Jack B. Brown, Division of Mathematics, Auburn University, AL 36849-5310

MEASURABLE DARBOUX FUNCTIONS

Consider the following "topologically defined" classes of functions f : [0,1] -> R,

EXT: f is extendable to a connectivity function g : [0,1]x[0,1] -> R,

AC: f is almost continuous, i.e. every open set containing the graph of f contains the graph of a continuous function with domain [0,1],

Conn: f is a connectivity function, i.e. $f|C$ is connected for every connected subset C of [0,1],

D: f is Darboux, i.e. f(C) is connected for every connected subset C of [0,1],

PC: f is peripherally continuous if for each x and each pair of open sets U and V containing x and f(x), respectively, there is an open subset W of U containing x such that f(bd(W)) is a subset of V,

PR: f has a perfect road at each point if for each x, there is a perfect set having x as a bilateral limit point such that $f|P$ is continuous at x,

Z_c: all sets of the form $\{f<t\}$ and $\{f>t\}$ are either empty or bilaterally c-dense in themselves,

Z_ω: all sets of the form $\{f<t\}$ an $\{f>t\}$ are either empty or bilaterally dense in themselves.

We discuss the relationships that are known to exist between these "topologically defined" classes of functions within certain "measurability classes":

B_1: Baire class 1,

R_1: pointwise limits of right-continuous functions,

J_1: pointwise limits of functions with only "jump-discontinuities",

G_δ: functions with G_δ graphs,

B: Borel functions,

U: universally measurable functions,

L: Lebesgue measurable functions,

B_r: functions with the Baire property in the restricted sense,

B_w: functions with the Baire property in the wide sense,

(s): Marczewski measurable functions.

Most of these relationships were established in paper [1], which also gives a discussion of previous work on these problems and a fairly complete bibliography. It should be noted that the PR -> PC and Z_c -> Z_ω implications were inadvertently omitted in the statement of Theorem 1 of [1].

The relationships within the class G_δ were not considered in [1], and these are as follows.

THEOREM: Within G_δ, the following implications hold

EXT -> AC -> Conn -> D -> PR = PC -> Z_c -> Z_ω , but

AC <-/- Conn <-/- D <-/- PR and PC <-/- Z_c <-/- Z_ω .

Several of the examples come from [2] and [3], and the other new implications and examples are not too difficult to work out.

As far as the relationships within the classes U, L, B_r, B_w, and (s) are concerned, it follows from the theorems in [1] that the only remaining question is whether AC, Conn, or D imply PR within U, B_r, or (s). A ZFC example which satisfies U, B_r, and AC but not PR would settle the question completely. We are unable to provide such an example. The example given in [1] which shows that L plus AC does not imply PR also satisfies B_w, but it is definitely not (s). We can provide a ZFC example which is L, B_w, (s), and D, but not PR, and we can provide a CH example that is U, B_r, and D, but not PR. Considering our previous luck in dealing with the classes U, B_r, and (s), it is probably going to turn out that there is a CH example which is U, B_r, and AC, but not PR, and also turn out to be consistent that (U or B_r) and D imply PR.

REFERENCES

[1] J. B. Brown, P. Humke, and M. Laczkovich, "Measurable Darboux functions", Proc. A. M. S. 102 (1989), 603-610.

[2] F. B. Jones and E. S. Thomas, Jr., "Connected G_δ graphs", Duke Math. J. 33 (1966), 341-345.

[3] M. H. Miller, Jr., "Discontinuous G_δ graphs", Studies in Topology (Proc. Topology Conf., U. N. C. Charlotte 1974), 383-392, Academic Press.