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MEASURE PRESERVING CONTINUOUS SMOOTHING OF FRACTIONAL DIMENSIONAL SETS

A set $E \subset \mathbb{R}^n$ is called an s–set if E has nonzero, finite Hausdorff s–dimensional measure. Call an s–set *smooth* if $\forall \delta > 0$, $s\text{–}m(E) = s\text{–}m_\delta(E)$. A necessary and sufficient condition for E to be a smooth s–set is easily seen to be that for each set A, $s\text{–}m(E \cap A) \leq \text{diam}(A)^s$. Some general examples are given in which there is a continuous, one–to–one function f which is measure preserving on E so that $f(E)$ is smooth (such an f will be called a *smoothing* of E). Each s–set is shown to be the countable union of sets for which there are smoothings.