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TWO INEQUALITIES INVOLVING GEOMETRIC MEANS

Cochran and Lee proved (1984) the inequality: if $p \geq 1$, $\kappa \geq 1$, and $0 \leq x_n \leq 1$, then

$$(1) \qquad \sum_{m=1}^{\infty} m^{\kappa-1}\left[\prod_{n=1}^{m} x_n^{pn^{p-1}}\right]^{1/m^p} \leq e^{\kappa/p} \sum_{m=1}^{\infty} m^{\kappa-1} x_m ,$$

generalizing Carleman's Inequality (1923, Hardy, Littlewood, and Pólya Theorem 334), which is the case $p = 1 = \kappa$. I have found that the expression

$$\sum_{m=1}^{\infty} m^{\kappa-1}\left[\prod_{n=1}^{m} x_n^{n^p-(n-1)^p}\right]^{1/m^p}$$

lies between the two sides of (1), and that the inequality

$$(2) \qquad \sum_{m=1}^{\infty} m^{\kappa-1}\left[\prod_{n=1}^{m} x_n^{n^p-(n-1)^p}\right]^{1/m^p} \leq e^{\kappa/p} \sum_{m=1}^{\infty} m^{\kappa-1} x_m$$

holds for $p > 0$, $\kappa \geq 1$, and $x_n \geq 0$, a wider range that that for which (1) holds. The constant $e^{\kappa/p}$ is best possible in (1) and (2).

Actually, (2) is a case of a more general inequality.

There are analogous integral inequalities; they are less trouble, and are given in Cochran and Lee (1984) and Love (1986).