Richard J. O'Malley, Department of Mathematics,
University of Wisconsin, Milwaukee, Wisconsin 53201

Selective Derivates

For each nondegenerate closed subinterval, I, of [0,1] pick a point from the interior of I and label it P_I. The collection of points obtained by this process is called a selection. For any fixed selection S we can define the selective Dini derivates of any function f: $[0,1] \to R$. For example, the lower bilateral selective derivate of f at x is

$$\liminf_{h \to 0} \frac{f(P_{[x,x+h]})-f(x)}{P_{[x,x+h]}-x} = {}_{S}f'(x).$$

In this paper the various selective derivates and finite selective derivative are studied. It is found that the bilateral selective derivates and finite selective derivative yield some information about functions but the one sided selective derivates are basically pathological.

Among other things it is found that

1) If f: $[0,1] \to R$ has ${}_{S}f' > 0$ for a fixed selection, then f is increasing.
2) If f: $[0,1] \to R$ has $-\infty < {}_{S}f' \leq {}^{S}f' < +\infty$ then f is Baire 1, Darboux.
3) Even if f is Baire 1, Darboux ${}_{S}f'(x)$ can be non-measurable.
4) If f: $[0,1] \to R$ has a finite selective derivative, sf', then:

a) There is a sequence of closed sets E_n whose union is [0,1] on each of which f is continuous relative to E_n.

b) The function has an approximate derivative $f'_{ap}=sf'$ for a.e. x.

c) The selective derivative has the Darboux property.

5) Even if f has a finite selective derivative, sf', this selective derivative need not be Baire 1, though it is measurable.

6) Under certain conditions on the selection a finite selective derivative is Baire 2. However it is not known whether a finite selective derivative is always Baire 2.

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