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More on Intersections of Continuous Functions and Smooth Functions

We present some improvements of known theorems and examples concerning intersections of continuous functions with smooth functions or intersections of smooth functions with smoother functions. We are particularly concerned with our ability to force the projection of the intersection to have an uncountable intersection with a given set M which is either large in measure or in category (or both). In particular, we consider such questions as they regard (1) the $C-C^1$ Intersection Theorem (i.e. every continuous function has an uncountable intersection with some C^1 function) of Laczkovich [3] and Agronsky, Bruckner, Laczkovich, and Preiss [1], (2) the Lip^1-C^1 Intersection Theorem of Federer [2], (3) the C^1-C^2 Intersection Theorem of Olevskii [4], and (4) the $C^{1+1}-C^2$ Intersection Theorem of Whitney [5], where C^{1+1} is the class of C^1 functions which have Lip^1 derivatives.

References

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- [4] A. Olevskii, *Ulam-Zahorski problem on free interpolation by smooth functions*, preprint, see summary in Real Anal. Exch. **16** (1990-91), 363-372.
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