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Some Remarks on the Density (Property) of O'Malley

This talk is based on work by the speaker and C. Freiling which appears in this issue under the title *The exact Borel class where a density completeness axiom holds*. A collection \mathbf{A} , of subsets of \mathbb{R} has the *O'Malley density property* if whenever a non-empty bounded set $A \in \mathbf{A}$ has right density 1 at each of its points, then there is point in A^c at which A has left density 1. We prove the following two theorems

Theorem 1 *The $G_{\delta\sigma}$ subsets of \mathbb{R} have the O'Malley density property.*

Theorem 2 *There is an open proper subset $A \subset (0, 1)$ such that for every $x \in [0, 1]$ if A has left density 1 at x , then A has right density 1 at x .*

It follows easily from this second result that the $F_{\sigma\delta}$ sets do not have the O'Malley density property.