Research Articles Real Analysis Exchange Vol. 17 (1991-92)

Andrzej Szymanski, Department of Mathematics, Slippery Rock University of Pennsylvania, Slippery Rock, PA 16057.

ON THE MEASURABILITY PROBLEM FOR CATEGORY BASES

In an effort at unifying properties which are common for measure and category, John C. Morgan II defined the concept of a category base. Namely, a family $\mathbb{C}$ is said to be a *category base* on X if $\mathbb{C}$ is a family of non-empty subsets of X, called *regions*, satisfying the following axioms: (1) $\cup\mathbb{C} = X$, (2) if $\mathcal{S} \subseteq \mathbb{C}$ and $A \in \mathbb{C}$ are such that $|\mathcal{S}| < |\mathbb{C}|$, $\mathcal{S}$ is pairwise disjoint and $A \cap B$ contains no region for each $B \in \mathcal{S}$, then there exists a region D such that $D \subseteq A - (\cup\mathcal{S})$. Although the definition given here differs from the original one (cf. [M1]), they are trivially equivalent. With respect to a given category base $\mathbb{C}$ on X, one can classify the subsets of X in the following way. A set $E \subseteq X$ is *singular* if each region contains a subregion disjoint from E. A set $M \subset X$ is *meager* if it is a union of countably many singular sets; the family of all meager sets for $\mathbb{C}$ is denoted by $\mathcal{M}(\mathbb{C})$. A set $B \subset X$ is *Baire* if every region contains a subregion A such that either $A \cap B$ or $A - B$ is meager; the family of all Baire sets for $\mathbb{C}$ is denoted by $\mathcal{B}(\mathbb{C})$.

A typical example of a category base is the family of measurable sets of positive measure with respect to a σ-finite complete measure. In this case, or even in the case of a *ccc* complete measure space, the meager sets coincide with the sets of measure zero and the Baire sets with measurable ones (see [M3]). K. Schilling [Sc] showed that there exists a complete and non-atomic measure space such that the family of all measurable sets is not the set of all Baire sets for any category base. In connection with this the following Measurability Problem was posed by J.C. Morgan II (cf [M2; p.379]): Characterize those complete measure structures for which the measurable sets coincide with the Baire sets for a suitable category base of measurable sets. We provide two such characterizations (Theorem 3 and Theorem 4) but with an additional stipulation that the measure zero sets coincide with the meager sets. The lack of this stipulation may yield measurable cardinals (see Proposition 2).

Our basic sources for related results and for used but undefined concepts are as follows: set-theoretical – Jech's book [J], measure-theoretical – Fremlin's survey paper [F] and category base theoretical – Morgan's book [M1].

The crucial role in our approach is played by the *ccc* decomposability of measure spaces. To justify its definition, we discuss first a parallel property for category bases. Let $\mathbf{C}$ be a category base on X. We say that a family $\mathcal{R}$ is a $\mathbf{C}$-*decomposition* of X if it satisfies the conditions:

(1) $\mathcal{R}$ is a disjoint family of non-meager Baire sets,

(2) $X - (\cup\mathcal{R})$ is meager,

(3) if Y is a non-meager Baire set, then there exists $A \in \mathcal{R}$ such that $Y \cap A$ is non-meager.

Proposition 1. *Let $\mathcal{R}$ be a $\mathbf{C}$-decomposition of X. If Y is a subset of X such that $Y \cap A$ is Baire for each $A \in \mathcal{R}$, then Y is Baire.*

Before proving it, let us notice the following well-known fact (cf. [M1] or [DS]).

Lemma. *Let $\mathbf{C}$ be a category base on X. If B is a region and A is a Baire set such that $A \cap B$ is non-meager, then there exists a region D such that $D \subseteq B$ and $D - A$ is meager.*

Proof of Proposition 1. Let B be a region. Without loss of generality we may assume that B is non-meager. There exists $A \in \mathcal{R}$ such that $B \cap A$ is non-meager. By virtue of the Lemma, there exists a region D such that $D \subseteq B$ and $D - (B \cap A)$ is meager. Since $Y \cap A$ is Baire, there exists a region E such that $E \subseteq D$ and either $E \cap (Y \cap A)$ is meager or $E - (Y \cap A)$ is meager. Hence $E \cap Y = E \cap (Y \cap A) \cup E \cap (Y \cap (X - A)) \subseteq E \cap (Y \cap A) \cup (E - A)$ is meager if $E \cap (Y \cap A)$ is meager or $E \cap (X - Y) \subseteq E - (Y \cap A)$ is meager if $E - (Y \cap A)$ is meager.

We say that a triple $(X, \mathcal{M}, \mu)$ is a *complete measure space* if $\mathcal{M}$ is a σ-field of subsets of X and μ is an extended real function on $\mathcal{M}$ satisfying the following conditions:

(i) $\mu(X) > 0$ and $0 \leq \mu(A) \leq \infty$ for all $A \in \mathcal{M}$

(ii) if $\mu(A) = 0$ and $B \subseteq A$, then $B \in \mathcal{M}$ (and $\mu(B) = 0$);

(iii) $\mu(\cup(A_n : n \in \omega)) = \sum_n \mu(A_n)$ for all pairwise disjoint sequences $\{A_n\}$ of elements of $\mathcal{M}$.

A complete measure space $(X, \mathcal{M}, \mu)$ is *ccc decomposable* if there exists a disjoint family $\mathcal{R} \subseteq \mathcal{M}$ of sets of positive measure such that:

(a) $X - (\cup\mathcal{R})$ is a set of measure zero;

(b) if $A \in \mathcal{R}$, then there are at most countably many disjoint subsets of A of positive measure (i.e., A is *ccc* for each $A \in \mathcal{R}$);

(c) if $\mu(A) > 0$, then there exists $B \in R$ such that $\mu(B \cap A) > 0$;

(d) if $Y \subseteq X$ and $Y \cap A$ is measurable for each $A \in \mathcal{R}$, then Y is measurable.

Theorem 1. *Let $(X, \mathcal{M}, \mu)$ be a complete measure space which is ccc decomposable. Then there exists a category base $\mathbf{C} \subseteq \mathcal{M}$ such that the Baire sets coincide with the measurable sets (i.e., $\mathcal{B}(\mathbf{C}) = \mathcal{M}$) and the meager sets coincide with the sets of measure zero (i.e., $\mathcal{M}(\mathbf{C}) = \mathcal{N} = \{A \in \mathcal{M} : \mu(A) = 0\}$).*

Proof. Let $\mathcal{R} \subseteq \mathcal{M}$ be a disjoint family of sets of positive measure satisfying the conditions (a) - (d). For any $B \in \mathcal{M}$ let $J_B = \{A \in \mathcal{R} : \mu(B \cap A) > 0\}$. We define $\mathcal{P} = \{B \in \mathcal{M} : 0 < |J_B| \leq \chi_0\}$ and then $\mathbf{C} = \{B \cap (\cup J_B) : B \in \mathcal{P}\} \cup \{X\}$. We shall show that $\mathbf{C}$ is a category base on X such that $\mathcal{M}(\mathbf{C}) = \mathcal{N}$ and $\mathcal{B}(\mathbf{C}) = \mathcal{M}$.

To see that $\mathbf{C}$ is a category base notice first that $\mathbf{C} \subseteq \mathcal{M}$ and that $\cup\mathbf{C} = X$. Now let $\mathcal{S}$ be a disjoint subfamily of $\mathbf{C}$ and let $A \in \mathbf{C}$ be such that $A \cap B$ contains no region for each $B \in \mathcal{S}$. We can obviously rule-out the possibilities that $A = X$ and $X \in \mathcal{S}$. Then $A = A' \cap (\cup J_{A'})$ for some $A' \in \mathcal{P}$. Since $\mathcal{P}$ is closed under measurable subsets of positive measure, $A \cap B$ must be a set of measure zero for each $B \in \mathcal{S}$. Moreover, only countably many are non-empty. For suppose $A \cap B \neq \emptyset$ for $B = B' \cap (\cup J_{B'}) \in \mathcal{S}$ and $B' \in \mathcal{P}$. Then $(\cup J_{A'}) \cap (\cup J_{B'}) \neq \emptyset$, so there exist $A'' \in J_{A'}$, $B'' \in J_{B'}$ with $A'' \cap B'' \neq \emptyset$. Because $A'', B'' \in \mathcal{R}$, this means $A'' = B''$. Consequently, $\mu((\cup J_{A'}) \cap B) \geq \mu(B'' \cap B) > 0$. Since each member of $\mathcal{R}$ is *ccc*, $\cup J_{A'}$ is *ccc*, as well, which implies that $|\{B \in \mathcal{S} : B \cap A \neq \emptyset\}| \leq \chi_0$. Hence there exists a set of measure zero N such that $A \cap (\cup\mathcal{S}) \subseteq N$. Thus $A - N \in \mathbf{C}$ and $A - N \subseteq A - (\cup\mathcal{S})$.

It is obvious that each set of measure zero is singular. To prove that $\mathcal{M}(\mathbf{C}) = \mathcal{N}$ it is enough to check that each singular set is of measure zero. So let $E \subseteq X$ be a singular set. If $A \in \mathcal{R}$, then $E \cap A$ is a singular set with respect to the category base of sets of positive measure from a *ccc* measure algebra on A, namely from the trace algebra $\mathcal{M}|A$. To see this let B be a subset of A of positive measure. Then $J_B = \{A\}$ and so $B \in \mathbf{C}$. There exists $B' \in \mathbf{C}$ such that $B' \subseteq B$ and $B' \cap E = \emptyset$. Hence B' is a set of positive measure which is a subset of B and disjoint from E. Thus $E \cap A$ is a set of measure zero for each $A \in \mathcal{R}$. By virtue of the property (d), the set E belongs to $\mathcal{M}$ and by (c), E is a set of measure zero.

It is obvious that each measurable set is Baire. To prove the converse, let $B \in \mathcal{B}(\mathbf{C})$ and $A \in \mathcal{R}$. It can be argued as above that then $B \cap A$ is a Baire set with respect to the category base of sets of positive measure from the trace algebra $\mathcal{M}|A$. Hence $B \cap A$ is measurable for each $A \in \mathcal{R}$. By virtue of the property (d), $B \in \mathcal{M}$.

We say that a measure space $(X, \mathcal{M}, \mu))$ has the *ccc (finite) subset property* if each subset of positive measure contains a *ccc* measurable subset of positive (and finite) measure. Notice that if $(X, \mathcal{M}, \mu)$ is *ccc* decomposable, then it has the *ccc* subset property.

Theorem 2. *Let $(X, \mathcal{M}, \mu)$ be a complete measure space that has the ccc subset property. If there exists a category base $\mathbf{C} \subseteq \mathcal{M}$ such that $B(\mathbf{C}) = \mathcal{M}$ and $\mathcal{M}(\mathbf{C}) = \mathcal{N}$, then $(X, \mathcal{M}, \mu)$ is ccc decomposable.*

Proof. Let $\kappa = |\mathbf{C}|$ and let $\mathbf{C} = \{A_\alpha : \alpha < \kappa\}$. For any $\alpha < \kappa$ we are going to define, by transfinite induction, $B_\alpha \in \mathbf{C}$ such that the following hold:

(1) each B_α is *ccc*,

(2) $\{B_\alpha : \alpha \leq \beta\}$ is a disjoint family for each $\beta < \kappa$,

(3) if $\beta < \kappa$, then there exists $\alpha \leq \beta$ such that $A_\beta \cap B_\alpha$ contains a region.

Our construction uses Morgan's method from [M1; Lemma 1.11.4]. To define B_0, let us consider A_0. If $\mu(A_0) = 0$ we set $B_0 = A_0$. If $\mu(A_0) > 0$ we take an arbitrary *ccc* $E \in \mathcal{M}$ such that $E \subseteq A_0$ and $\mu(E) > 0$. Hence E is a non-meager set contained in a region. By virtue of the Lemma, there exists a region B such that $B \subseteq A_0$ and $B - E$ is meager. Hence B is *ccc* and we set $B_0 = B$.

Suppose we have defined B_α for each $\alpha < \beta$, where $\beta < \kappa$. Let us take A_β and consider the intersections $A_\beta \cap B_\alpha$ for $\alpha < \beta$. If $A_\beta \cap B_\alpha$ contains a region for some $\alpha < \beta$, then we set $B_\beta = B_0$. If the opposite holds, then there exists a region D such that $D \subseteq A_\beta - (\cup\{B_\alpha : \alpha < \beta\})$. If $\mu(D) = 0$ we set $B_\beta = D$. If $\mu(D) > 0$ we take an arbitrary *ccc* $E \in \mathcal{M}$ such that $E \subseteq D$ and $\mu(E) > 0$. Hence E is a non-meager set contained in a region. By virtue of the Lemma, there exists a region B such that $B \subseteq D$ and $B - E$ is meager. Hence B is *ccc* and we set $B_\beta = B$. We shall show that the family $\mathcal{R} = \{B_\alpha : \mu(B_\alpha) > 0\}$ gives the required decomposition of our measure space.

It follows easily from (3) that $X - (\cup\mathcal{R})$ is meager, i.e., that $X - (\cup\mathcal{R})$ is a set of measure zero (see also [M1; Lemma 1.11.3]).

Let Y be a measurable set of positive measure. Hence Y is a non-meager set. By virtue of Morgan's Fundamental Lemma (cf. [M1]), there exists a region D

such that for any subregion B of D, $B \cap Y$ is non-meager. By virtue of (3), there exists $\alpha < \kappa$ such that $D \cap B_\alpha$ contains a region. Hence $\mu(Y \cap B_\alpha) > 0$.

Let $Y \subseteq X$ be such that $Y \cap B$ is measurable for each $B \in \mathcal{R}$. Notice that the three properties of the family $\mathcal{R}$ proved so far show that $\mathcal{R}$ is also a $\mathbf{C}$-decomposition of X. By virtue of Proposition 1, Y is Baire. Hence Y is measurable.

Both Theorem 1 and Theorem 2 give us the following characterization.

Theorem 3. *Let $(X, \mathcal{M}, \mu)$ be a complete measure space with the ccc subset property. There exists a category bases $\mathbf{C}$ such that $\mathcal{B}(\mathbf{C}) = \mathcal{M}$ and $\mathcal{M}(\mathbf{C}) = \mathcal{N}$ if and only if $(X, \mathcal{M}, \mu)$ is ccc decomposable.*

Since the original Measurability Problem does not require that $\mathcal{M}(\mathbf{C})$ be equal to $\mathcal{N}$, one can ask whether this condition is essential for the problem itself. The following proposition indicates an answer.

Proposition 2. *Let $(X, \mathcal{M}, \mu)$ be a complete measure space with the ccc subset property such that $\mu(\{x\}) = 0$ for each $x \in X$. Suppose that there exists a point-meager base $\mathbf{C}$ such that $\mathcal{B}(\mathbf{C}) = \mathcal{M}$ and such that each region contains a subregion of regular cardinality. If $\mathcal{M}(\mathbf{C}) \neq \mathcal{N}$, then it is consistent with ZFC that a measurable cardinal exists.*

Proof. Since $\mathcal{M}(\mathbf{C}) \neq \mathcal{N}$, either $\mathcal{M}(\mathbf{C}) - \mathcal{N} \neq \emptyset$ or $\mathcal{N} - \mathcal{M}(\mathbf{C}) \neq \emptyset$.

If $\mathcal{M}(\mathbf{C}) - \mathcal{N} \neq \emptyset$, then there exists a set E which is meager and, simultaneously, *ccc* of positive measure. Since each subset of a meager set is meager, each subset of E is measurable. Let $\mathcal{N}(E)$ be the set of all subsets of E of measure zero. Then $\mathcal{N}(E)$ is a σ-complete ideal on an uncountable cardinal $\lambda = |E|$, and since E is *ccc*, it is ω_1-saturated. Let κ be the additivity of the measure μ on E, i.e., $\kappa = \inf\{|\mathcal{S}| : \mathcal{S} \subseteq \mathcal{N}(E) \text{ and } \mu(\bigcup\mathcal{S}) > 0\}$. Then κ is a regular uncountable cardinal. There exists a disjoint family $\mathcal{S}$ of cardinality κ of subsets of E of measure zero such that $\mu(\bigcup\mathcal{S}) > 0$. Let $\mathcal{S} = \{E_\alpha : \alpha < \kappa\}$ and set $/ = \{Y \subseteq \kappa : \mu(\bigcup\{E_\alpha : \alpha \in Y\}) = 0\}$. Then $/$ is a κ-complete and ω_1-saturated ideal on κ, so by a result of Kunen [K], κ is measurable in some tansitive model of ZFC.

If $\mathcal{N} - \mathcal{M}(\mathbf{C}) \neq \emptyset$, then there is a non-meager set F of measure zero. Let A be a region such that F is abundant everywhere in A. There exists a subregion B of A such that $\kappa = |B|$ is a regular cardinal. Then F is abundant everywhere in B, too. Since $\mathbf{C}$ is point-meager, κ is regular and uncountable. Let us consider the family $\mathbf{C}|B = \{A \in \mathbf{C} : A \subseteq B\}$. It is a Baire base. We shall show that each subset of B is Baire. Let $Y \subseteq B$. We can break it up into two parts: $Y \cap F$ and $Y - F$. The first of them is Baire since it is a subset of a measure zero set F with respect to a complete measure. The second one is a subset of the set $B - F$ which is meager. Since $\mathcal{M}(\mathbf{C}|B) = \{E \cap B : E \in \mathcal{M}(\mathbf{C})\}$ and $\mathcal{B}(\mathbf{C}|B) = \{E \cap B : E \in \mathcal{B}(\mathbf{C})\}$, each subset of B is Baire with respect to the category base $\mathbf{C}|B$ on B. As was shown in [S], the existence of such a category base implies that ZFC is consistent with the existence of a measurable cardinal.

Let us give examples and compare *ccc* decomposable measures with some more familiar measures.

Following D. Fremlin [F], we say that a measure space $(X, \mathcal{M}, \mu)$ is *decomposable* if there is a partition $\mathcal{B} = \{b_i : i \in I\}$ of X such that:

(1) $\mathcal{M} = \{b : b \subseteq X \text{ and } b \cap b_i \in \mathcal{M} \text{ for each } i\}$,

(2) $\mu(b_i) < \infty$ for each i,

(3) $\mu(b) = \sum_{i \in I} \mu(b \cap b_i)$ for each $b \in \mathcal{M}$.

We recall that a measure μ_1 is *absolutely continuous* with respect to a measure $\mu_2, \mu_1 << \mu_2$, if $\mu_2(E) = 0$ implies $\mu_1(E) = 0$.

Proposition 3. *Let μ_1 and μ_2 be two complete measures on the same measure space $(X, \mathcal{M})$ such that $\mu_1 << \mu_2$ and $\mu_2(X) < \infty$. Then X is μ_1 ccc.*

Proof. Suppose to the contrary that there exists an uncountable family $\mathcal{S}$ of pairwise disjoint subsets of X positive μ_1-measure. According to a theorem by E. Hewitt and K. Stromberg [HS; (19;26) Lemma], since $\mu_2(X) < \infty$, X can be decomposed into two measurable sets G and H such that μ_1 is σ-finite on G, $\mu_1(K) = 0$ or ∞ for any measurable subset K of H, and $\mu_1(E) = 0$ implies $\mu_2(E) = 0$ for any $E \subseteq H$. Hence $\mathcal{S}$ contains an uncountable family $\mathcal{S}'$ such that $\mu_1(B \cap G) = 0$ for each $B \in \mathcal{S}'$. Hence $\mu_1(B \cap H) = \infty$ for each $B \in \mathcal{S}'$. In consequence, $\mu_2(B) > 0$ for each $B \in \mathcal{S}'$, which is a contradiction.

Proposition 4. *Let μ_1 and μ_2 be two complete measures on the same measure space $(X, \mathcal{M})$ such that $\mu_2 << \mu_1$ and $\mu_1 << \mu_2$. If μ_2 is decomposable, then μ_1 is ccc decomposable.*

Proof. Let $\mathcal{B}$ be a decomposition of the measure space $(X, \mathcal{M}, \mu_2)$ satisfying conditions (1), (2) and (3) and let $\mathcal{R} = \{b \in B : \mu_2(b) > 0\}$. Then $\mu_2(X - \cup\mathcal{R}) = \mu_2(\cup(\mathcal{B} - \mathcal{R}) = 0$, so $\mu_1(X - \cup\mathcal{R}) = 0$. It follows from Proposition 3 that $\mathcal{R}$ is a required family for $(X, \mathcal{M}, \mu_1)$ to be *ccc* decomposable.

Proposition 5. *Let $(X, \mathcal{M}, \mu)$ be a measure space with the finite subset property. If $(X, \mathcal{M}, \mu)$ is ccc decomposable, then it is decomposable.*

Proof. Let $\mathcal{R} \subseteq \mathcal{M}$ satisfy all conditions for $(X, \mathcal{M}, \mu)$ to be *ccc* decomposable. For any $A \in \mathcal{R}$ let $\mathcal{S}_A$ be a maximal pairwise disjoint family of measurable subsets of A of positive and finite measure. Since A is *ccc*, each $\mathcal{S}_A$ is countable and $A - \bigcup \mathcal{S}_A$ is of μ-measure zero. Hence the family $\mathcal{R}' = \bigcup\{\mathcal{S}_A : A \in \mathcal{R}\}$ is another family satisfying (a) through (d). Let us set $\mathcal{S} = R' \cup \{X - \bigcup \mathcal{R}'\}$. It is obvious that the family $\mathcal{S}$ satifies the first two conditions for the measure space $(X, \mathcal{M}, \mu)$ to be decomposable. To show that it satisfies also the third one, let us take $b \in \mathcal{M}$ and consider the family $\mathcal{P} = \{B \in \mathcal{S} : \mu(b \cap B) > 0\}$. If this family is countable, then the condition (3) holds. If this family is uncountable, then $\mu(b) = \infty = \sum_{B \in \mathcal{P}} \mu(b \cap B)$, so (3) holds again. Thus $(X, \mathcal{M}, \mu)$ is decomposable.

Both Propositions 4 and 5 show that among measures with the finite subset property, decomposable measures coincide with *ccc* decomposable ones. Known examples of decomposable measures are Haar measures on locally compact groups and Radon measures (see [F]).

At the end we would like to give another characterization. It is valid for measure spaces with the finite subset property and is an "external" characterization.

Theorem 4. *Let $(X, \mathcal{M}, \mu)$ be a complete measure space with the finite subset property. There exists a category base $\mathbb{C}$ on X such that $\mathcal{B}(\mathbb{C}) = \mathcal{M}$ and $\mathcal{M}(\mathbb{C}) = N$ if and only if there exists a topology $\mathcal{T}$ on X such that $\mathcal{B}(\mathcal{T}) = \mathcal{M}$ and $\mathcal{M}(\mathcal{T}) = \mathcal{N}$, where $\mathcal{B}(\mathcal{T}) = \{B \subseteq X : B$ has the Baire property in the topology $\mathcal{T}\}$ and $\mathcal{M}(\mathcal{T}) = \{B \subseteq X : B$ is of first category in the topology $\mathcal{T}\}$.*

Proof. The implication "$\Leftarrow$" is obvious. To prove the implication "$\Rightarrow$" notice that from our Theorem 2 it follows that μ is decomposable. The von Neumann-Maharam lifting theorem guarantees that $(X, \mathcal{M}, \mu)$ has a lifting, say λ. If $\mathcal{T}$ is the density topology induced by λ, i.e., $\mathcal{T} = \{V : V$ is the union of sets of the form $\lambda(A) - E$, where $A \in \mathcal{M}$ and $\mu(E) = 0\}$, then $\mathcal{T}$ is a required topology on X (see [R] or [O]).

Acknowledgment. The author thanks the referees for many valuable suggestions and improvements.

References

[DS] M. Detlefsen and A. Szymanski, *Category bases*, preprint.

[F] D. Fremlin, *Measure Algebras*, in Handbook of Boolean Algebras, Vol. III, edited by D. Monk with R. Bonnet, North-Holland, 1989, pp. 877-980.

[HS] E. Hewitt and K. Stromberg, *Real and Abstract Analysis*, Springer-Verlag 1969.

[J] T. Jech, *Set theory*, Academic Press 1978.

[K] K. Kunen, *Some applications of iterated ultrapowers in set theory*, Annals of Mathematical Logic 1 (1970), 179-227.

[M1] J.C. Morgan II, *Point Set Theory*, Marcel Dekker, Inc., 1989.

[M2] ____________, *On the general theory of point sets*, II, Real Analysis Exchange 12 (1986), 377-386.

[M3] ____________, *Measurability and the abstract Baire property*, Rendiconti del Circolo Matematico di Palermo 34 (1985), 234-244.

[O] J. Oxtoby, *Measure and Category*, 2nd edition, Springer-Verlag, New York 1980.

[R] M.M. Rao, *Measure Theory and Integration*, John Wiley & Sons, New York 1987.

[Sc] K. Schilling, a letter to the author, 1986.

[S] A. Szymanski, *Measurable cardinals and category bases*, preprint.

Received February 23, 1990