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Approximating Hausdorff Measures

This talk involved a few topics and questions in which Hausdorff measures and dimensions play a role. First, it appears to be still an open question as to whether these are $F\sigma$ subfields (or subrings) of the real numbers of dimensions larger than 0. Subgroups of any dimension s with $0 \leq s \leq 1$ and s-measures 0, in the case $0 < s \leq 1$, or s-measures ∞, in the case $0 \leq s < 1$, were constructed, for example in [1] using restrictions on the decimal expansions of numbers. In [2] it was shown that closed sets F of dimension s exist which satisfy for each $x, y \in F$, $\frac{1}{2}(x + y) \notin F$. Other algebrais combinations involving Hausdorff measure remain to be considered.

Secondly, the question of non-measurability of sets in Hausdorff m_δ^s measures was discussed. It seems clear from examples that every closed set of n-dimensional measure 0 in $\mathbf{R}^n$ is not m_δ^s measurable for $s < n$ and $\delta > 0$ unless its m_δ^s measure is 0. Proofs of this might involve further insight into the structure of sets of fractional measure.

Thirdly, some observations concerning the Cantor singular function (namely, that it satisfies a Lipschitz condition of order log2/log 3, the dimension of the Cantor set) lead to the consideration of its 'inverse' jump function $J(x)$ which satisfies a reverse Lipschitz condition $|J(x) - J(y)| \geq |x - y|^{1/\alpha}$. Such a reverse condition on a jump function implies that its image is of dimension $\leq \alpha$. The question of a converse to this observation is a natural one. (Namely, given a compact subset of the line of finite α-measure, is there a jump function which has this set as its image such that the jump function satisfies a reverse condition of order $1/\alpha$?)

Finally, several generalizations of generalized bounded variation to σ-finite length graphs and other dimension like restrictions (see [3] for references) were presented.

References

[1] J. Foran, Some relationships of subgroups at the real numbers to Hausdorff measures, J. Lond. Math. Soc., 7 (1974) p. 651-661.

[2] J. Foran, Nonaveraging sets, dimension and porosity, Can. Bulletin Math. 29, 1 (1986) p.60-63.

[3] C.M. Lee, Some Hausdorff variants of absolute continuity, Banorh's condition (S) and Lusin's condition (N), Real Analysis Exchange 13 #2 (1987-88) p.404-419.