces arose which has not been previously considered.

We quickly recapitulate the essential facts about these spaces: Let $\Lambda = \{\lambda_n\}$ be a nondecreasing sequence of positive real numbers such that

$$\sum \frac{1}{\lambda_n} = \infty.$$

A function $f$ defined on an interval (finite or infinite) is of $\Lambda$-bounded variation if $\sum |f(b_n) - f(a_n)|/\lambda_n$ converges for every sequence of nonoverlapping intervals $\{[a_n, b_n]\}$. The class of such functions is known as $\Lambda BV$. It may be shown that such functions are regulated, i.e., right and left limits exist at each point. We generally assume that $\lambda_n \nearrow \infty$, for otherwise, $\Lambda BV = BV$.

In the study of the Tauberian theorem we referred to, it seemed necessary to make the assumption that

$$\limsup \lambda_{n+1}/\lambda_n < \infty.$$

A question which arises naturally is

**Question 1** *Given a class* $\Lambda BV$ *for which* $\limsup \lambda_{n+1}/\lambda_n = \infty$, *is there a* $\Gamma = \{\gamma_n\}$, *with* $\limsup \gamma_{n+1}/\gamma_n < \infty$, *such that* $\Gamma BV = \Lambda BV$*?*

When $\Gamma BV = \Lambda BV$, we shall say that the sequences $\Gamma$ and $\Lambda$ are equivalent.

After we answered this question affirmatively, the next to come to mind was,

**Question 2** *For any given* $\Lambda$ *is there a* $\Gamma$ *equivalent to* $\Lambda$ *such that* $\lim \gamma_{n+1}/\gamma_n = 1$*?*

A sequence is called *smooth* if $\lim \gamma_{n+1}/\gamma_n = 1$.

Question 2 was also answered affirmatively. The method employed for Question 1 consisted of altering a subsequence of $\Lambda$ to form the desired $\Gamma$. The method employed for Question 2 is an amplification of the original argument. This method, although direct, is relatively complicated. Another question, which also has an affirmative answer is

**Question 3** *Is there a computationally simple method by which one can obtain a smooth $\Gamma$ equivalent to a given $\Lambda$?*