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## Decreasing Sequences of $\sigma$ -Fields

Let  $X$  be a Polish space,  $m$  a Borel probability measure on  $X$ ,  $\{\mathcal{A}_n\}_n$  a decreasing sequence of countably  $\sigma$ -generated  $\sigma$ -fields of Borel sets of  $X$ . A martingale  $(f_n, \mathcal{A}_n)_n$  is "adapted" to the sequence  $\{\mathcal{A}_n\}_n$  if  $f^{-1}(\mathcal{B}) = \mathcal{A}_n$  for each  $n$ , where  $\mathcal{B}$  denotes the family of Borel sets of  $\mathbb{R}$ . In the simple case in which  $X$  is a measuretheoretic product and the  $\mathcal{A}_n$ 's are the tail-fields, it is easily seen that there always exists an adapted martingale. As a further step to the general case, we settle the case of two fields; this leads to a solution for any *finite* decreasing sequence of fields of the stated type.