Real Analysis Exchange
Vol. 18(1), 1992/93, pp. 38–39

Tamás Keleti *, (student at the Eötvös Loránd University, Budapest) , Puli sétány 21, Budapest, H-1213, Hungary; e-mail: elek@ludens.elte.hu

The Mountain Climbers' Problem and the Complexity of Real Continuous Functions

The first part of this talk investigated the following problem:

> Two mountain climbers begin at sea level, at opposite ends of a (two-dimensional) chain of mountains. Can they find routes along which to travel, always maintaining equal altitudes, until they eventually meet?

If we now select a point of maximum altitude and reparametrize, we can formulate it as follows:

> **Problem 1)** Let f and g be continuous functions mapping $[0,1]$ to $[0,1]$ with $f(0) = g(0) = 0$ and $f(1) = g(1) = 1$. Are there continuous functions k and h: $[0,1] \to [0,1]$ satisfying $k(0) = h(0) = 0$, $k(1) = h(1) = 1$ and $f \circ k = g \circ h$?

J. V. Whittaker showed in [2] that the answer is "*yes*" if f and g are piecewise monotone but "*no*" in general. It is easy to construct a counter-example: let f be a monotone function which is constant in an interval, and let g be a function which oscillates around this value.

However, typical countinuous functions are climbable. If we assume that neither f nor g have an interval of constancy then the answer for Problem 1) is "*yes*". This is the main result[1] and in the talk we sketched the elementary proof. (The theorem and the proof are going to appear in [1].)

The talk also considered the following situation.

> Let $\mathcal{F} = \{f | f : [0,1] \to [0,1]$ continuous, $f(0) = 0$, $f(1) = 1$ and f has no interval of constancy $\}$. Let f and $g \in \mathcal{F}$. We will say that g is more complex than f or f is simpler than g (notation: $f \preceq g$) if there exists an $h \in \mathcal{F}$ such that $g = f \circ h$. We say that f is equivalent to g

*The author's participation in the conference was sponsored by the Pro Renovanda Cultura Hungariae Foundation.

[1] After the conference the author found this result in the paper of T. Homma, *A theorem on countinuous functions*, Ködai Math. Sem. Reports 1 (1952), p.13-16.

if $f \preceq g$ and $g \preceq f$. (This is the case when h is strictly increasing.) The $\preceq$ relation is a partial ordering over these equivalence classes.

The main result of the first part shows that any two element have a common upper bound. It is obvious that any two elements have a common lower bound, namely the identity function is simpler than any function in $\mathcal{F}$.

Problem 2) What else can we say about this quite natural structure? Is there least upper and (or) greatest lower bound for any 2 elements? In other words: is this a lattice?

References

[1] T. Keleti, *The Mountain Climbers' Problem*, Proc. Amer. Math. Soc., to appear

[2] J. V. Whittaker, *A Mountain-Climbing Problem*, Can. J. Math 18 (1966), 873-882