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# Derivates for Symmetric Functions

A real-valued function f defined on $I_0 = [0, 1]$ is said to be symmetric if for each $x \in I_0^o = (0, 1)$, $f(x+h) + f(x-h) - 2f(x) = o(1)$ as $h \to 0$. In [3], Neugebauer has studied the relation between continuity and symmetry and discovered properties that symmetric and continuous functions have in common. In particular, he has proved that if f is measurable and symmetric on $I_0$, then $\{x: \bar{f}^-(x) \neq \bar{f}^+(x)$ or $\underline{f}^-(x) \neq \underline{f}^+(x)\}$ is a set of the first category. This is an extension of a theorem obtained by him [2]. The purpose for the paper [5] is to prove that the sets $\{x: \bar{f}_{ap}^-(x) < \bar{f}^+(x)$ or $\bar{f}_{ap}^+(x) < \bar{f}^-(x)\}$ and $\{x: \bar{f}^s(x) \neq \bar{f}^+(x)\}$ are of the first category if f is measurable and symmetric. It follows easily from this work that

$$\bar{f}_{ap}^s(x) \leq \bar{f}_{ap}^+(x) = \bar{f}_{ap}^-(x) = \bar{f}^+(x) = \bar{f}^-(x) = \bar{f}^s(x)$$

holds except possibly for a set of the first category. This observation for continuous functions has been noted by Evans and Humke [1]. Here, $\bar{f}_{ap}^-(x)$, $\bar{f}_{ap}^+(x)$, $\bar{f}_{ap}^s(x)$, $\bar{f}^-(x)$, $\bar{f}^+(x)$, $\bar{f}^s(x)$ denote the various upper derivates of f at x. For these definitions we refer the reader to [6].

Theorem 1. <u>The set</u> $\{x: \bar{f}^-_{ap}(x) < \bar{f}^+(x)$ or $\bar{f}^+_{ap}(x) < \bar{f}^-(x)\}$ <u>is a set of the first category</u>.

Theorem 2. <u>The set</u> $\{x: \bar{f}^s(x) \neq \bar{f}^+(x)\}$ <u>is of the first category</u>.

The proof for these theorems is more complicated than the one used for continuous case in [4]. It is heavily based on the fact that the set $\{x: x \in I_0^o$ , f is continuous at $x\}$ has full measure on $I_0$ and its complement relative to $I_0$ is a set of the first category [3]. From Theorem 1, we conclude that $\{x: \bar{f}^-_{ap}(x) \neq \bar{f}^+_{ap}(x)\}$ is a set of the first category.

## References


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