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HOMOLOGY AND DIMENSION OF PROXIMITY SPACES

Dedicated to Professor Yukihiro Kodama on his sixtieth birthday

By

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Abstract. The classical homological characterization theorem of (Lebesgue) covering dimension for compact Hausdorff spaces of finite covering dimension is generalized to proximity spaces in the sense of Efremovič.

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§ 0 Introduction

It is a well-known fact that there is a characterization theorem of covering dimension for compact Hausdorff spaces of finite covering dimension by means of Čech homology (cf. J. Nagata [10]). In order to extend this result to a broader class of (not necessarily topological) spaces the following observations are useful:

1) The category **Prox** of proximity spaces (and δ-maps) in the sense of V. A. Efremovič [3] is isomorphic to the category of proximal nearness spaces (and uniformly continuous maps) in the sense of H. Herrlich [5], and the category of compact Hausdorff spaces can be embedded into **Prox**.
2) There is a well-behaved dimension function for proximity spaces investigated by Yu. M. Smirnov [14] which coincides with Lebesgue covering dimension for compact Hausdorff spaces.
3) Čech's homology theory for topological spaces can be generalized to nearness spaces (cf. H. L. Bentley [1] and D. Čzarcinsky [2]).

Thus, the following question arises: Is it possible to obtain an analogue to the above mentioned characterization theorem for the broader class of proximity spaces? It will be shown that the answer to this question is yes and that the resulting theorem contains the classical one as a corollary.

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§ 1. Preliminaries

1.1. Some notions from general topology

1.1.1. V. A. Efremovic [3] introduced proximity spaces by means of an axiomatization of the concept of nearness of two sets. There is an alternative description via uniform covers which can be explained in the realm of nearness spaces invented by H. Herrlich [5]. For the convenience of the reader we repeat some basic definitions.

1.1.2. DEFINITIONS. 1) A *nearness space* is a pair (X, μ), where X is a set and μ is a non-empty set of non-empty covers of X satisfying the following axioms:

N_1) If $\mathcal{A}$ refines $\mathcal{B}$ and $\mathcal{A}\in\mu$, then $\mathcal{B}\in\mu$.

N_2) If $\mathcal{A}\in\mu$ and $\mathcal{B}\in\mu$, then $\mathcal{A}\wedge\mathcal{B}=\{A\cap B: A\in\mathcal{A} \text{ and } B\in\mathcal{B}\}\in\mu$.

N_3) If $\mathcal{A}\in\mu$, then $\{int_\mu A: A\in\mathcal{A}\}\in\mu$, where $int_\mu A=\{x\in X: \{A, X\setminus\{x\}\}\in\mu\}$.

(The elements of μ are called *uniform covers.*)

If (X, μ) and (Y, η) are nearness spaces, then a map $f: X\to Y$ is called *uniformly continuous* iff $f^{-1}\mathcal{A}=\{f^{-1}[A]: A\in\mathcal{A}\}\in\mu$ for each $\mathcal{A}\in\eta$.

2) A nearness space (X, μ) is called

a) *topological* provided that $X=\cup\{int_\mu A: A\in\mathcal{A}\}$ implies $\mathcal{A}\in\mu$.

b) *uniform* provided that each $\mathcal{A}\in\mu$ is star-refined by some $\mathcal{B}\in\mu$.

c) *contigual* provided that each $\mathcal{A}\in\mu$ is refined by some finite $\mathcal{B}\in\mu$.

d) *proximal* provided that (X, μ) is contigual and uniform.

1.1.3. REMARKS. 1) If **Near** denotes the category of nearness spaces (and uniformly continuous maps), then the full subcategory **T-Near** of **Near** whose object class consists of all topological nearness spaces is bicoreflective in **Near** and isomorphic to the category R_0-**Top** of topological R_0-spaces (and continuous maps) [Note: A topological space X is called an R_0-space provided that $x\in cl\{y\}$ implies $y\in cl\{x\}$ for each pair $(x, y)\in X\times X$).].

2) If **U-Near** denotes the full subcategory of **Near** whose object class consists of all uniform nearness spaces, then **U-Near** is bireflective in **Near** and isomorphic to the category **Unif** of uniform spaces (and uniformly continuous maps) in the usual sense.

3) If **C-Near** denotes the full subcategory of **Near** whose object class consists of all contigual nearness spaces, then **C-Near** is bireflective in **Near** and isomorphic to the category **Cont** of contiguity spaces (and contiguity maps) in the sense of V. M. Ivanova and A. A. Ivanov [8]. Note: If (X, μ) is a nearness

space and μ_c is the set of all covers of X which are refined by some finite element of μ, then $1_X:(X, \mu)\to(X, \mu_c)$ is the bireflection of (X, μ) with respect to **C-Near**.

4) If **Pr-Near** denotes the full subcategory of **Near** whose object class consists of all proximal nearness spaces, then **Pr-Near** is bireflective in **C-Near** as well as in **U-Near**. Furthermore, **Pr-Near** is isomorphic to the category **Prox** of proximity spaces (and δ-maps) in the sense of V. A. Efremovič. Thus, we identify proximity spaces and proximal nearness spaces in the following.

1.2. Homology and cohomology of nearness spaces

Using methods developed by S. Eilenberg and N. Steenrod [4], we give some definitions and results due to H. L. Bentley [1] and D. Čzarcinski [2].

1.2.1. 1) Let $\mathbf{Near}_2$ be the category of pairs of nearness spaces: *Objects* of $\mathbf{Near}_2$ are pairs $((X, \mu), (Y, \mu_Y))$—shortly (X, Y)—where (X, μ) is a nearness space, Y a subset of X and $\mu_Y=\{\alpha\wedge\{Y\}:\alpha\in\mu\}$, i.e. (Y, μ_Y) is a subspace of (X, μ). *Morphisms* $f:(X, Y)\to(X', Y')$ are uniformly continuous maps $f:X\to X'$ such that $f[Y]\subset Y'$.

2) Let (K, L) be a simplicial pair (i.e. K is a simplicial complex and L a subcomplex, possibly empty), and let G be an abelian group. Then the group $C_q(K, L;G)$ (resp. $C^q(K, L;G)$) of q-dimensional chains (resp. q-dimensional cochains is defined in the usual way (cf. [4; VI. 4]). Thus the homology groups of the chain complex $\{C_q(K, L;G), \partial\}$ (resp. cochain complex $\{C^q(K, L;G), \delta\}$) may be defined and are called the homology groups of the pair (K, L) (resp. cohomology groups of the pair (K, L)) [notation: $H_q(K, L;G)$ (resp. $H^q(K, L;G)$)]. The boundary homomorphism $\partial: H_q(K, L;G)\to H_{q-1}(L;G)$ (resp. coboundary homomorphism $\delta: H^q(L;G)\to H^{q+1}(K, L;G)$) and the homomorphism f_*: $H_q(K, L;G)\to H_q(K', L';G)$ (resp. $f^*: H^q(K', L';G)\to H^q(K, L;G)$) for a simplicial map $f:(K, L)\to(K', L')$ are defined in the usual way.

3) Let $(X, Y)\in|\mathbf{Near}_2|$. For every uniform cover α of X let (X_α, Y_α) be the following simplicial pair: X_α is the nerve of the covering α (i.e. the vertices of X_α are the non-empty elements of α and the simplexes of X_α are those non-empty sets of vertices of X_α whose intersection is non-empty) and Y_α is a subcomplex of X_α which is decribed as follows: The vertices of Y_α are the elements of $\alpha'=\{A\in\alpha: A\cap Y\neq\emptyset\}$; a simplex of Y_α is a finite set of elements of α' whose intersection meets Y. Thus, Y_α is the nerve of $\alpha\wedge\{Y\}$ (up to an isomorphism).

4) If $\beta>\alpha$ (covering β is a refinement of covering α), then any projection

$\Pi_\alpha^\beta: (X_\beta, Y_\beta) \to (X_\alpha, Y_\alpha)$ defines a homomorphism $\Pi_{\alpha*}^\beta: H_q(X_\beta, Y_\beta; G) \to H_q(X_\alpha, Y_\alpha; G)$ (resp. $\Pi_\alpha^{\beta*}: H^q(X_\alpha, Y_\alpha; G) \to H^q(X_\beta, Y_\beta; G)$) which is independent of the choice of the projection Π_α^β. There results an inverse spectrum $\{H_q(X_\alpha, Y_\alpha; G); \Pi_{\alpha*}^\beta\}$ (resp. a direct spectrum $\{H^q(X_\alpha, Y_\alpha; G); \Pi_\alpha^{\beta*}\}$) whose limit group is designated by $\check{H}_q(X, Y; G)$ (resp. $\check{H}^q(X, Y; G)$) and called the *q-dimensional Čech homology group* (resp. *q-dimensional Čech cohomology group*) of the pair (X, Y) of nearness spaces. Using the same method as Eilenberg and Steenrod [4; IX, 4] one can show that any $\mathbf{Near}_2$-morphism $f: (X, Y) \to (X', Y')$ induces homomorphisms $\check{H}_q(f): \check{H}_q(X, Y; G) \to \check{H}_q(X', Y'; G)$ (resp. $\check{H}^q(f): \check{H}^q(X', Y'; G) \to \check{H}^q(X, Y; G)$). Thus, we obtain covariant functors $\check{H}_q: \mathbf{Near}_2 \to \mathbf{Ab}$ (resp. contravariant functors $\check{H}^q: \mathbf{Near}_2 \to \mathbf{Ab}$) from the category $\mathbf{Near}_2$ into the category $\mathbf{Ab}$ of abelian groups (and homomorphisms), the so-called *Čech homology functors* (resp. *Čech cohomology functors*).

The boundary operator $\partial_q^{(X,Y)}: \check{H}_q(X, Y; G) \to \check{H}_{q-1}(Y, \phi; G)$ (resp. coboundary operator $\delta_{(X,Y)}^q: \check{H}^q(Y, \phi; G) \to \check{H}^{q+1}(X, Y; G)$) is defined in the usual way (cf. [4; IX, 7]).

1.2.2. THEOREM (cf. [1] and [2]). *Let G be a fixed abelian group. For each integer q, let $\check{H}_q: \mathbf{Near}_2 \to \mathbf{Ab}$ (resp. $\check{H}^q: \mathbf{Near}_2 \to \mathbf{Ab}$) be the Čech homology functor (resp. Čech cohomology functor) and $\partial_q = (\partial_q^{(X,Y)})_{(X,Y) \in |\mathbf{Near}_2|}$ (resp. $\delta^q = (\delta_{(X,Y)}^q)_{(X,Y) \in |\mathbf{Near}_2|}$) the corresponding family of boundary operators (resp. coboundary operators). Then $\partial_* = (\partial_q)_{q \in Z}$ (resp. $\delta^* = (\delta^q)_{q \in Z}$) is a family of natural transformations $\partial_q: \check{H}_q \to \check{H}_{q-1} \circ T$ (resp. $\delta^q: \check{H}^q \circ T \to \check{H}^{q+1}$) with a functor $T: \mathbf{Near}_2 \to \mathbf{Near}_2$ defined by $T(X, Y) = (Y, \phi)$ and $T(f) = f | Y$ (=restriction of f to Y) for each $\mathbf{Near}_2$-morphism $f: (X, Y) \to (X', Y')$. Furthermore, the following are valid:*

1) *For any pair (X, Y) with inclusion maps $i: (Y, \phi) \to (X, \phi)$ and $j: (X, \phi) \to (X, Y)$ the homology sequence*

$$\cdots \to \check{H}_q(Y, \phi; G) \xrightarrow{\check{H}_q(i)} \check{H}_q(X, \phi; G) \xrightarrow{\check{H}_q(j)} \check{H}_q(X, Y; G) \xrightarrow{\partial_q^{(X,Y)}} \check{H}_{q-1}(Y, \phi; G) \to \cdots$$

is of order 2 (i.e. the composition of any two successive homomorphisms of the sequence is zero) and the cohomology sequence

$$\cdots \to \check{H}^q(X, Y; G) \xrightarrow{\check{H}^q(j)} \check{H}^q(X, \phi; G) \xrightarrow{\check{H}^q(i)} \check{H}^q(Y, \phi; G) \xrightarrow{\delta_{(X,Y)}^q} \check{H}^{q+1}(X, Y; G) \to \cdots$$

is exact.

2) *If $g: (X, Y) \to (Z, W)$ and $h: (X, Y) \to (Z, W)$ are uniformly homotopic (i.e. there exists a uniformly continuous map $F: (X \times I, Y \times I) \to (Z, W)$ such that $F(\cdot, 0) = g$ and $F(\cdot, 1) = h$, where I denotes the unit interval $[0, 1]$ with its usual uni-*

form (=topological nearness structure), then $\check{H}_q(g)=\check{H}_q(h)$ *and* $\check{H}^q(g)=\check{H}^q(h)$ *for each integer* q.

3) *If* Y *and* U *are subspaces of* (X, μ) *such that* $\{X\setminus U, Y\}\in\mu$ *then the inclusion map* $i:(X\setminus U, Y\setminus U)\to(X, Y)$ *induces isomorphisms*

$$\check{H}_q(i):\check{H}_q(X\setminus U, Y\setminus U\,;\,G)\longrightarrow\check{H}_q(X, Y\,;\,G)$$

and

$$\check{H}^q(i):\check{H}^q(X, Y\,;\,G)\longrightarrow\check{H}(X\setminus U, Y\setminus U\,;\,G)$$

for each integer q.

4) *If* P *is a nearness space with a single point, then*

$$\check{H}_q(P, \phi\,;\,G)=0=\check{H}^q(P, \phi\,;\,G) \qquad \textit{for each integer } q\neq 0 \textit{ and}$$

$$\check{H}_0(P, \phi\,;\,G)\cong G\cong\check{H}^0(P, \phi\,;\,G)$$

1.2.3. REMARKS. 1) Let G be a compact Hausdorff topological abelian group. In this case the homology sequence under 1.2.2.1) is exact for any pair (X, Y) of contigual nearness spaces [note, that each subspace Y of some contigual nearness space X is contigual since C-**Near** is bireflective in **Near** (cf. 1.1.3. 3))].

2) Let D_2 be the two-point discrete nearness space where discrete means that each cover is uniform. Then the following are equivalent for each non-trivial abelian group G and each nearness space (X, μ):

(1) (X, μ) is uniformly connected, i.e. each uniformly continuous map $f:(X, \mu)\to D_2$ from (X, μ) into D_2 is constant.

(2) $\check{H}_0(X, \phi\,;\,G)\cong G$.

(3) $\check{H}_0(X, \phi\,;\,G)\cong G$.

[Note: A topological nearness space is uniformly connected iff it is connected in the usual (topological) sense. The rationals endowed with its usual uniform structure are uniformly connected.]. This result is obtained by using similar arguments as in the topological case.

3) If (X, μ) is a nearness space and $j_x:(X, \mu)\to(X^*, \mu^*)$ its canonical completion (cf. [12; 6.2.4]), then

$$\check{H}_q(j_x):\check{H}_q(X, \phi\,;\,G)\longrightarrow\check{H}_q(X^*, \phi\,;\,G)$$

and

$$\check{H}^q(j_x):\check{H}^q(X^*, \phi\,;\,G)\longrightarrow\check{H}^q(X, \phi\,;\,G)$$

are isomorphisms for each integer q, where G denotes a fixed abelian group.

4) If X is a topological nearness space (i.e. a topological R_0-space) and Y is a closed subspace, then $\check{H}_q(X, Y\,;\,G)$ (resp. $\check{H}^q(X, Y\,;\,G)$) is isomorphic to the

usual q-dimensional Čech homology (resp. cohomology) group of the closed pair (X, Y) of topological spaces [4; IX, 8] (obviously the directed set of all open coverings of X is a cofinal subset of the directed set of all uniform covers of X).

1.2.4. CONVENTION. Let $\boldsymbol{R}$ be the additive group of real numbers, and $\boldsymbol{Z}$ its subgroup of all integers. We write $\check{H}_q(X, Y)$ (resp. $\check{H}^q(X, Y)$) instead of $\check{H}_q(X, Y; G)$ (resp. $\check{H}^q(X, Y; G)$) provided that G coincides with the factor group $\boldsymbol{R}/\boldsymbol{Z}$ (resp. with the group $\boldsymbol{Z}$). Furthermore, $\check{H}_q(X)$ (resp. $\check{H}^q(X)$) denotes $\check{H}_q(X, \phi)$ (resp. $\check{H}^q(X, \phi)$).

§2. Relation between homology and cohomology groups of a contiguity space (=contigual nearness space)

2.1. REMARKS. 1) If (X, μ) is a contigual nearness space, then the directed set $(\mu^f, <)$ of all finite uniform covers of X is a cofinal subset of the directed set $(\mu, <)$ of all uniform covers of X. Thus, for each subspace Y of X, the limit $\check{H}_q(X, Y)$ (resp. $\check{H}^q(X, Y)$) may be taken to be based on $(\mu^f, <)$.

2) As well-known, $\boldsymbol{R}/\boldsymbol{Z}$ is the character group of $\boldsymbol{Z}$ (up to isomorphism). In the following we will use some parts of the theory of character groups (cf. [11]).

2.2. PROPOSITION ([6; VIII. 2F)]. *Let G_1 and G_2 be abelian groups and $h: G_1 \to G_2$ a homomorphism. Further, let G_i^* be the character group of G_i for $i \in \{1, 2\}$ (i.e. the elements of G_i^* are the homomorphisms of G_i into $\boldsymbol{R}/\boldsymbol{Z}$). Then the following are equivalent:*

(1) *$h: G_1 \to G_2$ is surjective.*

(2) *$h^*: G_2^* \to G_1^*$ defined by $h^*(\chi) = \chi \circ h$ for each $\chi \in G_2^*$ is injective.*

2.3. PROPOSITION. *Let (X, μ) be a contigual nearness space and (Y, μ_Y) a subspace of (X, μ). Then $\check{H}_q(X, Y)$ is the character group of $\check{H}^q(X, Y)$.*

PROOF. It is a well-known fact that, for each finite uniform cover α of (X, μ), $H_q(X_\alpha, Y_\alpha)$ is the character group of $H^q(X_\alpha, Y_\alpha)$. Thus, the desired result follows immediately from [6; VIII. 4D)].

2.4. PROPOSITION. *Let (X, μ) and (Y, μ) be contigual nearness spaces and $f: (X, \mu) \to (Y, \mu)$ a uniformly continuous map. Then the following are satisfied:*

(1) $\check{H}_q(X) = (\check{H}^q(X))^*$ *and* $\check{H}_q(Y) = (\check{H}^q(Y))^*$

(2) $\check{H}_q(f) = (\check{H}^q(f))^*$.

PROOF. (1) follows from 2.3.
(2) is proved analogously to [6; VIII. 5F)].

§ 3. A homological characterization of (finite) δ-dimension

3.1. DEFINITION. Let (X, μ) be a nearness space. Then the *small dimension* $dim(X, \mu)$ of (X, μ) is said to be $\leqq n$ provided every finite uniform cover $\mathcal{U}$ of X has a (finite) refinement $\mathcal{V} \in \mu$ of order $\leqq n+1$ (i.e. each $x \in X$ is contained in at most $n+1$ elements of $\mathcal{V}$). The precise number is the smallest such n, or -1 for the special case that X is emty; and we write $dim(X, \mu)=\infty$ if there is no such n.

3.2. REMARK. For uniform spaces *dim* coincides with the uniform dimension δd of Isbell [7]. For proximal nearness spaces (=proximity spaces) *dim* coincides with the δ-dimension of Smirnov [14]. For normal topological R_0-spaces *dim* is identical with the Lebesque covering dimension.

3.3. A general theorem on normal nearness spaces of finite small dimension due to the author (cf. [12; 7.3.3.] or [13]) contains a result of Kodama [9] on normal topological R_0-spaces of finite covering dimension as well as the following theorem as corollaries:

THEOREM. *Let (X, μ) be a proximal nearness space of finite δ-dimension. Then the following are equivalent:*
(1) $dim(X, \mu) \leqq n$.
(2) $\check{H}^m(X, A)=0$ *for every integer* $m \geqq n+1$ *and every subspace* A *of* X.
(3) *For every integer* $m \geqq n$ *and every subspace* A *of* X *the homomorphism*

$$\check{H}^m(i) : \check{H}^m(X) \longrightarrow \check{H}^m(A)$$

induced by the inclusion map $i: A \to X$ *is a surjective mapping.*

3.4. The above theorem can be stated in terms of homology as follows:

THEOREM. *Let (X, μ) be a proximal nearness space of finite δ-dimension. Then the following are equivalent:*
(1) $dim(X, \mu) \leqq n$.
(2) $\check{H}_m(X, A)=0$ *for every integer* $m \geqq n+1$ *and every subspace* A *of* X.
(3) *For every integer* $m \geqq n$ *and every subspace* A *of* X *the homomorphism*

$$\check{H}_m(i) : \check{H}_m(A) \longrightarrow \check{H}_m(X)$$

induced by the inclusion map $i: A \to X$ in an injective mapping.

PROOF. (1)⇒(2). Apply 2.3. and 3.3.

(2)⇒(3). This implication is an immediate consequence of the fact that the homology sequence is exact (cf. 1.2.3.1).

(3)⇒(1). Apply 2.2, 2.4. and 3.3.

3.5. COROLLARY (cf. Nagata [10; Theorem VIII. 3]). *Let X be a compact Hausdorff space of finite covering dimension. Then the following are equivalent:*

(1) *$\dim X \leqq n$.*

(2) *$\check{H}_m(X, A)=0$ for every integer $m \geqq n+1$ and every closed subspace A of X.*

(3) *For every integer $m \geqq n$ and every closed subspace A of X the homomorphism*

$$\check{H}_m(i): \check{H}_m(A) \longrightarrow \check{H}_m(X)$$

induced by the inclusion map $i: A \to X$ in an injective mapping.

PROOF. If we don't make a notational distinction between X and its underlying set, we may consider X to be the nearness space (X, μ) (note: R_0-**Top**$\cong$ **T-Near**), where μ consists of all covers of X which are refined by some open cover of X. Then (X, μ) is uniform, topological, contigual and T_1.

(1)⇒(2). Since a closed (topological) subspace is a nearness subspace, this implication follows immediately from 3.4.

(2)⇒(3). Note, that for each $(X, Y) \in |\mathbf{Near}_2|$, $H_m(X, Y) \cong H_m(X, cl_X Y)$ (cf. [1; 9]). Thus, the desired implication follows from 3.4.

(3)⇒(1). In order to apply 3.4., it suffices to show that (3) is valid for each nearness subspace of (X, μ) provided (3) is valid for each closed subspace of X (=closed nearness subspace of (X, μ)): Let A be a subset of X and $\bar{A}$ the closure of A in X. If (A, μ_A) denotes the (nearness) subspace of (X, μ), then the inclusion map $i: (A, \mu_A) \to (\bar{A}, \mu_{\bar{A}})$ is uniformly continuous because (A, μ_A) is also a (nearness) subspace of the (nearness) subspace $(\bar{A}, \mu_{\bar{A}})$ of (X, μ). Since $(\bar{A}, \mu_{\bar{A}})$ is topological (and therefore complete), uniform, contigual and T_1, it is a complete separated uniform space (even a proximity space) and it contains the separated uniform space (A, μ_A) as a dense subspace. If $j_A: (A, \mu_A) \to (A^*, \mu_A^*)$ denotes the canonical completion of (A, μ_A) which coincides with the Hausdorff completion (=complete hull) of (A, μ_A) in the sense of A. Weil (cf. [12; 6.2.5.①]), then there exists an isomorphism $1_A^*: (\bar{A}, \mu_{\bar{A}}) \to (A^*, \mu_A^*)$ such that the diagram

$$\begin{array}{ccc} (A, \mu_A) & \xrightarrow{1_A} & (A, \mu_A) \\ {\scriptstyle i}\downarrow & & \downarrow{\scriptstyle j_A} \\ (\bar{A}, \mu_{\bar{A}}) & \xrightarrow{1_A^*} & (A^*, \mu_A^*) \end{array}$$

commutes. Since $\check{H}_q(j_A): \check{H}_q((A, \mu_A)) \to \check{H}_q((A^*, \mu_A^*))$ is an isomorphism (cf. 1.2.3.3)) it follows immediately from $\check{H}_q(1_A^* \circ i) = \check{H}_q(1_A^*) \circ \check{H}_q(i) = \check{H}_q(j_A)$ that $\check{H}_q(i): \check{H}_q((A, \mu_A)) \to \check{H}_q((\bar{A}, \mu_{\bar{A}}))$ is an isomorphism. Thus, (3) is also valid for arbitrary subspaces.

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