TOPICAL SURVEY *Real Analysis Exchange Vol. 5 (1979-80)*

# Non-Absolute Integrals: A Survey

P. S. Bullen

## 1. The Classical Integrals.

The classical primitive problem was solved in 1912 by A. Denjoy, [1], in a short note that began:

> "L'intégrale de Riemann a un sens quand la fonction intégrée est continue ou quand ses points de dis- continuité forment un ensemble de mesure nulle. L'intégrale de Lebesgue s'applique à toute fonction, d'une part mesurable, et d'autre part bornée, ou plus généralement sommable. L'une et l'autre integrales, prises entre a et x , sont fonctions continues de x , a restant fixe, et leur derivée est f , sauf en un ensemble de points de mesure nulle. Mais il est possible de former des fonctions derivées que ne sont ni intégrable selon Riemann ni sommables selon Lebesgue. J'indiquerai dans cette Note un mode de calcul qui s'applique en particulier avec succés à toute fonction derivée f et qui nous donne pour resultat une fonction ayant pour derivée f ."

Within three months N.N. Luzin, [1], gave another solution, and in 1915, H. Bauer, [1], adapted a definition of the Lebesgue integral of bounded measurable functions due to O. Perron, [1], to give a third solution. In 1957, J. Kurzweil, [1], gave a fourth solution based on a simple modification of the definition of the Riemann integral.

References: A.Bruckner, [1], Denjoy, [16], H. Lebesgue, [2], I.N. Pesin, [1].

1.1. Denjoy's Solution of the Primitive Problem.

The total, $T[f;a,b]$, of a function, $f: [a,b] \to \overline{\mathbb{R}}$, is constructed by transfinite induction, using the operations given below. If the construction is possible then $f$ is said to be totalisable on $[a,b]$.

(1) $\forall P \subset [a,b]$, perfect, the set of points of $P$, in the neighbourhood of which $f$ is not summable on $P$, is nowhere-dense in $P$.

(2) If $f \in \mathcal{L}[\alpha,\beta]$, $a \leq \alpha < \beta \leq b$ then $T[f;\alpha,\beta] = \mathcal{L}\text{-}\int_{\alpha}^{\beta} f$.

(3) If $\forall \alpha', \beta', \alpha < \alpha' < \beta' < \beta$, $T[f;\alpha',\beta']$ is defined then $\lim\limits_{\substack{\alpha'\to\alpha \\ \beta'\to\beta}} T[f;\alpha',\beta']$ exists and $T[f;\alpha,\beta]$ is defined to have this value.

Suppose $P \subset [a,b]$, perfect, has extremities $\alpha$ and $\beta$, $\alpha < \beta$, and contiguous intervals $[\alpha_n,\beta_n]$, $n \in N$. Suppose further that $\forall n \in N$, $\alpha_n', \beta_n', \alpha_n < \alpha_n' < \beta_n' < \beta_n$, $T[f;\alpha_n',\beta_n']$ is defined and let

$$\omega_n = \omega(T;\alpha_n,\beta_n) = \sup|T[f;\alpha_n',\beta_n']| .$$

(4) $\forall P \subset [a,b]$, perfect, the set of points of $P$, in the neighbourhood of which $\sum \omega_n = \infty$, is nowhere-dense in $P$.

(5) $\forall P \subset [a,b]$, perfect, if $f \in \mathcal{L}(P)$ and $\sum\limits_{n\in N} \omega_n < \infty$ then we define

$$T[f;\alpha,\beta] = \mathcal{L}\int_{P}^{-} f + \sum_{n\in N} T[f;\alpha_n,\beta_n] .$$

If $f$ is a finite derivative then it is totalisable and $(T[f;a,x])' = f(x)$. Examples of such an $f$ can be given that require the transfinite induction to proceed to any given number of steps, less than the first uncountable ordinal; Denjoy [4; p.206-236; 16].

Later this total become known as the restricted Denjoy integral;

we will write $f \in \mathcal{J}^*$ and $\mathcal{J}^* - \int_b^b f$ for $T[f;a,b]$ . (Generally integrals defined this way will be called totals and will be denoted by some notation involving $\mathcal{J}$ .)

(a) $\mathcal{L} \subsetneqq \mathcal{J}^*$ (b) $(\mathcal{J}^* - \int_a^x f)' = f$ a.e.

A. Hinčin, [1], noticed that the totalisation procedure is defined if (in (4) and (5) above) $\omega_n$ is replaced by $V_n$ , where

$$V_n = V_n(T;\ \alpha_n,\beta_n) = |T[f;\ \alpha_n,\beta_n]| \ .$$

This was also noted later by Denjoy [3]. However now the indefinite total is no longer differentiable almost everywhere. The correct generalisation of differentiation needed to restore this property was given by Hinčin and, following Denjoy, is called the approximate derivative. If $f$ is a finite approximate derivative of a continuous function it is totalisable in this weaker sense and $(T[f;\ a,x])'_{ap} = f(x)$ .

Later this total become known as the general Denjoy integral; we will write $f \in \mathcal{J}$ and $\mathcal{J} - \int_a^b f$ for $T[f;\ a,b]$ .

(a) $\mathcal{J}^* \subsetneqq \mathcal{J}$ (b) $(\mathcal{J} - \int_a^x f)'_{ap} = f(x)$ , a.e.

While this weakening of (4) leads to the solution of a different primitive problem, Hinčin found the right form of condition (4) that leads to a total that is differentiable almost everywhere. A necessary condition is that in (4) "$\Sigma\, \omega_n = \infty$" be replaced by "$\Sigma\, |V_n| = \infty$ or $\lim\limits_{\rho_n \to 0} \frac{\omega_n}{\rho_n} \neq 0$" , where $\rho_n$ = distance of the point $x$ of $P$ from $[\alpha_n,\beta_n]$ . Using this condition a third total is defined, now called the Denjoy-Hinčin integral; we will write $f \in \mathcal{J}^k$ for $f$ integrable in this sense.

(a) $\mathcal{J}^* \subsetneqq \mathcal{J}^k \subsetneqq \mathcal{J}$ . (b) $(\mathcal{J}^k - \int_a^x f)' = f(x)$ a.e.

(In all three cases the indefinite total is continuous.)

Full discussions can be found in the original papers of Denjoy, [1-4], and Hinčin, [1-3]. See also; E.W. Hobson, [1], T.H. Hildebrandt, [1], R.L. Jeffrey, [7], Lebesgue, [2], Pesin, [1].

Significant simplications of some of the fundamental lemmas were given by J.C. Burkill, [1] and J. Ridder, [1,2]. S. Saks, [2,3], laid bare the basic operations used in the transfinite induction; see also P. Natanson, [1]. P. Romanovskiĭ, [1], gave a lemma that enabled one to avoid the use of transfinite induction, but losing the constructive appeal of totalisation; see H. Kestelman, [1] and Y. Kubota, [14]. A direct construction of the $\mathcal{J}^*$-integral, avoiding the use of Lebesgue integration, was given in a very interesting paper by Lebesgue, [3]. See also D.E. Men'šov, [1], B.Ya Kozlov, [1], and I.P. Natanson and G.I. Natanson, [1].

## 1.2. Luzin's Solution of the Primitive Problem.

The following definitions are classical: if $f: [a,b] \to \mathbb{R}$ let

$$V(f; a,b) = |f(b) - f(a)| ,$$

$$\omega(f; a,b) = \sup_{a \le \alpha < \beta \le b} V(f; \alpha,\beta) .$$

Now $f$ is said to be of bounded variation on $[a,b]$ , $f \in BV$ , iff $\exists M$ s.t. $\forall$ subdivisions, $a \le a_0 < \dots \le a_n < b$ , $\sum_{k=0}^{n-1} V(f; a_k,a_{k+1}) < M$ .

If, in the sum, $V$ is replaced by $\omega$ then $f$ is said to be of bounded oscillation on $[a,b]$, $f \in BV^*$. It is known that $BV = BV^*$; Hobson, [1].

Given any set $E \subset [a,b]$, if the points $a_k$, $0 \le k \le n$, are required to be in $E$ then the existence of $M$ leads to the classes of functions of bounded variation in the wide sense on $E$, $f \in BV(E)$, and in the restricted sense, $f \in BV^*(E)$. The concepts are now of course distinct. If then $[a,b] = \bigcup_{n \in N} E_n$ and $f \in BV(E_n)$, $f \in BV^*(E_n)$, $n \in N$, then $f$ is said to be of generalised bounded variation in the wide sense, $f \in BVG$, in the narrow sense, $f \in BVG^*$. Similarly the classes of functions of generalised absolute continuity in the wide sense, $f \in ACG$, in the narrow sense $f \in ACG^*$, can be defined.

The classes $BVG$, $BVG^*$ retain some of the properties of the more classical $BV$. If $f$ is measurable then $f \in BVG^*$ implies $f'$ exists a.e., while $f \in BVG$ implies $f'_{ap}$ exists a.e.

These classes characterise three integrals as follows:

(1) $f \in \mathcal{D}^*$ iff $\exists F \in \mathcal{C}$ s.t. $F \in ACG^*$ and $F' = f$ a.e.;

(2) $f \in \mathcal{D}$ iff $\exists F \in \mathcal{C}$ s.t. $F \in ACG$ and $F'_{ap} = f$ a.e.;

(3) $f \in \mathcal{D}^k$ iff $\exists F \in \mathcal{C}$ s.t. $F \in ACG$ and $F' = f$ a.e.;

and in all cases $\int_a^x f = F(x) - F(a)$.

These descriptive definitions of integrals are obviously generalisations of the absolute continuity characterisation of the indefinite Lebesgue integral; and

(a) $\mathcal{J}^* = \mathcal{D}^*$ ; $\mathcal{J}^k = \mathcal{D}^k$ ; $\mathcal{J} = \mathcal{D}$.

Details can be found in Hinčin, [1-3], Jeffrey, [7], Luzin, [1,2], Pesin [1]. Denjoy gave his own characterisations of his totals; the totals; the exact connections with the above are not straightforward; see Saks, [4] and Ridder, [2].

The properties of the several classes of function introduced here were studied extensively, particularly by Saks; see, in addition to the above references, Saks, [3], and Ridder, [3]. An interesting extension to generalised upper and lower semi-absolute continuity in the wide and narrow sense was introduced by Ridder, [4]. Other references are: Bruckner, [1], M. Bruneau, [1], O. Haupt, [1].

## 1.3. The Perron-Bauer Solution of the Primitive Problem.

In 1912, Ch.-J. de la Vallée Poussin, [1], had shown that the indefinite Lebesgue integral could be approximated by continuous functions whose derivates majorised (minorised) the integrand; he called these functions majorants (minorants) of the integrand.

Perron's idea, [1], was to use this property to define an integral. This approach is very convenient for integrals regarded as primitives as it splits a defining property into two parts; thus defining two classes of functions and the integral is the unique function belonging to both classes.

Let $f: [a,b] \to \overline{\mathbb{R}}$, then $M$ is a major function of $f$, $M \in \tilde{\mathcal{M}}_f$ iff

(a) $M \in \mathcal{C}[a,b]$ ;

(b) $M(a) = 0$ ;

(c) $\underline{D}M \geq f$ , a.e.;

(d) $\underline{D}M > -\infty$ , n.e.; (nearly everywhere, except on a countable set);

$m$ is a minor function of $f$, $m \in \underset{\sim}{\mathcal{M}}_f$ iff $-m \in \tilde{\mathcal{M}}_{-f}$ .

Then f is Perron integrable, $f \in \mathcal{P}^*$ iff

(e) $\tilde{\mathfrak{M}}_f \neq \phi$ ; $\mathfrak{M}_{\tilde{f}} \neq \phi$ ;

(f) $\forall \varepsilon > 0 \ \exists M \in \tilde{\mathfrak{M}}_f$ , $m \in \mathfrak{M}_{\tilde{f}}$ , such that $M - m < \varepsilon$;

then $\displaystyle\int_a^b f = \inf_{M \in \tilde{\mathfrak{M}}_f} M(b) = \sup_{m \in \mathfrak{M}_{\tilde{f}}} m(b)$

Unlike totalisation this is an easy definition to work with and most properties of the integral follow immediately; in particular, if f is a finite derivative then $f \in \mathcal{P}^*$ and $(\mathcal{P}^*\text{-}\int_a^x f)' = f(x)$ ; for details see Kamke, [1].

However the definition poses some problems. How can we be sure about (e) ? In a scathing attack Denjoy, [15, p.677], showed that, given an f , the problem of finding an element of $\tilde{\mathfrak{M}}_f$ , was at least as difficult as totalising f , with the disadvantage that no one wanted an element of $\tilde{\mathfrak{M}}_f$ , whereas the total of f was just what the calculations were about. In fact, J.Marcinkiewicz, [Saks, 4, p.253] and G.P. Tolstov, [1], showed that for measurable f , $f \in \mathcal{P}^*$ iff (e) holds. The various technicalities in the definition, (a.e. in (c), n.e. in (d)), while convenient do not increase the generality of the integral defined; Bauer, [1], H. Looman, [2]. In fact, Tolstov, [2], pointed out that the lower derivates used in (c) and (d) can be replaced by exact derivatives. In the other direction Saks, [4], pointed out that (a) could be omitted, and D.N. Sarkhel, [3], weakened all of (a) - (d) and proved that the above quoted result of Marcinkiewicz-Tolstov still holds. Further we do not need to restrict the definition to measurable f but if f is $\mathcal{P}^*$-integrable then it will be measurable.

The relation of the Perron integral to those of Denjoy and Luzin took some time to elucidate. Six years after Bauer's paper, H. Hake, [1], proved that $\mathcal{J}^* \subset \mathcal{P}^*$, and about three years later, A. Aleksandrov, [1], and Looman, [2], independently, proved that $\mathcal{P}^* \subset \mathcal{J}^*$.

Rather surprisingly a proof of the integration by parts formula for the $\mathcal{P}^*$ - integral, (except by using its equivalence to the $\mathcal{D}^*$ - and $\mathcal{J}^*$ - integrals), was not found until 1967, L. Gordon and S. Lasher, [1]. It was this difficulty that led E.J.McShane, [1,2], to give a more elaborate, but equivalent, definition using the four Dini derivates in (c) and (d); he was then able to give a proof of the integration by parts formula; see also Jeffrey, [5], Ridder, [1]. The use of Dini derivates to define major and minor functions was explored further by C.T. Ionescu-Tulcea, [1]; he defined, in this way two integrals, strictly more general than the $\mathcal{P}^*$ - integral; he gave equivalent Luzin definitions for his integrals; they are strictly less general than the $\mathcal{D}$ - integral.

The question of obtaining a Perron integral equivalent to either the $\mathcal{D}^k$- , or the $\mathcal{D}$ - integral proved difficult. These integrals are in a sense less natural than the $\mathcal{D}^*$ - integral in that they mix approximate notions with standard ones and the exact mix was hard to find in the Perron approach. In 1932, Ridder, [4], gave a series of Perron definitions, using his idea of upper and lower semi-absolute continuity, (see 1.2 above), to split the property of primitives, and so to define major and minor functions, in a slightly different way. He obtained Perron integrals equivalent to the $\mathcal{R}$-, $\mathcal{L}$-, $\mathcal{D}^*$- and $\mathcal{D}^k$ - integrals. In a later paper, [7], he gave a Perron definition of an integral equivalent to the $\mathcal{D}$ - integral; Tolstov, [2], pointed that the generalisation of derivative that Ridder used was, from a certain point of view, unsatisfactory, and he gave a definition of a Perron integral equivalent to the $\mathcal{D}$ - integral that avoided this difficulty; see also Ridder [8,9], S.Izumi, [1], P. Malliavin, [1] and N. Jacquier-Bryssine and A. Pacquement, [1].

The lack of symmetry in the $\mathcal{D}^k$ - and $\mathcal{D}$- integrals and the simplicity of the Perron approach suggested the definition of an integral like the $\mathcal{P}^*$ - integral but which used approximate concepts throughout. There are several such definitions and not all the relationships between them seem to be known.

The first and simplest is due to Burkill, [3]; in (a), continuity is replaced by approximate continuity and in (c) and (d), the lower approximate derivate is used. The resulting integral is called the Burkill approximate Perron integral, the $B\mathcal{P}_{ap}$ - integral. Rather surprisingly there are $f \in \mathcal{D} \setminus B\mathcal{P}_{ap}$, Tolstov, [1]. Ridder, [6], gave a simple modification of the totalisation process in which the limits in (3) of 1.1 are replaced by approximate limits, the $\mathcal{J}_{ap}$ - integral (Ridder called it the $\beta$- integral); he then showed that $B\mathcal{P}_{ap} \subset \mathcal{J}_{ap}$ and $\mathcal{J} \subset \mathcal{J}_{ap}$, and so by the above example of Tolstov, $\mathcal{J} \subsetneq \mathcal{J}_{ap}$ : see also Ridder, [9]. S. Verblunsky, [1], gave a different generalisation of the totalisation process to define an integral, the $V\mathcal{J}_{ap}$ - integral and gave an equivalent Luzin definition; $\mathcal{J} \subset V\mathcal{J}_{ap}$ . This integral was generalised by S. Izumi, [2]. Y. Kubota, [3,5, 9-12], has defined a $\mathcal{P}_{ap}$ - integral, $\mathcal{P}_{ap} = \mathcal{J}_{ap}$ , and one of his theorems, [10, theorem 4], suggests that this one is the natural one in this context; see also Pacquement, [1-3].

A different kind of generalisation of the Perron definition has been given by M. Cotlar and Y. Frenkel, [1]. See also: Burkill and F.W. Gehring, [1], K. Iseki, [1] and Iseki and M. Maeda, [1].

1.4. A Riemann-like Solution to the Primitive Problem.

In 1909 Lebesgue, [1], had shown that his integral could be obtained as a limit of Riemann sums, and later Denjoy defined several integrals this way, [5,12]; see also S. Kempisty, [1-4].

Kurzweil, [1], gave a completely new definition of an integral by a simple modification of the Riemann approach; he chose the intervals of the subdivision after the arbitrary point, rather than before as in the classical theory.

(a) Let $\mathcal{S}[a,b] = \{S;\ S \subset \mathbb{R}^2$ s.t. $\forall u \in [a,b]\ \exists \delta = \delta(u) > 0$ s.t. $(u,v) \in S$ if $u - \delta(u) \leq v \leq u + \delta(u)\}$

(b) Let $A = (x_0, y_1, x_1, y_2, \ldots, y_n, x_n)$ be a finite set of numbers s.t. $a = x_0 < x_1 \ldots < x_n = b$ and $x_{k-1} \leq y_k \leq x_k$, $1 \leq k \leq n$ ; then $A$ is called a sub-division of $[a,b]$ subordinate to an $S \in \mathcal{S}[a,b]$, $A \in \mathcal{A}(S)$ iff $\forall t \in [x_{k-1}, x_k]$, $(y_k, t) \in S$, $1 \leq k \leq n$ .

(c) If $f\colon [a,b] \to \mathbb{R}$, $A \in \mathcal{A}(S)$ define

$$\phi(A) = \sum_{k=1}^{n} f(y_k)(x_k - x_{k-1}) ;$$

then $f$ is integrable on $[a,b]$ if $\forall \varepsilon > 0\ \exists\ S \in \mathcal{S}[a,b]$ s.t. $A_1, A_2 \in \mathcal{A}(S)$, $|\phi(A_1) - \phi(A_2)| < \varepsilon$ .

This integral was later to be called the Riemann complete integral, $\mathcal{R}C^*$ - integral, and its value is the unique $I$ s.t. $\forall \varepsilon > 0\ \exists\ S \in \mathcal{S}[a,b]$ s.t. $\forall A \in \mathcal{A}(S)$, $|I - \phi(A)| < \varepsilon$.

Kurzweil proved that $\mathcal{R}C^* = \mathcal{P}^*$.

This Riemann definition of the Perron integral was almost defined in passing by Kurzweil and remained unnoticed until it was rediscovered independently by R. Henstock, [6,7,9,10]. A similar definition has been developed by Ridder, [23-26]: his theory is more complicated than Henstock's and is sometimes slightly more general but not in the present situation; see Lee T-W, [2]. In its simplest form the definition of the $\mathcal{R}C^*$ - integral is as follows; see Henstock, [7-10], R.O. Davies and Z. Schuss, [1], Lee P-Y., [5,6], H.W. Pu, [2,3].

(d) If $f: [a,b] \to \mathbb{R}$ then $f$ is integrable with $\int_a^b f = I$ iff $\forall \varepsilon > 0$ $\exists \delta: [a,b] \to \mathbb{R}$, $\delta > 0$, s.t. whenever $a = x_0 < x_1 < \ldots < x_n = b$ and $x_{k-1} \leq y_k \leq x_k$, and $x_k - x_{ky} < \delta(y_k)$, $1 \leq k \leq n$, then $|I = \sum_{k=1}^{n} f(y_k)(x_k - x_{k-1})| < \varepsilon$.

The basic idea of replacing the constant $\delta$ of the classical definition by a function has been explored further by McShane, [3-5]. However the extension of these ideas to the other non-absolute integrals discussed above seems not to have been carried out in detail although Henstock says that the methods will include integrals equivalent to the $\mathcal{D}$- integral and the $B\mathcal{P}_{ap}$ - integral, [10]. See also D.C. Carrington and Pacquement, [1].

1.5. Other Solutions.

1.5.1. The Sequence Integrals.

In 1937, Jeffrey, [2], gave an interesting definition of an integral more general than the $\mathcal{J}$ - integral by a method that avoids the use of transfinite induction.

If $f$ is measurable on $[a,b]$ and if $\exists$ sequence of summable functions, $s_n$, $n \in N$, s.t. $\lim_{n\to\infty} s_n = f$ a.e., and a continuous function $F$, s.t. $\lim_{n\to\infty} \int_a^x f = F$, then $f$ is said to be sequence integrable to $F$; $f \in \mathcal{S}^a$, $\mathcal{S}\text{-}\int_a^b f = F$.

If in addition the sequence $s_n$, $n \in N$, can be so chosen so that $s_n = f\, 1_{E_n}$, where $E_n \subset E_{n+1}$, $\lim_{n\to\infty} |E_n| = b - a$, then $f$ is said to be totally sequence integrable to $F$; $f \in T\mathcal{J}$, $T\mathcal{J} - \int_a^b f = F$. Jeffrey showed that $\mathcal{J} \subset T\mathcal{J}$.

In a later paper, Jeffrey and M. Macphail, [1], defined an integral equivalent to the $\mathcal{J}$ - integral.

With the above notation put $F(E) = \lim_{n\to\infty} \int_E s_n$ : $F$ is the non-absolutely convergent integral of $f$ iff,

(a) $F([a,x])$ is continuous;

(b) $[a,b] = \bigcup_{n\in N} E_n$ where $n \in N$,

(i) $E_n$ is closed,

(ii) $F$ is additive on $E_n$,

(iii) if $I_n^k$, $k \in N$ are the contiguous intervals of $E_n$, $F(\bigcup_k I_n^k) = \sum_k F(I_n^k)$.

See also Jeffrey, [3], H.M. MacNeille, [1].

1.5.2. The Parametric Primitive

$F: [a,b] \to \mathbb{R}$ is said to have a parametric derivative $f$ iff $\exists$ a differentiable parametric representation of $y = F(x)$;

$$x = \phi(t), \quad y = F\circ\phi(t), \qquad \alpha \le t \le \beta,$$

$\phi$, increasing, and s.t.

$$\frac{dy}{dt} = f\circ\phi(t)\ \phi'(t).$$

This definition is due to Tolstov, [7,8], who showed that $f$ is a parametric derivative iff $f \in \mathcal{D}^*$ ; when $\mathcal{D}^* - \int_a^b f = F(b) - F(a)$ .

Since the parametric derivative has the basic three properties of the ordinary derivative:

(a) $(kF)' = kF'$ ; (b) $(F+G)' = F' + G'$ , (c) $F' = 0 \Rightarrow F$ constant;

this gives a simple classical approach to the $\mathcal{D}^*$ - integral. Properties (a) and (c) are simple consequences of the above definition; (b) was proved later by G.M. Armstrong, [1].

See also S. Nakanishi, [1], D. Butković, [1], for other definitions of non-absolute integrals.

## 2. The Coefficient Problem

Very soon after their introduction the $\mathcal{D}$ - and $\mathcal{D}^*$ - integrals were applied to theory of trigonometric series; see P. Nalli, [1], and also A. Zygmund, [1; II, p.83]. However sums of everywhere convergent trigonometric series, known to be not necessarily $\mathcal{L}$ - integrable, turned out also to be not necessarily $\mathcal{D}$ - integrable. Precisely: If $a_n > a_{n+1}$ , $n > 1$ , $\lim_{n\to\infty} a_n = 0$ , and $\sum_{n\geq 1} \frac{a_n}{n} = \infty$ ,

then if $f(x) = \sum_{n\geq 1} a_n \sin n x$ , $f \notin \mathcal{D}$ : see Denjoy, [15, p.42].

Since the coefficients of an everywhere convergent trigonometric series are uniquely determined by the sum, the coefficient problem, the problem of calculating the coefficients from the sum, or of finding an integral wide enough to make the usual Fourier formulae valid, remained open after the definition of the $\mathcal{D}$ - integral.

The problem is complicated by the fact that the "natural" primitive of the sum, the formally integrated trigonometric series, is not necessarily continuous, or even defined everywhere.

The problem was solved in 1921 by Denjoy, [7-11]; and various other solutions have been given since. All the solutions involve defining an integral by one of the three methods used in §1. However the relationships between the various integrals are not completely known; further in almost every case only one of the three methods was used and so the question of giving equivalent definitions by the other methods remains open.

A good survey of the field is given in R.D. James, [3], and Jeffrey, [8].

## 2.1. Denjoy's Solution of the Coefficient Problem

Denjoy avoided the difficulties of the "natural" primitive mentioned above by using the second order primitive, the sum of the twice integrated series. He pointed out, [15, p.1-8], that the coefficient problem is equivalent to the following generalisation of the classical primitive problem: find a continuous function given its Schwartz derivative: i.e. given

$$f(x) = D^2F(x) = \lim_{h\to 0} \frac{F(x+h) + F(x-h) - 2F(x)}{h^2} , \quad a < x < b ,$$

find $F$ , with $F(a) = F(b) = 0$ .

In a series of five notes Denjoy generalised his totalisation process to solve this problem, [7-11]: giving a new integral $S\mathcal{J}^2$- integral, the symmetric totalisation of second order. The classical totalisation used three basic limit processes in its transfinite induction; Lebesgue integration, 1.1(2); Cauchy integrals, 1.1(3); summation of infinite series, 1.1(5). To carry out the induction for the $S\mathcal{J}^2$ - integral, six extra, and much more complicated limit processes were required. Further, whereas in the classical situation only the basic properties of perfect sets were used, here a very detailed knowledge of the fine properties of such sets is needed; Denjoy, [6]. Another short note on the $S\mathcal{J}^2$ - integral appeared in 1933, [13], but full details were not published until 1941-1949, [15].

The process is too complicated to reproduce here, but a good summary can be found in [15, p.465-481], although there Denjoy is describing a more general integral, the $S\mathcal{J}^2_{ap}$ - integral, one in which the limits used to define the $S\mathcal{J}^2$ - integral are, where appropriate, replaced by approximate limits. Surprisingly this more general integral does not solve the primitive problem for the approximate Schwartz derivative; i.e. $\exists f$, $f = D^2_{ap} F$ and $f \notin S\mathcal{J}^2_{ap}$; V.A. Skvorcov, [2]. The following inclusions are known. Skvorcov, [1,2,5];

(a) $S\mathcal{J}^2 \subsetneq S\mathcal{J}^2_{ap}$ ; (b) $\mathcal{J}^* \subsetneq S\mathcal{J}^2$ ;

(c) $\mathcal{J} \not\subseteq S\mathcal{J}^2$ ; $\mathcal{J}$ and $S\mathcal{J}^2$ are not compatible.

Denjoy gave examples of trigonometric series needing all nine limit operations for the calculation of their coefficients from their sums. Also, as in the case of the classical primitive problem, examples were given to show that the transfinite induction may have to proceed as far as any ordinal less than the first uncountable ordinal, [15; p.483-595].

In passing Denjoy gave definitions of first order integrals more general than the $\mathcal{J}$ - integral in that the indefinite integrals only existed almost everywhere. Later he gave another first order totalisation, the $S\mathcal{J}$- integral, related to these integrals, [17]. Its construction required only five of the nine operations needed for the

$S\mathcal{J}^2$- integral. It solved the primitive problem for the derivative

$$DF(x) = \lim_{h\to 0} \frac{F(x+h) - F(x-h)}{2h} \; ;$$

$S\mathcal{J} \underset{=}{\not\subset} S\mathcal{J}^2$ ; $\mathcal{J} \underset{=}{\not\subset} S\mathcal{J}$. S.N. Mukhopadhyay, following James, [2], has given a Perron definition that also solves this primitive problem, [1]. P. Bhatacharyya, [1], has given a Perron integral that solves the primitive problem for the approximate derivative of this type.

## 2.2. The Work of J.C. Burkill.

The difficulty of the $S\mathcal{J}^2$- totalisation, and the delay in the appearance of full details led other authors to attempt solutions of the coefficient problem.

In 1932 Burkill, [4], defined the Cesàro derivative,

$$CDF(x) = \lim_{h\to 0} \mathcal{P}^* - \int_x^{x+h} \frac{1}{h} \{F(x+h) - F(x)\}dx \; .$$

He then, using the Perron approach, solved the primitive problem for this derivative. In two later papers, [5,7], he used the full scale of Cesàro means to define the derivatives $C_\alpha$ DF, $\alpha > 0$, and solved the associated primitive problems. The integrals obtained are called the Cesàro-Perron integrals, the $C_\alpha\mathcal{P}^*$ - integrals. They are strict generalisations of the $\mathcal{P}^*$ - integral, the scale of integrals is consistent in the following sense: if $\alpha < \beta$ then

(a) $C_\alpha\mathcal{P}^* \underset{\neq}{\subset} C_\beta\mathcal{P}^*$ ;

(b) $f \in C_\alpha\mathcal{P}^* \Rightarrow C_\alpha\mathcal{P}^* - \int_a^b f = C_\beta\mathcal{P}^* - \int_a^b f$ .

As is usual with Perron definitions these integrals are easy to use and Bürkill gave an integration by parts formula. As a result it was possible to give quick and easy solutions to variants of the coefficient problem; Burkill, [6]; see also G. Cross [2]. The same problems had been studied by Verblunsky, [1], using his $V\mathcal{J}_{ap}$ - integral; (see 1.3); but his solution took almost forty pages and appealed to sophisticated function theory.

This work of Burkill created a lot of interest. W.L.C. Sargent, [1,5], extended the Luzin approach to define $C_\alpha\mathcal{D}^*$ - integrals that are equivalent to the $C_\alpha\mathcal{P}^*$ - integrals; certain errors in her work and in Burkill's papers were corrected by Verblunsky, [2]. H.W. Ellis and Jeffrey, [1], extended the totalisation procedure to define $C\mathcal{J}^*$ - and $C_2\mathcal{J}^*$ - integrals, equivalent to the $C\mathcal{P}^*$- and $C_2\mathcal{P}^*$- integrals, respectively. Their methods broke down for other values of $\alpha$ ; the reasons for this were investigated in an interesting paper by Jeffrey and Miller, [1]. Their work influenced Henstock who gave Perron and Riemann definitions of a general integral that includes the $C_\alpha\mathcal{P}^*$ - scale of integrals as a special case; Henstock, [3,5]. Skvorcov, [3], proved a Markinciewicz-Tolstov result, [see 1.3], for the $C\mathcal{P}^*$- integral and showed that $C\mathcal{P}^* \subsetneq S\mathcal{J}^2_{ap}$ and that $\exists f \in \mathcal{D}\setminus C\mathcal{P}^*$. Kubota, [8], showed that the $C\mathcal{P}^*$- integral is the natural C-continuous extension of the $\mathcal{L}$- integral; in the same sense that the $\mathcal{P}^*$ - integral is the natural continuous extension; see Saks [2,4]. Ellis, [3], has shown that the $B\mathcal{P}_{ap}$ - integral and the $C\mathcal{P}^*$- integral are not compatible.

Despite its successes the $C\mathcal{P}^*$- integral did not solve the basic coefficient problem: although the indefinite integral was not continuous, (being only C-continuous), it existed everywhere. In 1951, Burkill, [8], introduced the symmetric Cesàro derivative and the associated symmetric Cesàro Perron integral, the $SC\mathcal{P}$- integral. With this integral he solved the coefficient problem and later, [9], applied it to related problems for summable series. Unfortunately his stated integration by parts formula was not proved and remains so to this day; see P.S. Bullen and Mukhopadhyay, [1]. Using a different approach H. Burkill, [2], showed that the $SC\mathcal{P}$- integral did however solve the coefficient problems; see also H. Burkill, [3], Skvorcov, [1,7], has shown that $\exists$ $f \in \mathcal{D}\setminus SC\mathcal{P}$ and that $\exists$ $g \in SC\mathcal{P} \setminus \mathcal{D}$.

Ridder, [14], has defined an approximate Cesàro Perron integral and Lee, C-M., [1], has defined a scale of such integrals, the $C_n\mathcal{P}_{ap}$- integrals, n = 1, 2, ..... This scale is however not consistent in that while the above property (a) holds, (b) does not. A definition, essentially the same as Ridder's, has been given by A. Jaiswal, P.L. Sharma and P.L. Singh, [1]. Lee has also defined a scale of symmetric Cesàro Perron integrals (the $SC_n\mathcal{P}$ -scale) that Cross has applied to coefficient problems associated with summable series; see Bullen and Lee, [2], and Cross, [9]. Ellis, [1] has defined a scale of integrals that generalizes $\mathcal{D}$- integral, rather than the $\mathcal{D}$* - integral as the case of the $C_\alpha\mathcal{P}$* -scale: he has given both a Luzin and totalisation definition; see also Skvorcov, [7], Kubota, [4].

Other references: Kubota, [13].

## 2.3. Other Solutions of the Coefficient Problem.

In 1936 Marcinkiewicz and Zygmund, [1], introduced an integral of Perron type that inverted the Borel derivative,

$$\lim_{h\to 0} \lim_{\delta\to 0} \mathcal{D}^* - \int_{\delta}^{h} \frac{F(x+t) - F(x-t)}{2t}\, dt ,$$

and used it to solve the coefficient problem. Skvorcov, [10] gave an example of a function integrable in this sense but not $SC\mathcal{P}$-integrable, but Lee showed that under certain natural restrictions these two integrals are equivalent; Bullen and Lee, [1]. Some work has been done on an integration by parts formula; see Bullen and Mukhopadhyay, [1]; see also Ridder, [15].

2.3.2. The Work of R.D. James.

In 1946 James gave what is the most natural Perron definition for the present problem by using the Schwartz derivative; his second order integral is called the $S\mathcal{P}^2$- integral; W.H. Gage and R.D. James, [1], James, [1] and also Zygmund, [1:, II, p.86]. Skvorcov, [1,3,4,6,7] and G. Cross, [3], have shown that

$$SC\mathcal{P} \subset S\mathcal{P}^2 ; \quad \exists f \in \mathcal{D} \setminus S\mathcal{P}^2 \text{ and } \quad \exists g \in S\mathcal{P}^2 \setminus \mathcal{D} :$$

but little is known about the relation between $S\mathcal{P}^2$ and $S\mathcal{J}^2$: see also Ellis, [5].

Skvorcov, [4], has defined an $S\mathcal{P}^2_{ap}$- integral, by using the approximate Schwartz derivative; its relationship to the $S\mathcal{J}^2_{ap}$- integral is not known but $\exists f \in \mathcal{D} \setminus S\mathcal{P}^2_{ap}$ and $\exists g \in S\mathcal{P}^2_{ap} \setminus \mathcal{D}$. See also V.A. Sklyarenko, [2].

2.3.3. S.J. Taylor, [1], gave an interesting Perron definition using the Abel derivative,

$$AD^2F(x) = \frac{1}{\pi} \lim_{r\to 1} \frac{\partial^2}{\partial x^2} \int_{-\pi}^{\pi} \frac{1 - r^2}{1+r^2-2r\cos t}\, F(x+t)dt .$$

The resulting integral, the $A\mathcal{P}^2$- integral, was then used to solve coefficient problems associated with Abel summable trigonometric series. Taylor showed that $\mathcal{D}^* \subset A\mathcal{P}^2$ and Skvorcov, [5], showed that $S\mathcal{P}^2 \subset A\mathcal{P}^2$, after making certain natural modifications so as to make these integrals comparable; Skvorcov [5] and Cross, [5], have shown that $SC\mathcal{P} \subset A\mathcal{P}^2$.

2.3.4. Cross, [4], has adapted Taylor's idea to define an integral for Cesaro summable series; see Sklyarenko, [1] and Skvorcov, [8]. Coefficient problems for other orthonormal sequences have not been used to introduce new integrals, except for Skvorcov, [9], who has considered an integral connected with the Haar system.

## 2.4. Higher Order Integrals

In the study of summable trigonometric series derivatives of order higher than two occur naturally, see Zygmund, [1; II, p.59], and various authors have defined integrals that invert these derivatives. There are three basic definitions of these higher order derivatives.

(1) The Riemann Derivatives. Let

$$V_r(F;x_k) = \sum_{k=0}^{r} \frac{F(x_k)}{w'(x_k)} ,$$

$$w(x) = \prod_{k=0}^{r} (x-x_k) ;$$

now suppose,

$$x_k = x + h_k , \quad 0 \leq k \leq r ,$$

$$0 \leq |h_o| < \dots < |h_r| ,$$

and define

$$D^r F(x) = \lim_{h_r \to 0} \dots \lim_{h_o \to 0} r!\, V_r(F;x_k) ,$$

if this iterated limit exists independently of the manner in which the $h_k$ tend to zero, subject only to the above restriction.

Various generalisations are possible by restricting the $h_k$ in some way. In particular we can require that the $x_k$ be symmetric with respect to $x$ ; then if $r = 1, 2$ we get derivatives mentioned earlier, (2.1).

(2) The Peano Derivatives. If it is true that

$$F(x+h) - F(x) = \sum_{k=1}^{r} \alpha_k \frac{h^k}{k!} + o(h^r) , \quad \text{as} \quad h \to 0 ,$$

where the $\alpha_k$ do not depend on $h$ , then $\alpha_k$ is called the kth Peano derivative of $F$ at $x$ , $F_{(k)}(x)$ : if $k = 1$ , $F_{(1)} = F'$ .

(3) The de la Vallée Poussin Derivatives. If it is true that

$$\frac{F(x+h) + F(x-h)}{2} = \sum_{k=0}^{r} \beta_{2k} \frac{h^{2k}}{(2k)!} + o(h^{2r+1}) , \quad \text{as} \quad h \to 0 ,$$

or

$$\frac{F(x-h) - F(x-h)}{2} = \sum_{k=0}^{r} \beta_{2k+1} \frac{h^{2k+1}}{(2k+1)!} + o(h^{2r+1}) , \quad \text{as} \quad h \to 0 ,$$

where the $\beta_k$ do not depend on $h$ , then $\beta_k$ is called the kth de la Vallée Poussin derivative of $F$ at $x$ , $D_k F(x)$ ; if $k = 1$ or $2$ , $D_k F = D^k F$ .

The Peano derivatives are closely related to the Cesàro derivatives, Burkill, [5]. This was used later by Sargent, [5], to give an alternative Luzin definition of her $C\mathcal{P}_n^*$ - integrals. A very detailed study of the problem of inverting nth order derivatives was made by Denjoy, [14], but his work has not been used in other discussions of this topic; for an application in the theory of distributions see S. Lojásiewicz, [1].

James, [2,4], studied a scale of integrals, the $S\mathcal{P}^n$- integrals, that invert the de la Vallée Poussin derivatives. The scale is consistent and James applied these integrals to the coefficient problems of Cesàro-summable trigonometric series; see also Cross, [1]. Various authors noted a flaw in the James' definition and suggested modifications; Cross, [6], J. Mařík, [2,3], and Mukhopadhyay, [1]. Lee, C.-M. has shown that $SC_n\mathcal{P} \subset S\mathcal{P}^{n+1}$. James, [2], specialised his results to obtain a scale of integrals that invert the Peano derivatives, the $\mathcal{P}^n$ - integrals. A direct and slightly modified definition was given by Bullen, [2], who then showed that $\mathcal{P}^n = C_{n-1}\mathcal{P}^*$: (in particular of course, $\mathcal{P}^1 = \mathcal{P}^*$) .

Both J.A. Bergin, [1], and Lee, C.-M. [1], have used higher order derivatives to define a scale of first order integrals. Bergin's scale is equivalent to the $C_n\mathcal{P}$- scale but Lee's is more general than his $SC_n\mathcal{P}$- scale but less general than the $S\mathcal{P}^{n+1}$- scale.

A completely different scale of derivatives has been used by Gordon, [1], to define a scale of Perron integrals with applications to trigonometric series.

See also J. Brille, [1], Jeffrey, [8], Bullen, [1], Ridder, [22], C. Kassimatis, [1].

## 3. Multiple Integrals

The construction of non-absolute multiple integrals goes back to Bauer's original paper, [1]. He extends his definition of the $\mathcal{P}^*$-integral to functions of n-variables and shows that for bounded functions this integral is equivalent to the $\mathcal{L}$- integral. A similar definition, at least of the major and minor functions, is given by Saks, [4].

In 1923 Looman, [1], defined a totalisation process that inverts $\frac{\partial^2 f}{\partial x \partial y}$ in the generalised form

$$\lim_{(h,k)\to(0,0)} \frac{f(x+h,y+k) + f(x,y) - f(x+h,y) - f(x,y+k)}{hk}$$

Almost ten years later M. Krzyżański, [1], gave both a Perron and an equivalent Luzin definition of an integral that also inverts this derivative, in the symmetric case, i.e. $h = k$. Almost immediately Kempisty, [5], generalised this work to the situation of a regular derivative, i.e. for some $\alpha > 0$, $\alpha \leq |\frac{h}{k}| \leq \frac{1}{\alpha}$. Here, and later [7,8], Kempisty made a most significant contribution to the theory of multiple integrals: he replaced the infinite sums in the totalisation process, or equivalently in the definition of generalised absolute continuity, by the much simpler Burkill integral, of interval functions; see Burkill, [2], Saks, [4, p.165], Haupt-Aumann-Pauc, [1; p.227] and Krzyżański, [2]. This idea was taken up by Romanovskiĭ, [2-4], who gave several Luzin and equivalent Perron definitions of multiple integrals. Although he did not quite give a complete treatment of totalisation this was done by D. Solomon, [1-4], who then showed that in the case of one variable Romanovskiĭ's totalisation was equivalent to $\mathcal{J}^*$- totalisation. Ridder, [11], using the same derivative as Looman, defined a totalisation, and gave equivalent Luzin and Perron definitions: his integral is less general than that of Krzyżański and Kempisty but was not compared with that of Looman.

In 1937 V.G. Čelidze, [1], defined an approximate partial derivative and in long paper, [2], gave a totalisation process that inverts this. Apparently unaware of the second paper Pacquement, [4], gave a Luzin definition of such an integral; the relation between the two is not known. Čelidze's integral has been studied by A.G. Džvaršeǐšvili, [2], and P. Pych, [3], has given a definition of an integral in the narrow sense using the methods of Čelidze; the relation of her integral to the integrals of Looman and Ridder is not known.

Jeffrey, [4], extended the ideas of his earlier paper on sequence integrals, [2], (see 1.5.1) and gave a Luzin definition of two integrals that in the case of one dimension reduce to the $\mathcal{D}$ - and $\mathcal{D}^*$ - integrals. Another approach using sequences of Lebesgue integrals to define a more general integral in higher dimensions has been given by S. Enomoto, [1,2]; he does not relate his integral to Jeffrey's work, or to any of the other integrals mentioned above.

See also Mařík, [1], R. Caccioppoli, [1], K. Karták, [1], Karták and Mařík, [1]; Džvaršeǐšvili, [7,8].

4. Stieltjes Integrals

Lebesgue, [2], defined the first Denjoy-Stieltjes integral that inverted derivatives with respect to functions of bounded variation. His approach depends, in part, on a transformation that changes Stieltjes' integrals into ordinary integrals. A more direct approach was taken by Jeffrey, [1], who defined two totals corresponding to the narrow and general totalisation processes of Denjoy. This author was to continue to study such integrals over a number of years; [9-11], Ellis and Jeffrey, [2]; see also P.C. Bhakta, [1,2] and M.C. Chakrabarty, [1-3], Sarkhel, [1,2]. Ridder, [12], gives Perron and Luzin definitions for this situation; Kempisty, [5], extended these integrals to higher dimensions as did Ridder, [20].

The first integral that was more general than both the Lebesgue-

Stieltjes integral and the Riemann-Stieltjes integral was defined using the Perron approach by A.J. Ward, [1], see also Saks [4]. Ridder, [16,17] investigated Ward's integral further, gave a Luzin definition and also defined an integral generalising the $\rho$- integral rather than the $\rho^*$ - integral. L. Nicolescu, [1], applied McShane's ideas of using four derivates to Ridder's integrals and so, as in McShanes earlier work, (see 1.3), was able to prove an integration by parts formula. Henstock combined the ideas of Ward and of Jeffrey and Miller, [1], to define a very general Perron-Stieltjes integral, [1-10].

A direct definition of a Cesàro-Perron Stieltjes integral was given by Kubota, [7], who did not investigate its properties or its relation to other Stieltjes integrals. A thorough investigation of another integral of this type has been given by D.K. Dutta, [1], see also M.K. Bose, [1].

See also C. Choquet, [1], Ridder, [18,19], Y. Hayashi, [1], Kurzweil, [4].

5. Axiomatic Theories.

It is immediately seen that the various non-absolute integrals have similar definitions and that all are generalisations of the Lebesgue integral. So it was natural to try to axiomatise and to fit the resulting theory into general measure theory and abstract differentiation, see Hayes and Pauc, [1].

The first attempt was made by Saks, [2; 4, p.254], who stripped the totalisation to its essentials. His method was used later to fit many non-absolute integrals into single approach; Solomon, [1], Bullen, [3]. Various authors gave more or less obvious axiomatisations of the Perron approach; Izumi, [3], Kubota, [6], Ridder, [13]; others gave interesting generalisations of the Luzin's definition; Ellis, [4], J. Foran, [1], Lee, C.-M., [2].

The first attempt at a full axiomatisation was made by Romanovskiĭ, [2-4]. His theory sets non-absolute integrals in what is now known as a Romanovskiĭ space. It is this setting that Solomon, [1-4], combining the approaches of Romanovskiĭ and Saks defines a very general non-absolute integral of vector-valued functions: see also Morrison, [1].

Mařik, [1], and Pfeffer, [1,2] developed a theory of Perron integrals in locally compact first countable Hausdorff spaces. Their theory placed awkward restraints on the domains of the functions but later work by Pfeffer, [3-8], removed these and generalized the setting to arbitrary topological spaces; see also Pfeffer and Wilbur, [1]; and Choquet, [1].

Lee, C.-M., has given an axiomatic theory that includes both that of Peffer and that of Romanovskiĭ as special cases; see Bullen and Lee, [1].

McShane's theory, [3-5] and Henstock's, [9,11,13] both define non-absolute integrals of vector values functions that relate to integrals known in functional analysis; see also Scanlon, [1], and Alexiewicz, [2], Lee, T.-W., [3].

All of these theories applies to non-absolute non-symmetric integrals. Trjitzinsky, [1-5], besides generalising totalisations to setting more general than Romanovskiĭ spaces, has also considered the problem of symmetric totalisations, that in particular apply to non-summable Laplaciens. Bullen, [2], has set James $S\rho^2$- integral in general harmonic spaces; see also Nasibov, [1].

Most of these theories have a large number of axioms and compare badly to general measure theory. However the correct comparison, with general measure theory in topological spaces, does a little to improve the comparison.

A more complete discussion of the various axiomatic theories can be found in Bullen, [5].

## 6. Integral Calculus and Applications

The basic results in calculus follow easily for integrals defined by a totalisation procedure since they follow by transfinite induction from similar properties of the Lebesgue integral. The integration by parts formula shows that for the Perron definitions this is not always the case; for a proof of this formula for the $\mathcal{J}^*$- integral see Hobson, [1; p.711], for the $\mathcal{D}^*$- integral, Saks, [4; p.246] or Denjoy, [16; p.343]: see also Henstock [12], Kurzweil, [3], Sargent, [3]. Further, since, in all cases, if $f \geq 0$ and is integrable in some non-absolute sense then $f \in \mathcal{L}$, the Lebesgue monotone convergence theorem holds, Denjoy, [16; p.342], Perron, [1] and Bauer, [1]. Further results on interchange of limits and integrals for sequences have been given by McShane, [2], Manougian, [1], Džvaršeišvili, [3,9,10], Kubota, [15], Henstock, [14], and Lee, P.-Y., [6]. Properties of the indefinite integral, basic of course to the whole theory, have been studied by many authors; Dubuc, [1] has given a simple proof of the fundamental theorem for the $\mathcal{P}^*$- integral; see also Bosanquet, [1,2], Cross, [8], Ellis, [2], Džvaršeišvili, [4,11], Grimshaw, [1], Jeffrey, [6], Kartak, [2], Krzyżewski, [1,2], Kubota, [1,2], Mugalov, [1,2], Pu, [1,2,4], Sargent, [2]. That Fubini's theorem does not always hold for non-absolute integrals was noted by Tolstov, [3]; but several authors have proved positive theorems of this type; Džvaršeišvili, [5], Kurzweil, [2], Lee, T.-W., [1].

Applications of non-absolute integrals to other parts of analysis have been made by several authors but the results are isolated, and not a part of the main stream of modern analysis; see H. Burkill, [1], Choquet, [2], Denjoy, [15], Džvaršeĭšvili, [1,10], Foglio, [1-3], Foglio and Henstock, [1], Henstock, [11,13], Lee, P.-Y., [1-4], Lukašenko, [1], Matyska, [1], McGill, [1], Mugalov, [3,4], Nalli, [1], Pych, [1-8], Sargent, [4,5], Tolstov, [5].

Finally a few attempts have been made to treat the various spaces of non-absolutely integrable functions from the point of view of functional analysis; Alexiewicz, [1], Džvaršeĭšvili, [6].

## Bibliography


Aleksandrov, A., 1. Über die Äquivalenz des Perronschen und des Denjoyschen Integralbegriffes, Math. Z., 20 (1924), 213-222.

Alexiewicz, A., 1. Linear functionals on Denjoy-integrable functions, Colloq. Math., 1 (1948), 289-293; M.R., 10, 717.

2. On Denjoy integrals of abstract functions, Soc. Sci. Lett. Varsovie C.R. Cl. III Sci. Math. Phys., 41 (1948), 97-129; M.R., 14, A27.

Armstrong, G.M., 1. A classical approach to the Denjoy integral by parametric derivatives, J. London Math. Soc., (2) 3 (1941), 346-349; M.R., 43, #4971.

Bauer, H., 1. Der Perronsche Integralbegriff und seine Beziehung zum Lebesgueschen, Monatsh. Math. Phys., 26 (1915), 153-198.

Bergin, J.A., 1. A new characterization of Cesàro-Perron integrals using Peano derivatives, Trans. Amer. Math. Soc., 228 (1977), 287-305; M.R., 55, #8272.

Bhakta, P.C., 1. On functions of bounded ω-variation, Riv. Mat. Univ. Parma, 6 (1955), 55-64; M.R., 36, #327.

2. On functions of bounded ω-variation II, J. Austral. Math. Soc., 5 (1965), 380-387; M.R., 33, #2771.

Bhattacharyya, P., 1. On modified Perron integral, Indian J. Math., 15 (1973), 103-117; M.R., 51, #824.

Bosanquet, L.S., 1. A property of Cesàro-Perron integrals, Proc. Edinburgh Math. Soc., (2) 6 (1940), 160-165; M.R., 2, 131.

2. Some properties of Cesàro-Lebesgue integrals, Proc. London Math. Soc., (2) 49 (1947), 40-62; M.R., 7, 280.

Bose, M.K., 1. Results on $AC^*$-ω (C-sense) and $ACG^*$-ω (C-sense) functions, Comment. Math. Prace Mat., 20 (1977), 7-28.

Brille, J., 1. Sur une propriété des fonctions présentant un certain caractère complexe de résolubilité, C.R. Acad. Sci. Paris, 192 (1931), 1191-1193; Zbl., 1, 329.

Bruneau, M., 1. Variation Totale d'une Fonction, Lecture Notes in Math., #413, Berlin-Heidelberg-New York, 1974.

Bullen, P.S., 1. Construction of primitives of generalized derivatives with applications to trigonometric series, Canad. J. Math., 13 (1961), 48-58; M.R., 22, #12186.

2. A general Perron integral, I, II, Canad. J. Math., 17 (1965), 17-30; 19 (1970), 457-473; M.R., 30, #4959; 36, #421.

3. A constructive definition of an integral, Tôhoku Math. J., 22 (1970), 597-603; M.R., 43, #6368.

4. The $P^n$-integral, J. Austral. Math. Soc., 14 (1972), 219-236; M.R., 47, #8783.

5. Axiomatisations of various non-absolute integrals, Bull. South-East Asian Math. Soc., to appear.

Bullen, P.S. and Lee, C.-M., 1. On the integrals of Perron type, Trans. Amer. Math. Soc., 182 (1973), 481-501; M.R., 49, #3057.

2. The $SC_nP$-integral and the $P^{n+1}$-integral, Canad. J. Math., 25 (1973), 1274-1284; M.R., 53, #13489.

Bullen, P.S. and Mukhopadhyay, S.N., 1. Integration by parts formulae for some trigonometric integrals, Proc. London Math. Soc., (3) 29 (1974), 159-173; M.R., 51, #825.

Burkill, H., 1. Cesàro-Perron almost periodic functions, Proc. London Math. Soc., (3) 2(1952), 150-174; M.R., 14, 162.

2. A note on trigonometric series, J. Math. Anal. Appl., 40 (1972), 39-44; M.R., 47, #2250.

3. Fourier series of SCP-integrable functions, J. Math. Anal. Appl., 57 (1977), 587-609.

Burkill, J.C., 1. The fundamental theorem of Denjoy integration, Proc.

Cambridge Philos. Soc., 21 (1923), 659-663.

2. Functions of intervals, Proc. London Math. Soc., 22 (1923), 273-310.

3. The approximately continuous Perron integral, Math. Z. 34 (1931), 270-278; Zbl., 2, 386.

4. The Cesàro-Perron integral, Proc. London Math. Soc., (2) 34 (1932), 314-322; Zbl., 5, 392.

5. The Cesàro-Perron scale of integration, Proc. London Math. Soc., (2) 39 (1935), 541-552, Zbl., 12, 204.

6. The expression of trigonometrical series in Fourier form, J. London Math. Soc., 11 (1936), 43-48; Zbl., 13, 260.

7. Fractional orders of integration, J. London Math. Soc., 11 (1936), 220-226; Zbl., 14, 258.

8. Integrals and trigonometric series, Proc. London Math. Soc., (3) 1 (1951), 46-57; M.R., 13, 126.

9. Uniqueness theorems for trigonometric series and integrals, Proc. London Math. Soc., (3) 1 (1951), 163-169; M.R., 13, 935.

Burkill, J.C. and Gehring, F.W., 1. A scale of integrals from Lebesgue's to Denjoy's, Quart. J. Math. Oxford Ser., (2) 4 (1953), 210-220; M.R., 15, 205.

Butković, D., 1. On Denjoy integrals which depend on a parameter, Proc. 5th Congress Mathematicians, Phys. and Astron. Yugoslavia (Ohrid, 1970), Vol. 1, 77-81; M.R., 56, #8765.

Caccioppoli, R., 1. L'integrazione e la ricerca delle primitive rispetto ad una funzione continua qualqunque. Ann. Mat. Pura Appl., 40 (1955), 15-34; M.R., 17, 954.

Carrington, D.C. and Pacquement, A., 1. Sur une extension du procédé d'integration de M.R. Henstock, C.R. Acad. Sci. Paris, 274 (1972), 1901-1904; M.R., 46, #3712.

Celidze, V.G., 1. Über derivierte Zahlen einer Funktionen zweier Variablen, C.R. Akad. Sci. U.R.S.S., 15 (1937), 13-15; Zbl., 16, 297.

2. Dvoĭnye integraly Denjoy, Akad. Nauk Gruzin. SSR Trudy Tbiliss.Mat. Inst., Razmadze 15 (1947), 155-242; M.R., 14, 735.

Chakrabarty, M.C., 1. Some results on ω-derivatives and BV-ω functions, J. Austral. Math. Soc., 9 (1969), 345-360; M.R., 40, #1547.

2. Some results on AC-ω functions, Fund. Math., 64 (1969), 219-230; M.R., 41, #5567.

3. On functions of generalized bounded ω-variation, Fund. Math., 66 (1969/70), 293-300; M.R., 41, #5568.

Choquet, G., 1. Primitive d'une fonction par rapport à une fonction à variation non bornée, C.R. Acad. Sci. Paris, 218 (1944), 495-497, M.R., 6, 204.

2. Application des propriétés descriptives de la fonction contingent à la théorie des fonctions de variable réelle et à la géométrie différentielle des variétés cartesiennes,

J. Math. Pures Appl., 26 (1947), 115-226; M.R., 9, 419.

Cotlar, M. and Frenkel, Y., 1. Mayorantes y minorantes no aditivas en la teoria de la integral de Perron-Denjoy, Rev. Acad. Ciene. Exact. Fis. Nat. Madrid, 44 (1950), 411-426; M.R., 13, 121.

Cross, G.E.,

1. The relation between two definite integrals, Proc. Amer. Math. Soc., 11 (1960), 578-579; M.R., 22, #8094.
2. The expression of trigonometrical series in Fourier form, Canad. J. Math., 12 (1960), 694-698; M.R., 22, #8275.
3. A relation between two symmetric integrals, Proc. Amer. Math. Soc., 14 (1963), 185-190; M.R., 26, #281.
4. An integral for Cesàro summable series, Canad. Math. Bull., 10 (1967), 85-97; M.R., 35, #4669.
5. On the generality of the AP-integral, Canad. J. Math., 23 (1971), 557-561; M.R., 44, #378.
6. The $P^n$-integral, Canad. Math. Bull., 18 (1975), 493-497; M.R., 53, #3224.
7. The representation of (C,k)-summable trigonometric series in Fourier form, Canad. Math. Bull., 21 (1978), 149-158.
8. Additivity of the $P^n$-integral, Canad. J. Math., 30 (1978), 783-796.
9. The $SC_{k+1}P$-integral and trigonometric series, Proc. Amer. Math. Soc., 69 (1978), 297-302; M.R., 57, #16494.

Davies, R.O. and Schuss, Z., 1. A proof that Henstock's integral includes Lebesgue's, J. London Math. Soc., (2) 2 (1970), 561-562; M.R., 42, #435.

de la Vallée Poussin, Ch.-J., 1. Sur l'unicité du developpement trigonometrique, Bull. Acad. Roy. Belgique, (1912), 702-708.

Denjoy, A., 1. Une extension de l'integrale de M. Lebesgue, C.R. Acad. Sci. Paris, 154 (1912), 859-862.

2. Calcul de la primitive de la fonction derivée la plus genérale, C.R. Acad. Sci. Paris, 154 (1912), 1075-1078.

3. Sur la dérivation et son calcul inverse, C.R. Acad. Sci. Paris, 162 (1916), 377-380.

4. Mémoire sur la totalisation des nombres derivés la plus générale, Ann. Sci. Ecole Norm. Sup., (3) 33 (1916), 127-236.

5. Sur l'intégration riemanienne, C.R. Acad. Sa. Paris, 169 (1919), 219-220.

6. Sur une propriété des séries trigonométrique, Verlag G.V. Wis.-en Natur., 29 (1920), 1- .

7. Sur un calcul de totalisation à deux degrés. C.R. Acad. Sci. Paris, 172 (1921), 653-655.

8. Sur la determination des functions presentant certain caractère de résolubilité, C.R. Acad. Sci. Paris, 172 (1921), 833-835.

9. Caractères de certaines fonctions intégrables et opérations correspondants, C.R. Acad. Sci. Paris, 172 (1921), 903-906.

10. Calcul des coéfficients d'une série trigonométrique convergente quelconque dont la somme est donnée, C.R. Acad. Sci. Paris, 172 (1921), 1218-1221.

11. Sur un mode d'intégration progressif et les caractères d'intégrabilité correspondants, C.R. Acad. Sci. Paris, 173 (1921),127-129.

12. Sur la définition riemanienne de l'intégrale de Lebesgue, C.R. Acad. Sci. Paris, 193 (1931), 695-698; Zbl., 3, 106.

13. Sur le calcul de coéfficients des séries trigonométriques, C.R. Acad. Sci. Paris, 196 (1933), 237-239; Zbl., 6, 302.

14. Sur l'intégration des coéfficients différentiels d'ordre supérieur, Fund. Math., 25 (1935), 273-326; Abl. 12, 346.

15. Lecons sur le Calcul de Coéfficients d'une Série Trigonométrique, I-IV, Paris 1941-1949; M.R., 8, 260; 11,99.

16. Mémoire sur la Dérivation et son Calcul Inverse, Paris, 1954; M.R., 16, 22; (The collection of Denjoy's three fundamental papers on this topic; it includes #4 above).

17. Totalisation des derivées premières generalisées, I, II; C.R. Acad. Sci. Paris, 241 (1955), 617-620, 829-832; M.R., 17, 853-4.

Dubuc, S., 1. La derivée de la primitive de l'intégrale de Perron Canad. Math. Bull., 12 (1969), 521-522; M.R., 40, #1550.

Dutta, D.K., 1. Cesàro-Perron-Stieltjes integral, Colloq. Math., 40 (1979), 291-304.

Džvaršeišvili, A.G., 1. On the representation by singular integrals of functions integrable in the sense of Denjoy-Perron, Soobšč. Akad. Nauk Gruzin. SSR, 11 (1950), 473-458; M.R., 14, 635.

2. On a double integral of Denjoy-Čelidze, Soobšč. Akad. Nauk Gruzin. SSR, 12 (1951), 193-199; M.R., 14, 28.

3. A posledovatel'nosti integralov v smysle Denjoy, Akad. Nauk Gruzin. SSR Trudy Tbilliss, Mat. Inst. Razmadze, 18, (1951), 221-236; M.R., 14-628.

4. Ob integrirovaniĭ i differencirovaniĭ pod znakom integrala Denjoy, Soobšč. Akad. Nauk. Gruzin. SSR, 12 (1951), 385-392; M.R., 14, 628.

5. On Fubini's theorem for double Denjoy integrals, Soobšč. Akad. Nauk Gruzin. SSR, 14 (1953), 393-398, M.R., 16, 345.

6. On the normed spaces of $D^*$-integrable functions, Akad. Nauk Gruzin. SSR Trudy Tbiliss. Mat. Inst. Razmadze, 19 (1953), 153-162; M.R., 16, 490.

7. On generalized absolutely continuous functions of two variables, Soobšč. Akad. Nauk Gruzin. SSR, 15 (1954), 129-133; M.R., 16, 1092.

8. On generalized absolutely continuous functions of two variables, Akad. Nauk Gruzin. SSR Trudy Tbiliss. Mat. Inst. Razmadze, 21 (1955), 77-110; M.R., 17, 954.

9. On a sequence of integrals, Soobšč. Akad. Nauk Gruzin. SSR, 17 (1956), 297-302; M.R., 18, 297.

10. Integral Denjoy i nekotorye voprosy iz analiza, Akad.

Nauk Gruzin. SSR Trudy Tbiliss. Mat. Inst. Razmadze, 25 (1958), 273-410; M.R., 21, #5711.

11. Ob integralahtipa Cauchy-Denjoy, Akad. Nauk Gruzin. SSR Trudy Tbiliss. Mat. Inst. Razmadze, 42 (1972), 34-51; M.R., 44, #417.

Ellis, H.W. 1. Mean-continuous integrals, Canad. J. Math., 1 (1949), 113-124; M.R., 10, 520.

2. Examples of integrals that are discontinuous on sets of positive measure, Trans. Roy. Soc. Canada, III (3) 44 (1950), 37-42; M.R., 12, 399.

3. On the compatibility of the approximate Perron and Cesàro-Perron integrals, Proc. Amer. Math. Soc., 2 (1951), 396-397; M.R., 13, 331.

4. Darboux properties and applications to non-absolutely convergent integrals, Canad. J. Math., 3 (1951), 471-485; M.R., 13, 332.

5. On the relation between the $P^2$-integral and the Cesàro-Perron scale of integrals, Trans. Roy. Soc. Canad., III (3) 46 (1952), 29-32; M.R., 14, 628.

Ellis, H.W. and Jeffrey, R.L., 1. Cesàro totalization, Trans. Roy. Soc. Canad., III (3) 36 (1942), 19-44; M.R., 4, 154.

2. Derivatives and integrals with respect to a base function of generalized bounded variation, Canad. J. Math., 19 (1967), 225-241; M.R., 35, #1725.

Enomoto, S., 1. Notes sur l'intégration, I-III, Proc. Japan. Acad., 30 (1951), 176-179, 289-290, 437-442; M.R., 16, 344-5.

2. Sur une totalisation dans les espaces de plusiers dimensions, I, II, Osaka Math. J., 1 (1955), 69-102, 157-178; M.R., 17, 246, 19, 399.

Foglio, S., 1. Absolute N-integration, J. London Math. Soc., 38 (1962), 87-88, M.R., 26, #3688.

2. The N-variational integral and the Schwarz distributions, Proc. London Math. Soc., 18 (1968), 337-348; M.R., 37, #3353.

3. The N-variational integral and the Schwarz distributions II, J. London Math. Soc., (2) 3 (1970), 14-18; M.R. 40, #6257.

Foglio, S. and Henstock, R., 1. The N-variational integral and the Schwarz distributions III, J. London Math. Soc., (2) 6 (1973), 693-700; M.R., 48, #2755.

Foran, J., 1. An extension of the Denjoy integral, Proc. Amer. Math. Soc., 49 (1975), 359-365; M.R., 51, #3370.

Gage, W.H. and James, R.D., 1. A generalized integral, Trans. Roy. Soc. Canad., III (3) 40 (1946), 25-35; M.R., 9, 19.

Gonçalves, V.J., 1. Sur la primitive des différentielles totales, Rev. Fac. Ci. Univ. Coimbra, 9 (1941), 65-68; M.R., 8, 141.

Gordon, L., 1. Perron's integral for derivatives in $L^r$, Studia Math., 28 (1967), 295-316; M.R., 36, #322.

Gordon, L. and Lasher, S., 1. An elementary proof of integration by parts for the Perron integral, Proc. Amer. Math. Soc., 18 (1971), 394-398; M.R., 35, #1726.

Grimshaw, M.E., 1. The Cauchy property of the generalised Perron integrals, Proc. Cambridge Philos. Soc., 30 (1934), 15-18; Zbl.,8 ,150.

Hake, H., 1. Über de la Vallée Poussins Ober-und Unterfunctionen einfacher Integrale und die Integraldefinitionen von Perron, Math. Ann., 83 (1921), 119-142.

Haupt, O., (mit Aumann, G. und Pauc, C.Y.), 1. Differential-und Integralrechnung III, Berlin, 155; M.R., 19, 1431.

Hayashi, Y., 1. A trial production on the integral, I-IV, Bull. Univ. Osaka Prefecture, Ser. A. 11 (1962), #1, 121-131; #2, 117-126, 12 (1963),#1, 111-126; #2, 127-138; M.R., 27, #2600; 28, #180, #5163.

Hayes, C.H. and Pauc, C.Y., 1. Derivation and Martingales, New York-Heidelberg-Berlin, 1970.

Henstock, R., 1. On Ward's Perron Stieltjes integral, Canad. J. Math., 9 (1957), 96-109; M.R., 18, 881.

2. A new descriptive definition of the Ward integral, J. London Math. Soc., 35 (1960, 43-48; M.R., 22, #1648.

3. The use of convergence factors in Ward integration, Proc. London Math. Soc., (3) 10 (1960), 107-121; M.R., 22 #12197.

4. The equivalence of generalized forms of the Ward, variational, Denjoy-Stieltjes, and Perron-Stieltjes integrals, Proc. London Math. Soc., (3) 10 (1960), 281-308; M.R., 22, #12198.

5. N-variation and N-variational integrals of set functions, Proc. London Math. Soc., (3) 11 (1961), 109-133; M.R., 23, #A995.

6. Definitions of Riemann type of the variational integrals, Proc. London Math. Soc., (3) 11 (1961) 402-418; M.R., 24.

7. Theory of Integration, London, 1963; M.R., 28, #1274.

8. Majorants in variational integration, Canad. J. Math., 18 (1966), 49-74; M.R., 32.

9. Linear Analysis, London, 1967; M.R., 54, #7725.

10. A Riemann-type integral of Lebesgue power, Canad. J. Math., 20 (1968), 79-87; M.R., 36, #2754.

11. Generalized integrals of vector-valued functions, Proc. London Math. Soc., (3) 19 (1969), 509-536; M.R., 40, #4420.

12. Integration by parts, Aequationes Math., 9 (1973), 1-18; M.R., 97, #3608.

13. Integration in product spaces, including Wiener and Feynman integration, Proc. London Math. Soc., (3) 27 (1973), 317-344; M.R., 49, #9145.

14. Additivity and the Lebesgue limit theorems, Proc. Carathéodory International Symposium, (1973), 22-241; M.R., 57, #6355.

Hildebrandt, T.H., 1. On integrals related to and extensions of the Lebesgue integrals, Bull. Amer. Math. Soc., 24 (1917), 113-144.

Hinčin, A. 1. Sur une extension de l'intégrale de M. Denjoy, C.R. Acad. Sci. Paris,162 (1916), 287-291.

2. Sur la dérivation asymptotique, C.R. Acad. Sci. Paris, 164 (1917), 142-144.

3. O processe integrirovaniya Denjoy, Dokl. Akad. Nauk SSSR, 30 (1918), 543-557.

Hobson, E.W., 1. The Theory of Functions of a Real Variable, I, 3rd Ed., Cambridge, 1927.

Ionescu Tulcea, C.T., 1. Sur l'intégration des nombres dérivés, C.R. Acad. Sci. Paris, 225 (1949), 558-560; M.R., 9, 179.

Iseki, K.I. 1. On quasi-Denjoy integration, Proc. Japan Acad., 38 (1962), 252-257; M.R. 26, #6348.

Iseki, K.I. and Maeda, M., 1. On a generalisation of Denjoy integration, Natur. Sci. Reports Ochanomizu Univ., 22 (1971), 101-110; M.R., 47, #2015.

Izumi, S., 1. A new concept of integrals, I, II, Proc. Imperial Acad. Japan, 9 (1933), 570-573; 10 (1934), 57-58; Zbl., 8, 248, 345.

2. On the Verblunsky's generalization of the Denjoy integrals, Tôhoku Univ. Sci. Reports, 24 (1934-36), 344-351; Abl., 12, 204.

3. An abstract integral, X, Proc. Imperial Acad. Japan, 18 (1942), 543-547; M.R. 9, 19.

Jacquier-Bryssine, N. and Pacquement, A., 1. Sur une extension du procédé d'intégration de O. Perron, C.R. Acad. Sci. 281 (1975), 839-842; M.R., 55, #8273.

Jaiswal, A., Sharma, P.L. and Singh, B., 1. An integral of Cesàro-Perron type, Math. Japon, 20 (1975), 107-111; M.R., 52, #8352.

James, R.D., 1. A generalized integral II, Canad. J. Math., 2 (1950), 297-306; M.R., 12, 94.

2. Generalized nth primitives, Trans. Amer. Math. Soc., 76 (1954), 149-176; M.R., 15, 611.

3. Integrals and summable trigonometric series, Bull. Amer. Math. Soc., 61 (1955), 1-15; M.R., 16, 692.

4. Summable trigonometric series, Pacific J. Math., 6 (1956), 99-110; M.R. 17, 1198.

Jeffrey, R.L. 1. Non-absolutely convergent integrals with respect to a base function of bounded variation, Trans. Amer. Math. Soc., 34 (1932) 645-675; Zbl., 4, 390.

2. Functions defined by sequences of integrals and the inversion of approximate derived numbers, Trans. Amer. Math. Soc., 41 (1937), 171-192; Zbl., 16, 158.

3. The equivalence of sequence integrals and non-absolutely convergent integrals, Bull. Amer. Math. Soc., 44 (1938), 840-845; Zbl. 20, 11.

4. Functions of bounded variation and non-absolutely convergent integrals in two or more dimensions, Duke Math. J., 5 (1939), 753-774; M.R., 1, 208.

5. Perron integrals, Bull. Amer. Math. Soc., 48 (1942), 714-717; M.R., 4, 75.

6. Non-absolutely convergent integrals, Proc. 2nd Canad. Math. Congress, (1949), 93-145; M.R., 13, 449.

7. Theory of Functions of a Real Variable, 2nd Ed., Toronto, 1953; M.R., 13, 216.

8. Trigonometric series, Canad. Math. Congress Lecture Series #2, Toronto, 1956; M.R., 20, #4137.

9. Generalized integrals with respect to functions of bounded variation, Canad. J. Math., 10 (1958), 617-626; M.R., 21, #113.

10. Derivatives and integrals with respect to a base function,Presidential address, Proc. 5th Canad. Math. Congress, 1963, 25-42.

11. Generalized integrals with respect to base functions which are not of bounded variation, Lecture Notes in Math., 419 (1970), 211-220; M.R., 54, #2898.

Jeffrey, R.L. and Macphail, M.S., 1. Non-absolutely convergent integrals, Trans., Roy. Soc. Canad., III (3) 35 (1941), 41-58; M.R., 3, 227.

Jeffrey, R.L. and Miller, D.S., 1. Convergence factors for generalized integrals, Duke Math. J., 12 (1945), 127-142; M.R., 6, 204.

Kamke, E., 1. Das Lebesgue-Stieltjes Integral, Leipzig, 1956; M.R., 18, 384.

Karták, K. 1. Zur Theorie des mehrdimensionalen Integrals, Časopis Pěst. Mat., 80 (1955), 400-414; M.R., 19, 640.

2. Ein Satz über die Subtitutionen in D-integralen, Časopis Pěst. Mat., 81 (1956), 410-419; M.R. 20, #2416.

Karták, K. and Mařik, J., 1. On some representations of Perron integrable functions, Czechoslovak. Math. J., 19 (1969), 745-749; M.R., 40, #2797.

Kassimatis, C., 1. The integration of generalized derivatives, Duke Math. J., 30 (1963), 101-105; M.R., 26, #3835.

Kempisty, S., 1. Un nouveau procédé d'intégration defonctions mesurables non-sommables, C.R. Acad. Sci. Paris, 180 (1925), 812-815.

2. Sur l'intégrale (A) de M. Denjoy, C.R. Acad. Sci. Paris, 185 (1927), 749-751.

3. Sur l'intégrale (A) de M. Denjoy, C.R. Acad. Sci. Paris, 192 (1931), 186-189; Zbl., 2, 20.

4. L'intégration des fonctions sommables, Ann. Soc. Polon. Math., 10 (1932), 1-11; Zbl., 6, 195.

5. Sur la totalisation des fonctions de deux variables, C.R. Acad. Sci. Paris, 198 (1934), 2060-2062; Zbl., 9, 208.

6. Intégrale Denjoy-Stieltjes d'une fonction de deux variables, C.R. Acad. Sci. Paris, 202 (1936), 1241-1244; Zbl., 13, 299.

7. Sur les fonctions absolument continues d'intervalle, Fund. Math., 27 (1936), 10-37; Zbl. 15, 105.

8. Fonctions d'Intervalle Non-Additives, Actual. Sci. Ind., #824, Ensembles et Fonctions III, Paris, 1939; M.R., 1, 207.

Kestelman, H., 1. Modern Theories of Integration, 2nd Ed., New York, 1959; M.R., 23, #A.282.

Kozlov, B.Ya. 1. Primer Gol'dovskogo, Mat. Sborn., 28 (1951), 197-204; M.R., 12, 599.

Krzyżański, M., 1. Sur les fonctions absolument continues généralisées de deux variables, C.R. Acad. Sci. Paris, 198 (1934), 2058-2060; Zbl., 9, 207.

2. Sur l'extension de l'operation intégrale de Denjoy aux fonctions de deaux variables, Bull. Sém. Math. Univ. Wilno, 2 (1939), 41-51; M.R., 1, 47.

Krzyżewski, K., 1. On change of variable in the Denjoy-Perron integral (I), (II), Colloq. Math. 9 (1962), 99-104, 317-323; M.R., 24, #A2652; 26, #283.

2. A note on the Denjoy integral, Colloq. Math., 19 (1968), 121-130; M.R., 36, #6559.

Kubota, Y., 1. On the definition of the Cesáro-Perron integrals, Tôhoku Math. J., 11 (1959), 266-270; M.R., 21, #7283.

2. The Cauchy property of the generalized approximately continuous Perron integral, Tôhoku Math. J., 12 (1960), 171-174; M.R., 22, #12199.

3. On the approximately continuous Denjoy integral,

Tôhoku Math. J., 15 (1963), 253-264; M.R., 27, #3772.

4. The mean continuous Perron integral, Proc. Japan. Acad., 40 (1964), 117-175; M.R., 29, #2355.

5. An integral of Denjoy type, I-III, Proc. Japan Acad., 40 (1964), 713-717; 42 (1966), 737-742; 43 (1967), 441-444; M.R., 31, #2371; 34, #7763; 36, #5276.

6. A generalized derivative and integrals of Perron type, Proc. Japan Acad., 41 (1965), 443-448; M.R., 32, #1321.

7. The Cesàro-Perron-Stieltjes integral I, Proc. Japan Acad., 42 (1966), 605-610; M.R., 33, #7762.

8. On a characterization of the CP-integral, J. London Math. Soc., 43 (1968), 607-611; M.R., 37, #4228.

9. On the compatibility of the AP- and the D-integrals, Proc. Japan Acad., 44 (1968), 330-333; M.R., 37, #2937.

10. A characterization of the approximately continuous Denjoy integral, Canad. J. Math., 22 (1970), 219-226; M.R.,41, #1943.

11. An approximately continuous Perron integral, Canad. Math. Bull., 14 (1971), 261-263; M.R., 47, #2010.

12. A constructive definition of the approximately continuous Denjoy integral, Canad. Math. Bull., 15 (1972), 103-108; M.R., 47, #6960.

13. An integral with basis and its application to trigonometric series, Bull. Fac. Sci. Ibaraki Univ. Ser. A, #5 (1973), 1-8; M.R. 48, #8707.

14. On Romanovski's general Denjoy integral, Bull. Fac. Sci. Ibaraki Univ., Ser. A, #8 (1976), 17-22;

15. Some theorems on the approximately continuous Denjoy integral, Math. Japon., 22 (1977), 289-293, M.R. 57, #3331.

Kurzweil, J.,

1. Generalized ordinary differential equations and continuous dependence on a parameter, Czechoslovak. Math. J., 7 (1957), 418-446; M.R., 22, #2735.
2. On Fubini theorem for general Perron integral, Czechoslovak. Math. J., 23 (1973), 286-297; M.R., 48, #11411.
3. On multiplication of Perron-integrable functions, Czechoslovak. Math. J., 23 (1973), 542-566; M.R., 49, #485.
4. The Perron-Ward integral and related topics, Appendix A, p. 515-533 of Measure and Integral, by K. Jacobs, New York, 1978.

Lebesgue, H.,

1. Sur les intégrales singulières, Ann. Fac. Sci. Univ. Toulouse, (3) 1 (1909), 25-117.
2. Lecons sur l'Intégration et la Recherche des Fonctions Primitives, 2nd Ed., Paris, 1926.
3. Sur la recherche des fonctions primitives, Acta Math., 49 (1926), 245-262.

Lee, C.-M.,

1. An approximate extension of Cesàro-Perron integrals, Bull. Inst. Math. Acad. Sinica, 4 (1976), 73-82; M.R., 54, #484.
2. An analogue of the theorem of Hake-Alexandroff-Loomen, Fund. Math., 100 (1978), 69-74.

Lee, P.-Y.,

1. An equality for variational integrals, Bull. Math. Soc. Nanyang Univ., (1965), 45-47; M.R., 33, #2824.
2. Integrals involving parameters, (I), (II), J. London Math. Soc., 40 (1965), 338-344; 41 (1966), 680-684; M.R., 310, #3956; 33, #2825.
3. A note on some generalisations of the Riemann-Lebesgue theorem, J. London Math. Soc., 41 (1966), 313-317; M.R., 34, #3157.
4. A note on an equality for variational integrals, J. Nanyang Univ., 1 (1967), 263-264; M.R., 38, #1232.
5. Some problems in integration theory, Math. Chronicle, 2 (1973), 105-116; M.R., 51, #826.
6. Uniqueness and convergence theorems in integration theory, Occasional Paper #4, (1976), Lee Kong Chan Inst. Math. Comp. Sci.

Lee, T.-W., 1. On an extension of Fubini's theorem, J. London Math. Soc., (2) 4 (1971), 519-522; M.R., 46, #315.

2. On the connection between Henstock's and Ridder's approaches to generalized Riemann integrals, J. London Math. Soc., (2) 5 (1972), 337-346; M.R., 47, #410.

3. On the generalized Riemann integral and stochastic integral, J. Austral. Math. Soc., 21 (1976), 64-71; M.R., 55, #8294.

Lojásiewicz, S. 1. Sur la valeur et la limite d'une distribution en un point, Studia Math., 16 (1957), 1-35; M.R., 19, 433.

Looman, H., 1. Sur la totalisation des derivées des fonctions continues de plusiers variables, Fund. Math., 4 (1923), 246-285.

2. Über die Perronsche Integral definition, Math. Ann., 93 (1925), 153-156.

Lukašenko, T.P., 1. The functions that are associated with D-H-integrable functions, Izv. Akad. Nauk SSSR Ser. Mat., 35 (1971), 381-407; M.R., 44, #379.

Luzin, N.N. 1. Sur les propriétés de l'intégrale de M. Denjoy, C.R. Acad. Sci. Paris, 155 (1912), 1475-1478.

2. Integral i Trigonometriceskiǐ Ayad, Moscow, 1915.

MacNeille, H.M., 1. A unified theory of integration, Nat. Acad. Sci. U.S.A., 27 (1941) 71-76; M.R., 2, 258.

Malliavin, P., 1. Majorantes et minorantes des fonctions simplement totalisables, C.R. Acad. Sci. Paris, 229 (1949), 286-287; M.R., 11, 90.

Manougian, M.N., 1. On the convergence of a sequence of Perron integrals, Proc. Amer. Math. Soc., 23 (1969), 330-332; M.R., 40, #278.

Marcinkiewicz, J. and Zygmund, A., 1. On the differentiability of functions and summability of trigonometric series, Fund. Math., 26 (1936), 1-43; M.R. 1, 8.

Mařik, J., 1. Foundation of the theory of an integral in Euclidean spaces, Časopis Pěst. Mat., 77, (1951), 1-51, 125-145, 267-30; M.R., 15 (2), 691.

2. Generalized derivative and generalized one dimensional integrals; unpublished notes.

3. Generalized integrals and trigonometric series; unpublished notes

Matyska, J., 1. On β-integration in $E_1$, Czechoslovak. Math. J., 18 (1968), 523-526; M.R., 37, #6413.

McGill, P., 1. Conditions for the Henstock integral to include the Lewis integral, Proc. Roy. Irish Acad. Sect. A, 73 (1973), 275-278; M.R., 48, #11438.

McShane, E.J., 1. On Perron integration, Bull. Amer. Math. Soc., 48 (1942), 718-726, M.R., 4, 75.

2. Integration, Princeton, 1944; M.R., 6, 43.

3. A Riemann Integral that Includes Lebesgue-Stieltjes, Bochner and Stochastic Integrals, Mem. Amer. Math. Soc., 88 (1969); M.R., 42, #436.

4. A unified theory of integration, Amer. Math. Monthly, 80 (1973), 349-359; M.R. 47, #6981.

5. Stochastic Calculus and Stochastic Models, New York, 1974; M.R., 56, #1457.

Men'šov, D.E., 1. Vzaimootnošenie meždu opredeleniyami integrala Borel'ya i Denjoy, Mat. Sborn., 30 (1916), 288-295.

Morrison, T.J., 1. A note on the Denjoy integrability of abstractly valued functions, Proc. Amer. Math. Soc., 61 (1976), 385-386.

Mugalov, A.G., 1. Integration and differentiation under the integral sign in the sense of Cesàro-Denjoy, Izv. Akad. Nauk Azerbaĭdžan. SSR., Fiz-Tehn. Mat. Nauk, (1966), 22-29; M.R., 35, #313.

2. O predel'nom perehode pod znakom integrala v smysle Cesàro-Denjoy, Izv. Akad. Nauk Azerbaĭdžan. SSR., Fiz-Tehn. Mat. Nauk, (1967); 33-41; M.R., 35, #5557.

3. Povtornye integraly, Special Questions in Differential Equations and Function Theory, 70-76; Izdat "Èlm", Baku, 1970; M.R., 43, #2171.

4. O krivolineĭnom integrale v smysle Cesàro-Denjoy, Dokl. Akad. Nauk Azerbaĭdžan. SSR., 32 (1976), 3-5.

Mukhopadhyay, S.N., 1. On the regularity of the $P^n$-integral and its application to summable trigonometric series, Pacific J. Math., 55 (1974), 233-247; M.R., 51, #10546.

Nakanishi, S., 1. L'intégrale de Denjoy et l'intégration au moyen des espaces rangés, I-IV, Proc. Japan. Acad., 32 (1956), 678-683; 33 (1957), 13-18, 265-270; 34 (1958), 96-101; M.R., 19, 256, 1167; 21, #1375.

Nalli, P., 1. Denjoy integral and trigonometric series, Rend. Circ. Math. Palermo, (1915).

Nasibov, M.H., 1. Reconstruction of a function of two variables from its Schwarzian derivative, Izv. Akad. Nauk Azerbaĭdžan. SSR, Fiz.-Tehn. Mat. Nauk, (1964), 25-34; M.R., 29, #2339.

Natanson, I.P., 1. Theory of Functions of a Real Variable, II, 2nd Ed., revised, New York, 1960.

Natanson, I.P. and Natanson, G.I., 1. K vzaimootnošeniyu meždu uzkim i širokim integralami Denjoy, Uspehi Mat. Nauk, 12 (1957), 161-168; M.R., 20, #949.

Nicolescu, L.J., 1. Intégrales Perron-Stieltjes, Acad. P.R. Romîne Bul. Şti. Secţ. Mat. Fiz., 6 (1954), 755-770; M.R., 17, 353.

Pacquement, A., 1. Sur une définition de l'intégrale, généralisant celle de M. Denjoy, C.R. Acad. Sci. Paris, 269 (1969), 1128-1131; M.R., 40, #7398.

2. Sur le calcul inverse de la dérivation approximative, C.R. Acad. Sci. Paris, 271 (1970), 80-83; M.R., 42, #3236.

3. Sur un cas de determination d'une fonction numérique par la connaissance de la derivée approximative, Ann. Univ. Madagascar Sér. Sci. Nature. Math., 7 (1970), 41-47; M.R., 52, #5899.

4. Intégration indéfinie dans $\mathbb{R}^2$, Ann. Univ. Madagascar Sér. Sci. Math., 8 (1971), 7-13; M.R., 52, #10972.

Perron, O., 1. Über den Integralbegriff, Sitzber. Heidelberg, Akad. Wiss. Abt. A. 16 (1914), 1-16.

Pesin, I.N., 1. Classical and Modern Integration Theory, New York, 1970; M.R., 32, #7764.

Pfeffer, W.F., 1. The Perron integral in topological spaces, Časopis Pěst. Mat., 88 (1963), 322-348; M.R., 32, #2546.

2. On a definition of the integral in topological spaces, I, II, Časopis Pěst. Mat., 89 (1964), 129-147, 257-277; M.R., 32, #1318.

3. A note on the lower derivative of a set function and semi-hereditary systems of sets, Proc. Amer. Math. Soc., 18 (1967), 1020-1025; M.R., 36, #1589.

4. On the lower derivates of a function, Canad. J. Math., 20 (1968), 1489-1498; M.R., 38, #1229.

5. An integral in topological spaces, Bull. Amer. Math. Soc., 75 (1969), 433-439; M.R., 39, #400.

6. An integral in topological spaces, I, J. Math. and Mech., 18 (1969), 953-972; M.R., 39, #2938.

7. An integral in topological spaces II, Math. Scand. 27 (1970), 77-104; M.R., 43, #461.

8. Singular integrals are Perron integrals of a certain type, Canad. J. Math., 22 (1970), 260-264; M.R., 41, #1963.

Pfeffer, W.F. and Wilbur, W.J., 1. On the measurability of Perron integrable functions, Pacific J. Math., 34 (1970), 131-144; M.R., 43, #458.

Pu, H.W., 1. A Cauchy criterion and a convergence criterion for Riemann-complete integral, J. Austral. Math. Soc., 13 (1971), 21-24; M.R., 45, #5287.

2. Concerning Riemann-complete integral, Bull. Soc. Roy. Sci. Liége, 40 (1971), 419-423; M.R., 46, #308.

3. Another proof of that Riemann-complete integral includes the Lebesgue integral, Bull. Soc. Roy. Sci. Liége, 41 (1972), 250-251; M.R., 46, #9276.

4. On the derivative of indefinite Riemann-complete integral, Colloq. Math., 28 (1973), 105-110; M.R., 49, #486.

Pych, P., 1. Theorems of Romanovski type for the Denjoy-Perron integral, Comment. Math. Prace. Mat., 16 (1972), 125-131; M.R. 48, #476.

2. The Denjoy integral in some approximation problems, I, Functiones et Approximatio, 1 (1974), 91-105; M.R., 53, #3571.

3. The Denjoy integral in some approximation problems, II, Fasc. Math., (1975), 53-69, M.R., 53, #3572.

4. The Denjoy integral in some approximation problems, III, Proc. Conference Constructive Theory of Functions, Cluj, 1973.

5. Some properties of double Denjoy integrals, Functiones et Approximatio, 2 (1976), 207-217; M.R., 56, #5812.

6. The Denjoy integral in some approximation problems, IV, Ann. Soc. Math. Polonae; Ser. 1, Comm. Math., 19 (1976), -17-125; M.R., 56, #950.

7. Theorems for Denjoy-integrable functions posessing derivatives of positive orders, Functiones et Approximatio, 4 (1976), 61-69; M.R. 56, #7683.

8. The Denjoy integral in some approximation problems, Rev. Anal. Numér. Théorie Approximation, 5 (1976), 79-86; M.R., 57, #10336.

Ridder, J.,

1. Sur un théorème dans la theorie de la totalisation, Nieuw Arch. Wisk., 16 (1929), 76-79.

2. Sur deux classes de fonctions introduites par M. Denjoy, Nieuw Arch. Wisk., 17 (1931), 33-50; Zbl. 2, 20.

3. Quelques théorèmes sur les fonctions primitives, Nieuw Arch. Wisk., 17 (1932), 169-172; Zbl., 4, 206.

4. Über den Perronschen Integralbegriff und seine Beziehung zu den R-, L- und D-Integralen, Math. Z., 34 (1932), 234-2 69; Zbl., 2, 386.

5. Über die Bedingung (N) von Lusin und das allgemeine Denjoysche Integral, Math. Z., 35 (1932); Zbl., 4, 206.

6. Über approximativ stetige Denjoy-Integrale, Fund. Math., 21 (1933), 1-10; Zbl., 8, 109.

7. Über das allgemeine Denjoysche Integral, Fund. Math., 21 (1933), 11-19; Zbl., 8, 109.

8. Der Perronsche Integralbegriff, Math. Z., 37 (1933), 161-169; Zbl., 7, 58.

9. Über die gegenseitigen Beziehungen verschiedener approximativ stetiger Denjoy-Perron Integrale, Fund. Math., 22 (1934), 136-162; Zbl., 9, 57.

10. Über die T- und N-Bedingungen und die approximativ stetigen Denjoy-Perron Integrale, Fund. Math., 22 (1934), 163-179; Zbl., 9, 58.

11. Über Denjoy-Perron Integration von Funktionenzweier Variablen, C.R. Soc. Sci. Lettres Varsovie, 28 (1935), 5-16; Zbl., 13, 7.

12. Über Perron-Stieltjessche und Denjoy-Stieltjessche Integrationen, Math. Z., 40 (1935), 127-160; Zbl. 11, 108.

13. Denjoysche und Perronsche Integration, Math. Z., 41 (1936), 184-199; Zbl., 14, 55.

14. Cesàro-Perron Integration, C.R. Soc. Sci. Lettres Varsovie, 29 (1937), 126-152; Zbl., 17, 60.

15. Über die gegenseitigen Beziehungen einige "trigonometrischen" Integrationen, Math. Z., 42 (1937), 322-336; Zbl., 15, 400.

16. Das spezielle Perron-Stieltjessche Integral, Math. Z., 43 (1938); Zbl. 18, 249.

17. Das allgemeine Perron-Stieltjessche Integral [(PS)-Integration II], Math. Ann., 116 (1938), 76-103; Zbl., 19, 202.

18. Sur la totalisation par rapport à une fonction à variation bornée generalisée, C.R. Acad. Sci. Paris, 209 (1939), 623-629; M.R., 1, 110.

19. Nouvelles propriétés de la totalisation par rapport à une fonction à variation bornée generalisée, C.R. Acad. Sci. Paris, 209 (1939), 670-672; M.R., 1, 110.

20. Denjoy-Stieltjessche und Perron-Stieltjessche Integration im k-dim. Euklidischen Raum, Neiuw Arch. Wisk. (2) 21 (1943), 212-241; M.R., 7, 421.

21. Ueber Definitionen von Perron-Integralen I, II, Indag. Math., 9 (1947), 227-235, 280-289; M.R., 8, 506; 9, 19.

22. Integrationen von Differentialkoeffizienten höherer Ordnung, Indag. Math., 19 (1957), 364-368; M.R., 20, #950.

23. Ein einheitliches Verfahren zur Definitionen von

Absolut-und Bedingt-Konvergenten Integralen, I-IX, Indag. Math., 27 (1965), 1-13, 14-30, 31-39, 165-177, 365-375, 705-745; 28 (1960), 248-257; 29 (1967), 1-7, 8-17, 305-316; 31 (1969), 10-17; M.R., 31 #308; 32, #187, 5836; 35, #4355; 36, #323; 39, #1619.

24. Die allgemeine Riemann-Integration in topologischen Räumen, A-D, Indag. Mat., 30 (1968), 12-23, 137-148, 239-252, 363-376; M.R., 3-7, #1553.

25. Äquivalenzen von Integraldefinitionen im Sinne von Denjoy, von Perron und von Riemann, Indag. Math., 31 (1969), 201-212; M.R., 40, #290.

26. Anwendung von Riemann-Summen in Definitionen von Integralen, Indag. Math., 31 (1969), 309-326; M.R., 41, #1945.

Roger, F., 1. Sur l'extension à l'ordre n des théorèmes de M. Denjoy sur les nombres derivés du premier ordre, C.R. Acad. Sci. Paris, 209 (1939), 11-14; M.R., 1, 47.

Romanovskiĭ, P., 1. Essai d'une exposition de l'intégrale de Denjoy sans nombres transfinis, Fund. Math., 19 (1932), 38-44; Zbl., 5, 392.

2. Intégrale de Denjoy dans les espaces abstraits, Mat. Sb., 51 (1941), 67-120; M.R., 2, 354; M.R., 2, 354.

3. Intégrale de Denjoy dans l'espace à n dimensions, Mat. Sb., 51 (1941), 281-306; M.R., 2, 354.

4. Intégrale relative à un reseau, Mat. Sb., 51 (1941), 309-316; M.R., 2, 354.

Saks, S.,

1. La condition (N) et l'intégrale de MM Denjoy-Perron, Fund. Math., 13 (1923), 218-227.
2. Sur l'intégrale de M. Denjoy, Fund. Math., 15 (1930), 242-262.
3. Sur certaines classes de fonctions continues, Fund. Math., 17 (1931), 124-151; Zbl., 3, 109.
4. Theory of the Integral, 2nd Ed. revised, New York, 1937, Zbl., 17, 300.

Sargent, W.L.C.,

1. A descriptive definition of Cesàro-Perron integrals, Proc. London Math. Soc., (2) 47(1941), 212-247; M.R., 3, 228.
2. On sufficient conditions for a function integrable in the Cesàro-Perron sense to be monotonic, Quart. J. Math. Oxford Ser., 12 (1941), 148-153; M.R., 3, 228.
3. On the integrability of a product, I, II, J. London Math. Soc., 23 (1948), 28-34; 26 (1951), 278-285; M.R., 10, 108; 13, 449.
4. On fractional integrals of a function integrable in the Cesàro-Perron sense, Proc. London Math. Soc. (2) 51 (1948), 46-80; M.R., 10, 516.
5. On the continuity (C) and integrability (CP) of fractional integrals, Proc. London Math. Soc., (2) 52 (1951), 253-270; M.R., 13, 599.
6. On generalized derivatives and Cesàro-Denjoy integrals, Proc. London Math. Soc., (2) 52 (1951), 365-376; M.R., 12, 811.

Sarkhel, D.N., 1. On ω-approximately continuous Denjoy-Stieltjes integral, Colloq. Math., 28 (1973), 111-131; M.R., 49, #487.

2. On ω-approximately continuous Perron-Stieltjes and Denjoy-Stieltjes integral. J. Austral. Math. Soc., 18 (1974), 129-152; M.R., 58, #1055.

3. A criterion for Perron integrability, Proc. Amer. Math. Soc., 7 (1978), 109-112.

Scanlon, H.W., 1. Additivity and indefinite integration for McShane's Perron integral, Proc. Amer. Math. Soc., 39 (1973), 129-134; M.R., 47, #3609.

Schwakik, Š., 1. On the relation between Young's and Kurzweil's concept of Stieltjes integrals, Časopis Pěst. Mat., 98 (1973), 237-251; M.R., 48, #477.

2. On a modified sum integral of Stieltjes type, Časopis Pěst. Mat., 98 (1973), 274-277, 315; M.R., 48, #478.

Sklyarenko, V.A., 1. Svyaz' meždu $(H^k)$-i (P)-integralami, Vestnik Moskov. Univ. Ser. I Mat. Meh., (2) (1971), 69-77.

2. Nekotorye svoĭstva $P^2$-primitivnoĭ, Mat. Zametki, 12 (1972), 693-700; Engl. transl: Math. Notes, 12 (1972), 856-860, (1973); M.R. 47, #8785.

Skvorcov, V.A., 1. Vzaimootnošenie meždu obščim integralom Denjoy i totalizaceĭ $(T_{2s})_o$, Mat. Sb. 52 (1960), 551-558; M.R., 22, #12194.

2. K vzaimootnošeniya meždu D-integration i totalizaceĭ

($T_{2s}$), Vestnik Moskov. Univ. Ser. I Mat. Meh., 6 (1962), 20-25; M.R., 26, #3870.

3. Nekotorye svoǐstva CP-integral, Mat. Sb., 60 (1963), 304-324; Engl. transl. Amer. Math. Soc. Translations, (2) 54 (1966), 231-254; M.R., 27, #264.

4. Ob integrirovanii točnoǐ asimptotičeskoǐ proizvodnoǐ Schwartza, Mat. Sb., 63 (1964), 329-349; M.R. 28, #4073.

5. Vzaimosvyaz' meždu AP-integralom Taylora, $P^2$-integralom Jamesa, Mat. Sb., 70 (1966), 380-393; M.R., 34, #305.

6. Po povodu opredeleniiP$^2$-i SCP-integralov, Vestnik Moskov. Univ. Ser. I Mat. Meh., 21 (1966), 12-14; M.R., 34, #7765.

7. Svyaz' meždu nekotorymi integralami, Vestnik Moskov. Univ. Ser. I Mat. Meh., 22 (1967), 68-72; M.R., 36, #324.

8. Obščnost' $H^k$-integrala Crossa, Vestnik Moskov. Univ. Ser. I. Mat. Meh., 23 (1968), 19-22; M.R., 40, #7714.

9. Nekotorye obobščenie integrala Perrona, Vestnik Moskov. Univ. Ser. I Mat. Meh., 24 (1969), 48-51; M.R., 40, #5802.

10. Integral Marcinkiewicz-Zygmund i ego isvyaz' i SCP-integralom Burkillya, Vestnik Mosk. Univ. Ser. I Mat. Meh., 27 (1972), 78-82; Engl. transl. Moscow Univ. Bull. 27 (1972), 66-70, (1973); M.R., 49, #5251.

Solomon, D.W., 1. On a constructive definition of the restricted Denjoy integral, Trans. Amer. Math. Soc., 128 (1967), 248-256;

M.R., 35, #1728.

2. Denjoy Integration in Abstract Spaces, Mem. Amer. Math. Soc., #85, (1969); M.R., 39, #404.

3. On separation in measure and metric density in Romanovski spaces, Duke Math. J., 36 (1969), 81-90; M.R., 38, #6019.

4. On non-measurable sets, Duke Math. J., 36 (1969), 183-192; M.R., 39, #7050.

Sunouchi, G. and Utagawa, M., 1. The generalized Perron integrals, Tôhoku Math. J., 1 (1949), 95-99; M.R., 11, 90.

Taylor, S.J., 1. An integral of Perron's type defined with the help of trigonometric series, Quart. J. Math. Oxford Ser., (2) 6 (1955), 255-274; M.R., 19, 255.

Thomson, B.S., 1. Constructive definition for non-absolutely convergent integrals, Proc. London Math. Soc., (3) 20 (1970), 699-716; M.R., 42, #3248.

2. Construction of measures and integrals, Trans. Amer. Math. Soc., 160 (1971), 287-296; M.R., 43, #6385.

3. On the Henstock strong variational integral, Canad. Math. Bull., 14 (1971), 87-91; M.R., 45, #8799.

4. A theory of integration, Duke Math. J., 39 (1972), 503-510; M.R., 46, #3736.

5. On McShane's vector-valued integral, Duke Math. J., 39 (1972), 511-519; M.R., 46, #3742.

Tolstov, G.P., 1. Sur l'intégrale de Perron, Mat. Sb., 5 (1939), 647-659; M.R., 1, 208.

2. La méthode de Perron pour l'intégrale de Denjoy, Mat. Sb., 8 (1940), 149-167; M.R., 2, 232.

3. Nespravedlivost' teoremy Fubini dlya mnogomernogo regulyarnogo integrala Denjoy, Mat. Sb., 24 (1949), 263-278; M.R., 10, 690.

4. Integral kak primitivnaya, Dokl. Akad. Nauk SSSR, 73 (1950), 659-662; M.R., 12, 167.

5. O krivolineinom i povtornom integrale, Trudy Mat. Inst. Steklov., 35 (1950), 1-101; M.R., 13, 448.

6. Majorants and minorants in various problems, Studies

of Modern Problems of Constructive Theory of Functions, 308-311, Fizmatgiz, Moscow, 1961; M.R., 32, #2535.

7. The definition of the restricted Denjoy integral by classical means, Studies of Modern Problems of Constructive Theory of Functions, 312-313, Fizmatgiz, Moscow, 1961; M.R., 32, #2548.

8. Parametričeskoe differencirovanie i uzkiĭ integral Denjoy, Mat. Sb., 53 (1961), 387-392; M.R., 25, #3144.

9. Proizvodnaya i integral s obscei tocki zreniya, Dokl. Akad. Nauk SSSR, 142 (1962), 1040-1042; Engl. transl: Soviet Math., 3 (1962), 253-255; M.R., 25, #154.

10. Tri tipe abstraktnyh integralov, Dokl. Akad. Nauk SSSR, 158 (1964), 536-539; Engl. Transl: Soviet Math., 5 (1965), 1275-1278; M.R., 30, #2118.

11. Proizvodnaya i integral (aksiomatičeskiĭ podhod), Mat. Sb., 66 (1965), 608-630; M.R., 30, #4895.

Trjitzinsky, W.J., 1. Les Problèmes de Totalisation se rattachant aux Laplaciens Non-Sommables, Mémor. Sci. Math., 125 (1954); M.R., 16, 471.

2. Les Laplaciens généralisees non-sommables, J. Math. Pures Appl., 34 (1955), 1-136; M.R., 20, #3389.

3. Totalisations dans les Espaces Abstraits, Mémor. Sci. Math., 155 (1963); M.R., 28, #4078.

4. La Totale-D de Denjoy et la Totale-S Symétrique, Mémor. Sci. Math., 166 (1968); M.R., 39, #1604.

5. Totalisations abstraites dans espaces vectoriels, Ann. Mat. Pura Appl., 82(1969), 275-379; M.R., 55, #8274.

Verblunsky, S. 1. On the theory of trigonometric series, VII, Fund. Math., 23 (1934), 193-236; Zbl., 10, 19.

2. On a descriptive definition of Cesàro-Perron integrals, J. London Math. Soc., (2) 2 (1971), 326-333; M.R., 44, #4161.

Ward, A.J., 1. The Perron-Stieltjes integral, Math. Z., 4 (1936), 578-604; Zbl., 14, 397.

Whyburn, W.M., 1. On the integration of unbounded functions, Bull. Amer. Math. Soc., 38 (1932), 123-131; Zbl., 4, 55.

Young, W.H., 1. On non-absolutely convergent, not necessarily continuous integrals, Proc. London Math. Soc., (2) 16 (1918), 175-218.

Zygmund, A., 1. Trigonometric Series, I, II, Cambridge, 1959; M.R., 21, #6498.

Mathematics Department
The University of British Columbia
#121 - 1984 Mathematics Road
Vancouver, British Columbia V6T 1Y4
Canada