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POINTS OF APPROXIMATE CONTINUITY, APPROXIMATE SYMMETRY, AND L–POINTS

All functions considered here will be measurable real valued functions defined on the interval [0,1]. For such a function f let

$$AC(f) = \{x : f \text{ is approximately continuous at } x\},$$

$$AS(f) = \{x : f \text{ is approximately symmetric at } x\}$$

$$= \{x : \text{ap-}\lim_{h \to 0} f(x+h) + f(x-h) - 2f(x) = 0\},$$

and

$$L(f) = \{x : \lim_{h \to 0} \frac{1}{h}\int_0^h f(x+t)dt = f(x)\}.$$

Furthermore, let

$$\mathscr{AC} = \{f : AC(f) = [0,1]\},$$

$$\mathscr{AS} = \{f : AS(f) = [0,1]\},$$

and

$$\mathscr{L} = \{f : L(f) = [0,1]\}.$$

Then $b\mathscr{AC}$ will denote those bounded functions in $\mathscr{AC}$, and similarly for $b\mathscr{AS}$ and $b\mathscr{L}$ By equipping these sets with the sup norm, each becomes a Banach space.

The results announced here are a continuation of the work begun in [2].

Theorem 1. For a measurable $f : [0,1] \to \mathbb{R}$, the set $AS(f) \setminus AC(f)$ is of first category.

It should be noted that in the event that $f \in \mathcal{AS}$ we have the stronger result that $AS(f) \setminus C(f) = [0,1] \setminus C(f)$ is a first category set, where $C(f)$ denote the set of points of ordinary continuity for f. This follows from the fact that functions in $\mathcal{AS}$ must belong to the first Baire class [3]. In general, however, $AS(f) \setminus C(f)$ need not be of first category, as is exhibited by the characteristic function of the rationals.

In [2] the set $AS(f) \setminus L(f)$ was shown to be first category, but not necessarily σ–porous. However, the proof given for Theorem 2 in [2] shows that the following is true.

Theorem 2. If $f : [0,1] \to \mathbb{R}$ is bounded, then $AS(f) \setminus L(f)$ is σ–porous.

On the other hand we have

Theorem 3. The typical function f in $b\mathcal{AS}$ has the property that the set $[0,1] \setminus L(f)$ is uncountably dense in every interval.

In [1] it was shown that the typical function f in $b\mathcal{S}$ has the property that the set $[0,1] \setminus AC(f)$ is everywhere dense; that is, the typical bounded derivative has a dense set of approximate discontinuities. We observe the following strengthening of this fact:

Theorem 4. The typical function f in $b\mathscr{L}$ has the property that the set $[0,1] \setminus AC(f)$ is non–σ–porous in every interval.

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