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The Fractal Analysis of Products and Projections of Measures

Given a Borel measure μ in $\mathbb{R}^d$, Cutler [1,2] showed that

$$\check{\mu}(x) = \liminf_{r\downarrow 0} \frac{\log \mu B(x,r)}{\log r}, \quad \hat{\mu}(x) = \limsup_{r\downarrow 0} \frac{\log \mu B(x,r)}{\log r}$$

relate directly to the Hausdorff and packing dimensions of measure theoretic supports for μ. We say that μ is a *fractal measure* if $\check{\mu}(x) = \hat{\mu}(x)$ for μ a.e. x. Using known and new results about the dimension properties of Cartesian products of sets and projections onto subspaces, we find the corresponding results for measures. In particular, if μ_1, μ_2 are Borel measures in $\mathbb{R}$ and $\mu = \mu_1 \times \mu_2$, then μ_1, μ_2 fractal implies that $\mu_1 \times \mu_2$ is fractal. Also, if μ_θ denotes the measure in $\mathbb{R}$ obtained by projecting μ in $\mathbb{R}^2$ onto a straight line of direction θ, then μ fractal implies that μ_θ is fractal for a.e. θ. These results are a corollary to an analysis of the connection between fractal properties of the support sets for μ and those for μ_θ; they extend results of Haase [3].

References

[1] C. D. Cutler, *The Hausdorff dimension distribution of finite measures in Euclidean space*, Can. J. Math. **38** (1986), 1459-1484.

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[3] H. Haase, *On the dimension of product measures*, Mathematika **37** (1990), 316-232.