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A SURVEY OF MY JOINT WORK WITH CASPER GOFFMAN

In "Basic sequences in the space of measurable functions" (PAMS, 1960) we gave a very short proof of a result of Talalyan. We showed that if $\{x_n\}$ is a total sequence in M , the space of measurable real functions on $[0,1]$, then there is a "universal" series $\{a_n x_n\}$ such that $f \in M$ implies the existence of $N_k \nearrow$ such that $\sum_1^{N_k} a_n x_n \rightarrow f$ a.e. . In our proof we showed that the result follows from the fact that the dual of M is trivial. In "A remark concerning universal series" (J.M. Anal. & Appl., 1972) we extended this result to nonatomic σ -finite measure spaces.

In "On upper and lower limits in measure" (FM. 1960) we gave a new definition for these limits, first defined by Menchoff. Our definition was based on the consideration of the space of measurable functions as a linear lattice. The properties of these limits can be easily deduced from our definition.

In "Approximately continuous transformations" (PAMS, 1961) we discussed the density topology on R^n and showed that approximately continuous transformations from R^n to a metric space have a generalized Darboux property.

The paper "Functions whose Fourier series converge for every change of variable" (PAMS, 1968) gave a characterization of a continuous function f such that the Fourier series of $f \circ h$ converges everywhere for every homeomorphism h of $[0, 2\pi]$ onto itself. In a later paper (JLMS, 1975) we extended this result to summable f showing that, except on a set of universal measure zero, f must equal a function which satisfies the condition of the previous paper.

A function of bounded variation can be written as the sum of a continuous function and the "function of the jumps". In "The structure of regulated functions" (with G. Moran, PAMS 1970) we showed that this can be extended to regulated functions if they are sufficiently discontinuous. We also showed that the set of regulated functions is the union of all classes of ϕ -BV functions.

In "The localization principle for double Fourier series" (submitted) we define a notion of generalized bounded variation for real functions on \mathbb{R}^n which appears to solve the localization problem for $n = 2$. A note describing this result has appeared (PNASUSA, 1978). In "A remark on the space $V_{\Lambda, \alpha}^p$ " (with Fon-Che Liu, submitted) we solve a problem concerning the measurability of certain functions which appear in the definition of the spaces introduced in our study of localization.

In concluding, I made the following remarks:

When Casper arrived at Purdue, a relationship began which has had a profound effect on my life.

Before I began working with Cas, I was primarily interested in Fourier Analysis from a function theoretic point of view. I viewed Real Analysis as a tool, essential and interesting but of secondary importance. This attitude changed radically with the first things Cas and I did together.

Working with Cas was an eye-opening experience for me in still another way. Anyone who has discussed mathematics with Cas knows how fertile and inventive his mind can be. I found that through our association my own abilities began to develop and broaden. I began to be able to see vital interconnections between areas I had thought of as separate compartments and to understand how new concepts and workable hypotheses about them are formulated.

Cas has always been very generous with his time and his aid to his students and friends. In fact, Cas has always been generous with everything he had to share. I can recall many students and colleagues who thought of themselves as almost part of his family and of his home as their second home. For some of them it was their only home.

Cas, on his part, was fiercely protective and supportive

toward his students and friends. I have seen him worry and hover over them with all the fussiness of a mother hen and, at times, protect them with all the ferocity of a mother lion. Those of you who were his students are not as aware of this as I am; usually he hid his concern from you, but he would tell me about it and discuss how he planned to help you.

Ferocity and toughness may seem strange words to apply to Cas if you have never seen these aspects of his personality. They are only rarely shown. But when he perceives events as a threat to his students, friends, or family, or when he feels that some important principle is being compromised, he can become very tough indeed.

I am sure that some of you have seen him in action on such an occasion and more of you have seen the strange phenomenon which occurs afterward. He will then step back and regard his own behavior with some surprise and say "You know, I'm really a tough guy".

Cas is a multifaceted personality far beyond my ability to describe or explain. In my life he has been an enormous force for good and I am just grateful that he was there. I am sure that he has had some similar effect on the lives of many of you. In my case perhaps the effect has been greater because I've been privileged to have known him better.

Thanks for everything Cas.