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Change of Variable in Fourier Analysis

The topic of this lecture is the analysis of creating, keeping, and d estroying "good" properties of Fourier series by (smooth) homeomorphisms of the circle, $\mathbf{T} = \mathbf{R}/2\pi\mathbf{Z}$ on itself. We present a survey of new results and open problems in the following main directions.

I. *Reducing of a given function or its families into some spaces close to the Wiener algebra* $A(\mathbf{T})$.

This topic goes up to the Pal–Bohr theorem on the space $U(\mathbf{T})$ (uniformly convergent Fourier series) and the Luzin problem on $A(\mathbf{T})$ (see [1]).

II. *Beuring–Helson type theorems.*

Here we discuss what homeomorphisms ϕ *operate* in a given space $\mathcal{F}$. (This means that $f \in \mathcal{F}$ implies $f \circ \phi \in \mathcal{F}$.)

Theorem 1 *If* $\phi \in C^2$ *operates in* $U(\mathbf{T})$*, then it reduces to a shift and symmetry.*

(For analytic ϕ this was proved by Alpar in 1974.) It is unknown if it is possible to remove the smoothness condition at all. But for some other important spaces it is impossible.

Theorem 2 *(obtained jointly with Lebedev)*

1. *There exist nontrivial Lipschitz homeomorphisms which operate in the space* $A_p(\mathbf{T})$ $(= \hat{l}_p(\mathbf{Z}))$ $(p > 1)$.
2. *If* $\phi \in C^1$ *operates in* A_p $(p \neq 2)$*, then it is affine.*

References

[1] A. Olevskii, Homeomorphisms of the circle, modifications of functions and Fourier series, *Proc. I.C.M. Berkeley 1986*, vol. II, 976–989.