Real Analysis Exchange Vol. 3 (1977-78)
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A Symmetric Condition for Monotonicity

If f is a real valued function defined on the real line R and if $x \in R$, the lower symmetric derivate of f at x is defined by

$$\underline{f}^s(x) = \liminf_{h\to 0} \frac{f(x+h) - f(x-h)}{2h}.$$

The upper symmetric derivate $\overline{f}^s(x)$ is defined analogously by replacing lim inf by lim sup. C.E. Weil [2] recently proved that if f is a Baire class one, Darboux function with $\underline{f}^s(x) \geq 0$ for each x, then f must be nondecreasing. The question arises concerning the necessity of the Baire class one, Darboux assumption. The purpose of the paper [1] being abstracted here is to characterize the class of measurable functions for which the non-negativity of $\underline{f}^s$ is sufficient to insure monotonicity. This class is denoted by M_{-1} to be somewhat consistant with Z. Zahorski's [3] notation for classes of functions and is defined as follows:

DEFINITION. *A function f: $R \to R$ is said to belong to class M_{-1} if it is measurable and has the property that for each x*

$$\liminf_{t\to x} f(t) \leq f(x) \leq \limsup_{t\to x} f(t).$$

THEOREM. *Let* f: R→R *be a function such that* $\underline{f}^s(x) \geq 0$ *for each* x. *Then* f *is nondecreasing if and only if* f *belongs to class* M_{-1}.

This result can then easily be generalized to yield

COROLLARY. *Let* f: R→R *be a function such that* $\underline{f}^s(x) > -\infty$ *for each* x *and* $\overline{f}^s(x) \geq 0$ *for almost every* x. *Then* f *is nondecreasing if and only if* f *belongs to class* M_{-1}.

Several straightforward consequences of these results are also presented in [1].

REFERENCES

[1] M. J. Evans, *A symmetric condition for monotonicity*, Bull. Inst. Math. Acad. Sinica (to appear).

[2] C. E. Weil, *Monotonicity, convexity, and symmetric derivates*, Trans. Amer. Math. Soc. 221(1976), 225-237.

[3] Z. Zahorski, *Sur la primiere derivee*, Trans. Amer. Math. Soc. 69(1950), 1-54.

Received April 6, 1978