TSUKUBA J. MATH.
Vol. 22 No. 2 (1998), 333–342

ON FOUR-MANIFOLDS FIBERING OVER SURFACES

By

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Abstract. We study closed connected topological or smooth 4-manifolds fibering over a surface in terms of classifying spaces, characteristic classes, and intersection forms.

1. Introduction.

Let F and X be closed connected oriented surfaces of genus h and g, respectively. We are going to study closed connected 4-manifolds M which admit a fibration

$$F \longrightarrow M \xrightarrow{\pi} X \tag{1}$$

with base X and fiber F.

It was shown by Meyer [11] that for a fixed $h \geq 3$ any integer $4m \in \boldsymbol{Z}$ may appear as signature of such a manifold M. So these manifolds provide an interesting class of 4-manifolds (see also [1] and [2] for related examples).

More recently Hillman ([5] and [6]) has proved that the necessary conditions:

$$\chi(M') = \chi(X)\chi(F)$$

and

$$\Pi_1(M') \quad \text{is an extension of} \quad \Pi_1(F) \quad \text{by} \quad \Pi_1(X)$$

are sufficient for the closed 4-manifold M' is homotopy equivalent to a 4-manifold M with fibration structure (1). Here χ denotes the Euler characteristic as

1991 *Mathematics Subject Classification*. 57R40, 57R67, 57N10.
Key words and phrases. Four-manifolds, homotopy type, obstruction theory, fibrations, classifying spaces, intersection forms, characteristic classes, spectral sequences.
Work performed under the auspices of the G.N.S.A.G.A. of the C.N.R. (National Research Council) of Italy and partially supported by the Ministero per la Ricerca Scientifica e Tecnologica of Italy within the projects *Geometria Reale e Complessa* and *Topologia* and by the Ministry for Science and Technology of the Republic of Slovenia Research Grant No. J1-7039-0101-95.
Received December 8, 1996
Revised May 9, 1997

usual. Moreover, if the homotopy equivalence is simple, then M' and M are topologically s-cobordant.

In this paper, we are going to study the problem of uniqueness of the fibration structure (1). If for example $M = X \times F$, then there are at least two such structures. But even if we fix the base X and the fiber F, the following question arises:

Does M admit non-isomorphic fibration

$$F \xrightarrow{i} M \xrightarrow{\pi} X \quad \text{and} \quad F \xrightarrow{i'} M \xrightarrow{\pi'} X?$$

We are trying to construct invariants of such fibrations which are not invariants of the manifold M. A complete set of such invariants will give a classification of closed 4-manifolds fibering as in (1) in terms of homotopy classes of maps from X to certain classifying spaces described below. As a general reference on the algebraic theory of closed 4-manifolds see for example [6]. For a standard text on differential topology of fiber bundles we refer to [8].

2. The description in terms of classifying spaces.

Let X and F be given as above. We fix an orientation on $F = F_h$, where h is the genus of F.

Let

$$G = \begin{cases} \mathrm{Aut}^+(F) & \text{with the compact-open topology} \\ \mathrm{Diff}^+(F) & \text{with the } C^\infty\text{-topology,} \end{cases}$$

where $\mathrm{Aut}^+(F)$ (resp. $\mathrm{Diff}^+(F)$) is the group of orientation preserving homeomorphisms (resp. diffeomorphisms) of F.

Any fiber bundle (1), $F \to M \to X$, is classified by a map $f : X \to BG$. Isomorphism classes of bundles (1) correspond bijectively to homotopy classes of maps $X \to BG$, i.e. to elements of $[X, BG]$.

The component $G_0 \subset G$ of the identity is (weakly) contractible if $h \geq 2$ (see [3] and [4]). Classical results, due to Nielsen, Dehn, and Birman, imply the following canonical group isomorphisms:

$$G/G_0 \cong \mathrm{Aut}^+(\Pi_1(F_h))/\mathrm{Inn}(\Pi_1(F_h)) \cong E^+(F_h),$$

where

$$E^+(F_h) = \{[\varphi] \in [F_h, F_h] : \varphi \text{ orientation preserving homotopy equivalence of } F_h\}.$$

Note that $\Gamma_h := E^+(F_h)$ is just the *Teichmüller group* of F_h.

It follows that G/G_0 is a discrete group. Since $G_0 \simeq \{*\}$ (at least weakly), the fibration

$$G/G_0 \longrightarrow BG_0 \longrightarrow BG$$

implies that $BG \simeq K(\Gamma_h, 1) = B\Gamma_h$.

Assuming that the genus of X is ≥ 2, i.e. $X = K(\Pi_1(X), 1) = B\Pi_1(X)$, we obtain

$$[X, BG] \xrightarrow{\cong} [B\Pi_1(X), B\Gamma_h] \cong \mathrm{Hom}(\Pi_1(X), \Gamma_h)/\Gamma_h,$$

where Γ_h acts on $\mathrm{Hom}(\Pi_1(X), \Gamma_h)$ by setting $(\gamma\alpha)(c) = \gamma\alpha(c)\gamma^{-1}$ for any $\gamma \in \Gamma_h$, $\alpha \in \mathrm{Hom}(\Pi_1(X), \Gamma_h)$, and $c \in \Pi_1(X)$. The last isomorphism holds for discrete groups $\Pi = \Pi_1(X)$ and $\Gamma = \Gamma_h$. Under this isomorphism the classifying map $f : X \to BG$ of the fibration (1) goes to the induced homomorphism

$$f_* : \Pi_1(X) \to \Pi_1(BG) \cong \Pi_0(G/G_0) \cong \Gamma_h.$$

In particular, we note that the obvious map

$$\mathrm{Diff}^+(F) \to E^+(F) \tag{2}$$

is a homotopy equivalence (see [4]).

On the other hand, any fibration $F \to M \to X$ defines an element in

$$\mathrm{Ext}(\Pi_1(F), \Pi_1(X))$$

by the sequence:

$$1 \longrightarrow \Pi_1(F) \longrightarrow \Pi_1(M) \longrightarrow \Pi_1(X) \longrightarrow 1.$$

Conversely, given

$$[1 \to \Pi_1(F) \to \Pi \to \Pi_1(X) \to 1] \in \mathrm{Ext}(\Pi_1(F), \Pi_1(X)),$$

where $h =$ genus $F \geq 2$ and $g =$ genus $X \geq 2$, we obtain a homotopy fibration $F \to B\Pi \to X$ and a classifying map $f_1 : X \to BE^+(F)$. Using (2), we find a unique DIFF fiber bundle (up to bundle isomorphism) $F \to E \to X$ inducing the sequence:

$$1 \longrightarrow \Pi_1(F) \longrightarrow \Pi_1(E) = \Pi \longrightarrow \Pi_1(X) \longrightarrow 1.$$

Let $f : X \to BG$ be its classifying map. Now it is well-known that the set of extensions

$$[1 \to \Pi_1(F) \to \Pi \to \Pi_1(X) \to 1] \in \mathrm{Ext}(\Pi_1(F), \Pi_1(X)),$$

inducing the same homomorphism $f_* : \Pi_1(X) \to \Pi_1(BG) \cong \Gamma_h$, is isomorphic to $H^2(X; \zeta\Pi_1(F))$, where $\zeta\Pi_1(F)$ denotes the center of $\Pi_1(F)$ (see [10], p. 128). But $\zeta\Pi_1(F) = \{1\}$ if $h \geq 3$. Hence we conclude that the fibration structure $F \to M \to X$ is uniquely defined by the element

$$(3) \qquad [1 \to \Pi_1(F) \to \Pi \to \Pi_1(X) \to 1] \in \mathrm{Ext}(\Pi_1(F), \Pi_1(X))$$

if $h =$ genus $F \geq 3$.

In this case our question is equivalent to:

In how many elements of $\mathrm{Ext}(\Pi_1(F), \Pi_1(X))$ *can a given group* Π *occur* (*as in* (3))?

As pointed out by Hillman in [7], a closely related question was considered by Johnson in [9] from a purely algebraic point of view. He proved that if the homomorphism f_* is not injective but has infinite image, then the extension is unique; if f_* has finite image, there are at most two distinct extension structures, and that there are such groups Π with two extension structures. If Π has one extension structure with f_* injective, then all have this property, but he does not settle the question completely in this case. Note also that $\mathrm{Wh}(\Pi) = 0$ in all cases (see for example [6], V.1, p. 68).

3. Characteristic classes.

Let $F \to M \xrightarrow{\pi} X$ be given as in Section 1. Let $\xi \subset TM$ be the subbundle of vertical vectors, i.e. vectors tangent to the fibers. We assume that $\xi \to M$ is an orientable bundle. Such fibrations $F \to M \to X$ are called *orientable* in [12] and [13]. Let $e(\xi) \in H^2(M; \mathbf{Z})$ be the Euler class of ξ. Note that $e(M) = e(\xi)\pi^*(e(X))$, where $e(M) \in H^4(M; \mathbf{Z})$ and $e(X) \in H^2(X; \mathbf{Z})$ denote the Euler classes of M and X, respectively.

Following [12], we define $e_1(\xi) = \mathcal{G}_*(e(\xi)^2) \in H^2(X; \mathbf{Z})$, where $\mathcal{G}_*$ is the Gysin homomorphism

$$\begin{array}{ccc} H^4(M;\mathbf{Z}) & \xrightarrow{\mathcal{G}_*} & H^2(X;\mathbf{Z}) \\ \mathrm{PD}\downarrow\cong & & \cong\downarrow\mathrm{PD} \\ H_0(M;\mathbf{Z}) & \xrightarrow[\pi_*]{} & H_0(X;\mathbf{Z}). \end{array}$$

It is clear that the higher classes $e_j = \mathcal{G}_*(e^{j+1}) = 0$ in our case. However there is another characteristic class. The classifying map $f_* : \Pi_1(X) \to \Gamma_h$ composes with

$\sigma_* : \Gamma_h \to \mathrm{Sp}(2h; \boldsymbol{Z})$, which is induced by the action of

$$\Gamma_h = \mathrm{Aut}(\Pi_1(F))/\mathrm{Inn}(\Pi_1(F))$$

on $H^1(F; \boldsymbol{Z})$. The composition of the maps

$$\Pi_1(X) \to \Gamma_h \to \mathrm{Sp}(2h; \boldsymbol{Z}) \subset \mathrm{Sp}(2h; \boldsymbol{R}) \tag{4}$$

induces $X \to B\,\mathrm{Sp}(2h; \boldsymbol{R})$. Here we continue to assume that the genus of X is ≥ 2.

Let us consider the bundle η associated to the fiber bundle $F \to M \to X$. For every $x \in X$, the fiber of η over x is the real cohomology of the fiber F_x. Since the unitary group $\mathcal{U}(h)$ is a maximal torus in $\mathrm{Sp}(2h; \boldsymbol{R})$, the structure group of this bundle can be reduced to the unitary group, i.e. η can be considered as a complex vector bundle over X.

Then we have the first Chern class $c_1(\eta) \in H^2(X; \boldsymbol{Z})$.

Now from [12], p. 555, it follows that $e_1(\xi) = -12c_1(\eta)$.

One of the results proved by Meyer in [11], p. 246, is

$$\mathrm{Sign}(M) = -\langle 4c_1(\eta), [X] \rangle,$$

where $\mathrm{Sign}(M)$ denotes the signature of M.

Furthermore, assuming $h \geq 3$, any class $c_1(\eta)$ can be realized by a fibration $F \to M \to X$. Here $h \geq 3$ is fixed, but X may vary.

Summarizing we have

PROPOSITION 3.1. *The characteristic classes $c_1(\eta)$ and $e(\xi)$ of the fibration*

$$F \to M \to X$$

are uniquely defined by the signature of M. Moreover, since $\xi \to M$ is a bundle of dimension two, it is also defined by the signature (*because it is defined by $e(\xi)$*).

Now we can state the following problem: *define characteristic classes of the fibration $F \to M \to X$ using other representation of $\Pi_1(X)$ instead of* (4).

4. The spectral sequence and the intersection form.

The spectral sequence of the fibration $F \xrightarrow{i} M \xrightarrow{\pi} X$ is of the following type:

$$E_2^{pq} = H^p(X; \overline{H^q(F)}) \Rightarrow H^{p+q}(M; \boldsymbol{Z}),$$

where $\overline{H^q(F)} = \tilde{X} \times_{\Pi_1(X)} H^q(F; \boldsymbol{Z})$ is the coefficient system over X induced by the homomorphism $f_* : \Pi_1(X) \to \mathrm{Sp}(2h; \boldsymbol{Z})$.

Since we continue to assume that $F \to M \to X$ is oriented, we have

$$E_2^{0\,2} = H^0(X; \overline{H^2(F)}) \cong H^2(F)^{\Pi_1(X)} \cong H^2(F) \cong \mathbf{Z}$$

and

$$E_2^{p\,0} = H^p(X; \overline{H^0(F)}) \cong H^p(X; H^0(F)) \cong H^p(X; \mathbf{Z}).$$

The only possible non-trivial differentials are

$$d_2^{0\,2} : E_2^{0\,2} \cong \mathbf{Z} \to E_2^{2\,1} \quad \text{and} \quad d_2^{0\,1} : E_2^{0\,1} \to E_2^{2\,0} \cong H^2(X; \mathbf{Z}).$$

There is the following commutative diagram

$$\begin{array}{ccc} H^2(M;\mathbf{Z}) & \xrightarrow{i^*} & H^2(F;\mathbf{Z}) \cong \mathbf{Z} \\ \text{epi} \downarrow & & \| \\ E_\infty^{0\,2} & \longrightarrow & E_2^{0\,2} \cong \mathbf{Z}. \end{array}$$

Since $i^*(e(\xi)) = e(F_h) \neq 0$, $h \geq 2$, it follows that $E_\infty^{0\,2} \cong \mathbf{Z}$, which implies $d_2^{0\,2} = 0$.
From this we obtain

(5) $$H^2(M; \mathbf{Z}) \cong H^2(F; \mathbf{Z}) \oplus H^1(X; \overline{H^1(F)}) \oplus E_2^{2\,0}/\mathrm{Im}\, d_2^{0\,1}.$$

Since $E_2^{2\,0} = H^2(X; \mathbf{Z}) \cong \mathbf{Z}$, there are three possibilities:

$$E_2^{2\,0}/\mathrm{Im}\, d_2^{0\,1} \cong \begin{cases} \mathbf{Z} \\ \mathbf{Z}/k\mathbf{Z} \\ 0. \end{cases}$$

Meyer has proved in [11] that

$$\mathrm{Sign}(M) = \mathrm{Sign}(H^1(X; \overline{H^1(F)}), \text{with respect to the obvious pairing}) = 4m$$

for some integer m. This implies that rank $H^2(M; \mathbf{Z})$ and rank $H^1(X; \overline{H^1(F)})$ are even. Then it follows from (5) that $E_2^{2\,0}/\mathrm{Im}\, d_2^{0\,1} \cong \mathbf{Z}$, hence $d_2^{0\,1} = 0$.
So we have proved the following result.

THEOREM 4.1. *If $F \to M \to X$ is an oriented fibration (i.e. the vertical subbundle $\xi \subset TM$ is oriented), then the spectral sequence*

$$E_2^{pq} = H^p(X; \overline{H^q(F)}) \Rightarrow H^{p+q}(M; \mathbf{Z})$$

collapses. In particular, we have

$$H^1(M;\boldsymbol{Z}) \cong H^1(F;\boldsymbol{Z})^{\Pi_1(X)} \oplus H^1(X;\boldsymbol{Z})$$

and

$$H^2(M;\boldsymbol{Z}) \cong H^2(F;\boldsymbol{Z}) \oplus H^1(X;\overline{H^1(F)}) \oplus H^2(X;\boldsymbol{Z}).$$

We remark that in [12] Morita proved that the spectral sequence of the rational cohomology of any surface bundle collapses.

As a consequence, the homomorphism $\pi^* : H^2(X;\boldsymbol{Z}) \to H^2(M;\boldsymbol{Z})$ is not trivial, and hence the quadratic form on $H^2(M;\boldsymbol{Z})$ is *indefinite*. Then we must distinguish two cases according to the intersection form is even or odd (in the second case there exists an element $x \in H^2(M;\boldsymbol{Z})$ such that $x^2 \neq 0 \pmod 2$).

Let us consider now the second Stiefel-Whitney class

$$w_2(M) = w_2(TM) \in H^2(M;\boldsymbol{Z}_2).$$

Let $\xi \subset TM$ be as above the subbundle of vectors tangent to the fibers of $M \xrightarrow{\pi} X$. Then we have $TM/\xi \cong \pi^*(TX)$, i.e. $w_2(M) = w_2(\xi) + \pi^*(w_2(X)) = w_2(\xi)$. Since $w_2(\xi) \equiv e(\xi) \pmod 2$, we conclude with the following implications:

$$\begin{aligned} w_2(M) = 0 &\Rightarrow e(\xi) = 2x \in H^2(M;\boldsymbol{Z}) \\ &\Rightarrow e_1(\xi) = \mathcal{G}_*(e(\xi)^2) \equiv 0 \ (\text{mod } 4) \text{ in } H^2(X;\boldsymbol{Z}) \cong \boldsymbol{Z} \\ &\Rightarrow c_1(\eta) \equiv 0 \ (\text{mod } 4) \\ &\Rightarrow \text{Sign}(M) = -4\langle c_1(\eta), [X]\rangle \equiv 0 \ (\text{mod } 16). \end{aligned}$$

In particular, one reobtains Rohlin's theorem for the special class of 4-manifolds fibering over surfaces.

THEOREM 4.2. *The closed TOP or DIFF* 4*-manifolds M considered above satisfy the Rohlin theorem, i.e.* $w_2(M) = 0$ *implies that* $\text{Sign}(M) \equiv 0$ (*mod* 16). *In this case the integral intersection form* μ_M *is always even.*

On the other hand, if $w_2(M) \neq 0$, then $e(\xi)^2 \neq 0 \pmod 2$, hence the intersection form μ_M is odd, i.e. it is of type:

$$\mu_M \cong (1) \oplus \cdots \oplus (1) \oplus (-1) \oplus \cdots \oplus (-1).$$

Finally, let us calculate the Euler characteristics for oriented fibrations

$$F \to M \to X,$$

i.e. ξ is oriented.

We have

$$\chi(M) = \chi(F)\chi(X) = (2 - 2h)(2 - 2g)$$

and

$$\chi(M) = 2 - 2(\operatorname{rank} H^1(F)^{\Pi_1(X)} + 2g) + \operatorname{rank} H^1(X; \overline{H^1(F)}) + 2,$$

hence

$$4gh - 4h = \operatorname{rank} H^1(X; \overline{H^1(F)}) - 2\operatorname{rank} H^1(F)^{\Pi_1(X)} \geq \operatorname{rank} H^1(X; \overline{H^1(F)}) - 4h.$$

Thus we have the following

PROPOSITION 4.3. rank $H^1(X; \overline{H^1(F)}) \leq 4gh$ *and equality holds if and only if* $H^1(F)^{\Pi_1(X)} \cong H^1(F)$, *i.e. if and only if the classifying map*

$$\Pi_1(X) \xrightarrow{f_*} \Gamma_h \xrightarrow{\sigma_*} \mathrm{Sp}(2h; \mathbf{Z})$$

vanishes.

5. The Pontryagin and Euler classes.

In dimension four the Hirzebruch formula for the signature writes as follows:

$$\mathrm{Sign}(M) = (1/3)\langle p_1(M), [M]\rangle.$$

Now the formula of Meyer [11]

$$\mathrm{Sign}(M) = -4\langle c_1(\eta), [X]\rangle$$

and the above calculated relations

$$\mathcal{G}_*(e(\xi)^2) = e_1(\xi) = -12c_1(\eta)$$

give

$$\mathrm{Sign}(M) = (1/3)\langle e_1(\xi), [X]\rangle.$$

The commutative diagram in Section 3 implies that

$$\pi_*\langle e(\xi)^2, [M]\rangle = \langle e_1(\xi), [X]\rangle,$$

where $\pi_* : H_0(M; \mathbf{Z}) \xrightarrow{\cong} H_0(X; \mathbf{Z})$.

But $\pi_* =$ identity via the identification $H_0(M; \mathbf{Z}) \cong \mathbf{Z} \cong H_0(X; \mathbf{Z})$, hence

$$(1/3)\langle e(\xi)^2, [M]\rangle = \mathrm{Sign}(M) = (1/3)\langle p_1(M), [M]\rangle,$$

i.e.

$$p_1(M) = e(\xi)^2.$$

This is a well known relation between p_1 and e^2.

For the Euler class we have

$$e(M) = e(\xi)\pi^*(e(X)).$$

Now recall the product $\mathrm{BSO}(4) = \mathrm{BSO}(3) \times \mathrm{BSU}(2)$ induced by the fibration (which has a section):

$$\mathrm{SO}(3) \longrightarrow \mathrm{SO}(4) \longrightarrow \boldsymbol{S}^3 = \mathrm{SU}(2).$$

Hence we have $p_1 \in H^4(\mathrm{BSO}(3); \boldsymbol{Z}) \cong \boldsymbol{Z}$ and $H^4(\mathrm{BSO}(4); \boldsymbol{Z}) \cong \boldsymbol{Z}[p_1] \oplus \boldsymbol{Z}[e]$, so $p_1(M)$ and $e(M)$ determine the tangent bundle TM.

REMARK 1: If $e(\xi) = 0$, then $p_1(M) = e(M) = 0$. Since $i^*(e(\xi)) = e(\xi|_F) = 0$, it follows that $h = 1$, i.e. $F = \boldsymbol{S}^1 \times \boldsymbol{S}^1$. In this case we have a map

$$\Pi_1(X) \to \Gamma_1 = \mathrm{SL}(2; \boldsymbol{Z}) = \mathrm{Sp}(2; \boldsymbol{Z}).$$

Assume that the composition $\Pi_1(X) \to \Gamma_1 = \mathrm{SL}(2; \boldsymbol{Z}) \to \mathrm{SL}(2; \boldsymbol{R})$ induces the constant map. Does it follow that $\Pi_1(X) \to \mathrm{SL}(2; \boldsymbol{Z})$ is trivial? (in other words: Is then $M = X \times F$?). As remarked by Hillman in [7], M need not be a product. Let N be an orientable $\boldsymbol{S}^1$-bundle over the torus T with nonzero Euler class. Assume that $\Pi_1(T)$ acts trivially on the fibre. Then $N \times \boldsymbol{S}^1$ is a T-bundle over T of the requested type.

REMARK 2: It is well-known that an aspherical 4-manifold M which fibres over a surface admits the geometry $H^2 \times H^2$ if and only if the map $f_* : \Pi_1(M) \to \Pi_1(\mathrm{BG})$ (our notation in Section 2) has finite image (see for example [6]).

OPEN PROBLEM (J. A. Hillman) Find examples of aspherical surface bundles over surfaces which admit one of the geometries H^4 *or* $H^2(\boldsymbol{C})$ (note that f_* must be injective in these cases).

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