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KENMOTSU TYPE REPRESENTATION FORMULA FOR SPACELIKE SURFACES IN THE DE SITTER 3-SPACE

By

Reiko AIYAMA and Kazuo AKUTAGAWA

Introduction

In [10], Kenmotsu proved that surfaces in the Euclidean 3-space $\boldsymbol{E}^3$ can be represented by means of the mean curvature and the Gauss map. In [3] and [4], we gave the Kenmotsu type representation formulas for surfaces in the hyperbolic 3-space (cf. [11]) and the Riemannian 3-sphere. For each Riemannian 3-space form N^3 and a surface M^2 in N^3, we can consider an adapted frame on M^2 as a map from M^2 to the isometry group $\mathrm{Isom}(N^3)$. The 'Gauss map' of M^2 to $\boldsymbol{S}^2(=SO(3)/SO(2))$ is defined from the 'rotational part' (i.e., $SO(3)$-part) of the adapted framing map. (For example, $\mathrm{Isom}(\boldsymbol{E}^3)=\boldsymbol{R}^3\rtimes SO(3)$.)

On the other hand, Nishikawa and the second author [8] proved the Lorentzian version of the Kenmotsu representation formula for spacelike surfaces in the Minkowski 3-space $\boldsymbol{L}^3$ (cf. [12]). Here $\mathrm{Isom}(\boldsymbol{L}^3)=\boldsymbol{R}^3\rtimes SO_0(1,2)$ and hence the Gauss map is a map to the upper hyperboloid $\boldsymbol{H}^2$ $(=SO_0(1,2)/SO(2))$. In this paper, we introduce the Kenmotsu type representation formula for spacelike surfaces in the Lorentzian 3-space form of constant curvature 1, that is, the de Sitter 3-space $\boldsymbol{S}_1^3$. A similar formula in the anti-de Sitter 3-space has been already given in [6].

1. De Sitter 3-space $\boldsymbol{S}_1^3$

The de Sitter 3-space $\boldsymbol{S}_1^3$ is defined as the semi-sphere in the Minkowski 4-space $\boldsymbol{L}^4$ of radius 1. As in [9] and [1], it is convenient to use the complex special linear group $SL(2;\boldsymbol{C})$, which is the double cover of $SO_0(1,3)$, as the group of isometries of $\boldsymbol{S}_1^3$. Put

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$$\underline{e_0} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \underline{e_1} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

$$\underline{e_2} = \sqrt{-1}J = \sqrt{-1}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \underline{e_3} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Identify $\boldsymbol{L}^4$ with the space $\mathrm{Herm}(2) = \{\underline{x} = x_0\underline{e_0} + \cdots + x_3\underline{e_3} \mid x_0, \dots, x_3 \in \boldsymbol{R}\}$ of 2×2 Hermitian matrices with the metric $\langle \underline{x}, \underline{x} \rangle = -\det \underline{x}$. $SL(2; \boldsymbol{C})$ acts isometrically on $\boldsymbol{L}^4$ by

$$g \cdot \underline{x} = g\underline{x}g^* \quad (g \in SL(2; \boldsymbol{C}), \underline{x} \in \boldsymbol{L}^4 = \mathrm{Herm}(2)).$$

Hence it acts on $\boldsymbol{S}_1^3$ isometrically and transitively. Then we can regard $\boldsymbol{S}_1^3$ as the symmetric space

$$\boldsymbol{S}_1^3 = SL(2; \boldsymbol{C})/SU(1,1) = \{g\underline{e_3}g^* | g \in SL(2; \boldsymbol{C})\},$$

where $SU(1,1) = \{h \in SL(2; \boldsymbol{C}) \mid h\underline{e_3}h^* = \underline{e_3}\}$.

Divide $SL(2; \boldsymbol{C})$ into three subsets G_-, G_0, G_+ according to the signature of the indefinite Hermitian metric $\langle g_2, g_2 \rangle_{\boldsymbol{C}_1^2} = g_{21}\overline{g_{21}} - g_{22}\overline{g_{22}}$ for the second row complex vector $g_2 = (g_{21}, g_{22})$ of $g \in SL(2; \boldsymbol{C})$. Then we can also divide $\boldsymbol{S}_1^3$, which is diffeomorphic to $S^2 \times \boldsymbol{R}$, into three components as follows:

$$S_- = \{g\underline{e_3}g^* | g \in G_-\} = \{\underline{x} \in \boldsymbol{S}_1^3 \mid x_0 - x_3 < 0\} (\cong \boldsymbol{R}^3),$$

$$S_0 = \{g\underline{e_3}g^* | g \in G_0\} = \{\underline{x} \in \boldsymbol{S}_1^3 \mid x_0 - x_3 = 0\} (\cong S^1 \times \boldsymbol{R}),$$

$$S_+ = \{g\underline{e_3}g^* | g \in G_+\} = \{\underline{x} \in \boldsymbol{S}_1^3 \mid x_0 - x_3 > 0\} (\cong \boldsymbol{R}^3).$$

Take a coordinate (y_0, y_1, y_2) on $S_\mp$ defined by $(y_0, y_1, y_2) = (1, x_1, x_2)/|x_0 - x_3|$, the metric on $\boldsymbol{S}_1^3$ is written as $ds^2 = (1/y_0^2)\, ds_0^2$, where $ds_0^2 = -dy_0^2 + dy_1^2 + dy_2^2$ is the Minkowski metric. We denote by $\boldsymbol{RS}_1^3$ the upper half space model $(\boldsymbol{R}_+^3, ds^2)$ of each $S_\mp \subset \boldsymbol{S}_1^3$.

The Gram-Schmidt procedure for row complex vectors of each matrix $g \in SL(2; \boldsymbol{C})$ with respect to the indefinite Hermitian metric $\langle \cdot, \cdot \rangle_{\boldsymbol{C}_1^2}$ gives the decomposition

$$G_- = S \cdot SU(1,1) \quad \text{and} \quad G_+ = S \cdot J \cdot SU(1,1), \tag{1.1}$$

where S is the Lie subgroup consisting of upper triangular matrices

$$\begin{pmatrix} a & \zeta \\ 0 & 1/a \end{pmatrix} \quad (a > 0, \zeta \in \boldsymbol{C}).$$

Then we can identify each component S_-, S_+ with S, that is,

$$S_- = \{\mathrm{s}\underline{\mathrm{e}_3}\mathrm{s}^* | \mathrm{s} \in S\}, \quad S_+ = \{-\mathrm{s}\underline{\mathrm{e}_3}\mathrm{s}^* | \mathrm{s} \in S\}.$$

Note that $S(\cong S_\mp)$ is diffeomorphic to $\boldsymbol{RS}_1^3$ under the map

$$\boldsymbol{RS}_1^3 \ni (y_0, y_1, y_2) \mapsto \begin{pmatrix} \sqrt{y_0} & \mp(y_1 + \sqrt{-1}y_2)/\sqrt{y_0} \\ 0 & 1/\sqrt{y_0} \end{pmatrix} \in S.$$

2. Normal Gauss Maps of Spacelike Surfaces in $\boldsymbol{S}_1^3$

Let f be a conformal immersion from a Riemann surface M into $\boldsymbol{S}_1^3$, whose image is a spacelike surface in $\boldsymbol{S}_1^3$. We can choose an adapted framing $\mathscr{E} : M \to SL(2; \boldsymbol{C})$ of f locally (that is, on each contractible neighborhood) and uniquely up to a right multiplication of $U(1)$-valued map. This implies that $f = \mathscr{E}\underline{\mathrm{e}_3}\mathscr{E}^*$, $\mathscr{E}\underline{\mathrm{e}_0}\mathscr{E}^*$ is a unit normal vector field of f and $\mathscr{E}(\underline{\mathrm{e}_1} - \sqrt{-1}\underline{\mathrm{e}_2})\mathscr{E}^*$ is a vector field of type $(1,0)$, where

$$U(1) = \left\{ \begin{pmatrix} e^{\sqrt{-1}\theta} & 0 \\ 0 & e^{-\sqrt{-1}\theta} \end{pmatrix} \middle| \theta \in \boldsymbol{S}^1 \right\}.$$

We define the *normal Gauss map* $\mathscr{G} : M \to \hat{\boldsymbol{C}} := \boldsymbol{C} \cup \{\infty\}$ by

$$\mathscr{G} = \frac{\mathscr{E}_{21}}{\overline{\mathscr{E}_{22}}}, \quad \text{where } \mathscr{E} = \begin{pmatrix} \mathscr{E}_{11} & \mathscr{E}_{12} \\ \mathscr{E}_{21} & \mathscr{E}_{22} \end{pmatrix}.$$

It should be pointed out that the normal Gauss map $\mathscr{G}$ is globally defined on M. On the open set $U_- := f^{-1}(S_-)$ (resp. $U_+ := f^{-1}(S_+)$) in M, the image of $\mathscr{G}$ is contained in the unit open disk $D := \{z \in \boldsymbol{C} | |z| < 1\}$ (resp. in $\hat{\boldsymbol{C}} \backslash D$). Then $\mathscr{G}(f^{-1}(S_0)) \subset \boldsymbol{S}^1 = \partial D$. We also remark that the union $U_- \cup U_+$ is an open dense subset in M.

As mentioned in Introduction, the normal Gauss map $\mathscr{G}$ of f is also obtained from the 'rotational part' of the adapted framing $\mathscr{E}$ as follows: The upper and lower hyperboloids $\boldsymbol{H}_\pm^2$ in the linear space $\boldsymbol{R}^3$ are given by

$$\boldsymbol{H}_\pm^2 = \{\underline{\mathrm{x}} = x_0\underline{\mathrm{e}_0} + x_1\underline{\mathrm{e}_1} + x_2\underline{\mathrm{e}_2} \,|\, \det \underline{\mathrm{x}} = 1, \mathrm{sgn}(x_0) = \pm 1\}.$$

The subgroup $SU(1,1)$ in $SL(2; \boldsymbol{C})$ acts transitively on each hyperboloid $\boldsymbol{H}_\pm^2$, and then $\boldsymbol{H}_\pm^2 = SU(1,1)/U(1)$. Decomposing $\mathscr{E}|_{U_\mp} : U_\mp \to G_\mp$ corresponding to the decomposition (1.1) of $G_\mp$, we obtain an $SU(1,1)$-valued map h and an S-valued map $\mathscr{S}$ (defined locally) on each $U_\mp$:

$$\mathscr{E}|_{U_-} = \mathscr{S}h, \quad \mathscr{E}|_{U_+} = \mathscr{S}Jh. \tag{2.1}$$

By using h, $\mathscr{G}_\mp : U_\mp \to \boldsymbol{H}_\pm^2$ is determined as follows:

$$\mathcal{G}_- = h\underline{e}_0 h^*, \quad \mathcal{G}_+ = -\underline{e}_1 h\underline{e}_0 h^*\underline{e}_1.$$

We denote by P the stereographic projection of $\boldsymbol{H}^2_+ \cup \boldsymbol{H}^2_-$ from the south pole $-\underline{e}_0 \in \boldsymbol{H}^2_-$. Then the normal Gauss map $\mathcal{G}$ on $U_\mp$ is just $P \circ \mathcal{G}_\mp$:

$$\mathcal{G} = \begin{cases} P \circ \mathcal{G}_- = p/q : U_- \to D, \\ P \circ \mathcal{G}_+ = q/p : U_+ \to \hat{\boldsymbol{C}} \backslash D, \end{cases} \quad \text{where } h = \begin{pmatrix} q & \bar{p} \\ p & \bar{q} \end{pmatrix}.$$

On each $U_\mp$, $\mathcal{G}$ can be also interpreted geometrically as follows: Consider $f|_{U_-}$ (resp. $f|_{U_+}$) to be a conformal immersion into $\boldsymbol{RS}^3_1 = (\boldsymbol{R}^3_+, ds^2)$ and $\boldsymbol{RS}^3_1$ to be a conformally embedded domain $\boldsymbol{R}^3_+$ in the Minkowski 3-space $\boldsymbol{L}^3 = (\boldsymbol{R}^3, ds_0^2)$. Let $N(z)$ be the future-pointing (resp. past-pointing) unit normal timelike vector at each point $f(z)$ in $\boldsymbol{L}^3$. Parallel translating $N(z)$ to the origin in $\boldsymbol{L}^3$, then we again obtain the normal Gauss map $\mathcal{G}_- : U_- \to \boldsymbol{H}^2_+$ (resp. $\mathcal{G}_+ : U_+ \to \boldsymbol{H}^2_-$) of f on U_- (resp. U_+).

Each $\mathcal{S} : U_\mp \to S$ in (2.1) is a (local) framing map of $f : M \to \boldsymbol{S}^3_1$, that is, $f|_{U_-} = \mathcal{S}\underline{e}_3\mathcal{S}^*$ and $f|_{U_+} = -\mathcal{S}\underline{e}_3\mathcal{S}^*$. In the same way as in [3] (cf. [9]), we can show that $\mathcal{S}$ satisfies the following differential equation (2.2) of first order by means of (the lift h of) $\mathcal{G}$.

Take an isothermal coordinate z and $(1,0)$-form ϕ on M such that the induced metric $f^* ds^2 = \phi \cdot \bar{\phi}$. Let β be the $\mathfrak{sl}(2;\boldsymbol{C})$-valued $(1,0)$-form on $U_- \cup U_+$ written locally as

$$\beta = \begin{pmatrix} \mathcal{G} & 1 \\ \mathcal{G}^2 & \mathcal{G} \end{pmatrix} \omega := \begin{cases} -\frac{1}{2} h(\underline{e}_1 - \sqrt{-1}\underline{e}_2) h^* \phi & \text{on } U_-, \\ -\frac{1}{2} \underline{e}_1 h(\underline{e}_1 - \sqrt{-1}\underline{e}_2) h^* \underline{e}_1 \phi & \text{on } U_+, \end{cases}$$

then $\beta|_{U_\mp} \in \Gamma(T^{*(1,0)} M|_{U_\mp} \otimes \mathcal{G}_\mp^{-1} T^{(1,0)} \boldsymbol{H}^2_\pm)$. We can write the differential equation for $\mathcal{S}$ by using β as follows:

$$\mathcal{S}^{-1} d\mathcal{S} = \begin{cases} \frac{1}{2}(\beta + \beta^*)\underline{e}_3 + \frac{1}{4}[\underline{e}_3, \beta + \beta^*]\underline{e}_3 & \text{on } U_-, \\ \frac{1}{2}\underline{e}_3(\beta + \beta^*) + \frac{1}{4}\underline{e}_3[\underline{e}_3, \beta + \beta^*] & \text{on } U_+. \end{cases} \tag{2.2}$$

We denote by H the mean curvature of f and by Φ its Hopf differential. It then follows from Proposition 6.1 in [1] combined with (2.2) that

$$f^* ds^2 = \frac{4|\mathcal{G}_{\bar{z}}|^2}{\{(1 + |\mathcal{G}|^2) + H(1 - |\mathcal{G}|^2)\}^2} |dz|^2,$$

$$\Phi = \frac{4\mathcal{G}_z(\overline{\mathcal{G}})_z}{\{(1 + |\mathcal{G}|^2) + H(1 - |\mathcal{G}|^2)\}(1 - |\mathcal{G}|^2)} dz \cdot dz.$$

Moreover, we obtain the following

PROPOSITION 1. *The normal Gauss map* $\mathscr{G} : M \to \hat{\boldsymbol{C}}$ *of a spacelike surface with mean curvature* H *in* $\boldsymbol{S}_1^3$ *satisfies*

$$(2.3)\quad (1-|\mathscr{G}|^2)\{(1+|\mathscr{G}|^2)+H(1-|\mathscr{G}|^2)\}\mathscr{G}_{z\bar{z}}+2\{|\mathscr{G}|^2+H(1-|\mathscr{G}|^2)\}\bar{\mathscr{G}}\mathscr{G}_z\mathscr{G}_{\bar{z}} = H_z(1-|\mathscr{G}|^2)^2\mathscr{G}_{\bar{z}}.$$

If we replace the ambient space $\boldsymbol{S}_1^3$ by the de Sitter 3-space $\boldsymbol{S}_1^3(c^2)$ of constant curvature c^2 $(c>0)$, then the above equation will change to

$$(1-|\mathscr{G}|^2)\{c(1+|\mathscr{G}|^2)+H(1-|\mathscr{G}|^2)\}\mathscr{G}_{z\bar{z}}+2\{c|\mathscr{G}|^2+H(1-|\mathscr{G}|^2)\}\bar{\mathscr{G}}\mathscr{G}_z\mathscr{G}_{\bar{z}} = H_z(1-|\mathscr{G}|^2)^2\mathscr{G}_{\bar{z}}.$$

Putting $c=0$ in it, we can obtain the generalized harmonic map equation for Gauss maps of spacelike surfaces in $\boldsymbol{L}^3$ ([8]).

PROPOSITION 2. *For a CMC (constant mean curvature) H conformal immersion* $f : M \to \boldsymbol{S}_1^3(c^2)$, *the normal Gauss map* $\mathscr{G}$ *is a non-holomorphic harmonic map from* M *to* $\hat{\boldsymbol{C}}$ *equipped with the following metric* $h'_{c,H}$:

$$h'_{c,H} = \frac{4|d\zeta|^2}{|(1-|\zeta|^2)\{c(1+|\zeta|^2)+H(1-|\zeta|^2)\}|}.$$

REMARK 1. (1) When $|H|>c$, $h'_{c,H}$ restricted on the unit open disk D is deformed to a hyperbolic metric $4|d\zeta|^2/(|H|(1-|\zeta|^2)^2)$ as c goes to 0 for a fixed nonzero H.

(2) When $|H|<c$, there exists a CMC H conformal immersion $\tilde{f}$ from M to the hyperbolic 3-space of constant curvature $-c^2$ such that the pair of $\tilde{f}$ and f forms a kind of Bonnet pair (cf. Appendix II in [3]). Then the normal Gauss maps f and $\tilde{f}$ satisfy the same harmonic map equation, up to the coordinate change of a homothety in $\hat{\boldsymbol{C}}$. (For the study of the metric $h'_{c,H}$ and harmonic maps to $(D, h'_{c,H})$, see also [5].)

3. Kenmotsu Type Representation Formula in $\boldsymbol{S}_1^3$

Conversely, we can show that (2.3) is the integrability condition for the framing equation (2.2). We then obtain the following Kenmotsu type representation formula in $\boldsymbol{S}_1^3$.

THEOREM 3. *Let M be a simply connected Riemann surface with a reference point $z_0 \in M$, and take an isothermal coordinate z on M. Give a smooth function H on M. Let $v : M \to D$ be a non-holomorphic smooth map satisfying the equation* (2.3):

$$\frac{(1+|v|^2)+H(1-|v|^2)}{1-|v|^2}v_{z\bar{z}}+\frac{2\{|v|^2+H(1-|v|^2)\}\bar{v}}{(1-|v|^2)^2}v_z v_{\bar{z}}=H_z v_{\bar{z}}.$$

Define a 1*-form ω on M as follows and assume that it is smooth on M*:

$$\omega=\frac{2(\bar{v})_z}{\{(1+|v|^2)+H(1-|v|^2)\}(1-|v|^2)}dz.$$

Put a Lie(S)-valued 1*-form μ on M by*

$$\mu=\frac{1}{2}(\beta+\beta^*)\underline{e_3}+\frac{1}{4}[\underline{e_3},\beta+\beta^*]\underline{e_3},\quad \beta=\begin{pmatrix} v & 1 \\ v^2 & v \end{pmatrix}\omega.$$

Then there exists uniquely a smooth map $\mathcal{S} : M \to S$ such that $\mathcal{S}(z_0)=\underline{e_0}$ and $\mathcal{S}^{-1}d\mathcal{S}=\mu$. Put $f=\mathcal{S}\underline{e_3}\mathcal{S}^*$, then $f : M \to S_- \subset \boldsymbol{S}_1^3$ is a conformal immersion outside $\{w \in M|\omega(w)=0\}$ with prescribed mean curvature H and the normal Gauss map $\mathcal{G}=v$.

REMARK 2. If we regard the immersion f constructed in Theorem 3 as an immersion $f=(f_0,f_1,f_2) : M \to \boldsymbol{RS}_1^3$, then f is given by the following path integral:

$$f_0(z)=\exp\left(2\mathrm{Re}\int_{z_0}^{z}v\omega\right),\quad f_1(z)+\sqrt{-1}f_2(z)=\int_{z_0}^{z}f_0(\omega+\bar{v}^2\bar{\omega}).$$

REMARK 3. For a spacelike surface in $\boldsymbol{S}_1^3$ with CMC H of range $|H|>1$ (resp. $|H|=1$), we have obtained the Kenmotsu-Bryant type (resp. Weierstrass-Bryant type (cf. [9])) representation formula by means of its adjusted Gauss map [1], which is a non-holomorphic harmonic map (resp. holomorphic map) to the hyperbolic disk $(D,4|d\zeta|^2/(1-|\zeta|^2)^2)$. By a similar adjusting theory to the one in [3], we can also deform the normal Gauss map to the adjusted Gauss map through a one-parameter family of integrable differential equations of first order.

REMARK 4. It has been proved in [7] and [13] that any complete spacelike surface in $\boldsymbol{S}_1^3$ with CMC H of range $|H| \leqq 1$ is totally umbilic. We also note that any totally umbilic complete spacelike surface of range $|H|<1$ is never contained

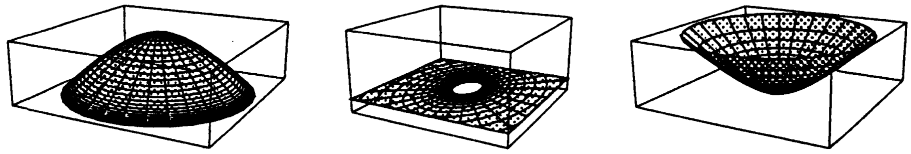

Figure 1: Totally umbilic spacelike surfaces in $\boldsymbol{RS}_1^3(\cong S_-) : |H| > 1, |H| = 1, |H| < 1$

in $S_-(\subset \boldsymbol{S}_1^3)$. (See the third example in Figure 1). Then any CMC H ($|H| < 1$) spacelike surface in S_- is not complete.

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Institute of Mathematics
University of Tsukuba
Ibaraki 305-8571
Japan
E-mail: aiyama@sakura.cc.tsukuba.ac.jp

Department of Mathematics
Shizuoka University
Shizuoka 422-8529
Japan
E-mail: smkacta@ipc.shizuoka.ac.jp