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Dense Extendable Connectivity Functions

A function $F : X \to Y$ is a *connectivity* function if for every connected subset C of X, the graph of the restriction, $F|C$, is connected. A function $g : I \to I$, where $I = [0,1]$, is *extendable* if there exists a connectivity function $G : I \times I \to I$ such that $G(x,0) = g(x)$ for all $x \in I$. Brown [1] has characterized dense connectivity functions in terms of negligible sets for connectivity functions. In the theorem below, we give an analogous characterization of dense extendable functions. Suppose $g : I \to I$ is an extendable function. We say that a subset M of I is *g-negligible* if $f : I \to I$ is extendable whenever $f = g$ on $I \setminus M$.

Theorem 1 *If $g : I \to I$ is extendable, then the following are equivalent:*

(i) The graph of g is dense in $I \times I$.

(ii) Every nowhere dense subset M of I is g-negligible.

(iii) There exists a G_δ subset A of I which is g-negligible.

According to Bruckner [2], a class K of real valued functions f defined on an interval is said to be characterized in terms of associated sets if there is a family P of subsets of $\mathbb{R}$ such that $f \in K$ if and only if for each $\alpha \in \mathbb{R}$, the *associated* sets $E^\alpha(f) = \{x : f(x) < \alpha\}$ and $E_\alpha(f) = \{x : f(x) > \alpha\}$ belong to P. Cristian and Tevy [3] used Brown's result to show that the class of connectivity functions $g : I \to I$ cannot be characterized in terms of associated sets.

Question 1 *Can the class of extendable functions be characterized by associated sets?*

References

[1] J. B. Brown, *Negligible sets for real connectivity functions*, Proc. Amer. Math. Soc. **24** (1970), 263–269.

[2] A. M. Bruckner, *On characterizing classes of functions in terms of associated sets*, Canad. Math. Bull. **10** (1967), 227–237.

[3] B. Cristian and I. Tevy, *On characterizing connected functions*, Real Anal. Exch. **6** (1980–81), 203–207.