TSUKUBA J. MATH.
Vol. 12 No. 1 (1988). 259-268

CYCLIC-PARALLEL REAL HYPERSURFACES OF A COMPLEX SPACE FORM

By

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Introduction.

In 1973 Takagi [14] classified homogeneous hypersurfaces of a complex projective space P_nC by proving that all of them could be divided into six types, and he [15], [16] showed also that if a real hypersurface M has two or three distinct constant principal curvatures, then M is congruent to one of the homogeneous hypersurfaces of type A_1, A_2 and B among these ones. This result is generalized by Kimura [6], who gives the complete classification that a real hypersurface M of P_nC has constant principal curvatures and FC is principal if and only if M is congruent to one of homogeneous examples, where C denotes the unit normal and F is the almost complex structure. The study of real hypersurfaces of type A_1, A_2 and B of P_nC was originated by Cecil and Ryan [1], Kimura [7], Kon [8], Maeda [10], Okumura [13] and so on.

Real hypersurfaces with cyclic-parallel Ricci tensor of a complex space form $M^n(c)$ have recently been classified by Kwon and Nakagawa [9] in the case where FC is principal. They also gave another characterization of real hypersurfaces of type A_1 and A_2 of P_nC.

On the other hand, many subjects for real hypersurfaces of a complex hyperbolic space H_nC were investigated from different points of view ([2], [3], [11], [12] etc.) one of which, done by Chen, Ludden and Montiel [3], asserts that a real hypersurface M of H_nC is of cyclic-parallel if and only if the structure tensor J induced on M and the shape operator A derived from the unit normal commute each other, that is, $JA=AJ$. In particular, real hypersurfaces of H_nC, which are said to be of type A, similar to those of type A_1 and A_2 of P_nC, were treated by Montiel and Romero [12].

The purpose of the present paper is to show that a real hypersurface of a complex space form $M^n(c)$, $c\neq 0$, is of cyclic-parallel if and only if $JA=AJ$, and to give a complete classification of such hypersurfaces by using those examples constructed in [9], [12] and [15].

Received May 15, 1987. Revised August 6, 1987.
* Partially supported by KOSEF.

The author wishes to express here his sincere gratitude to the referee for providing kind and helpful advices.

1. Preliminaries.

We begin by recalling fundamental properties on real hypersurfaces of a Kaehlerian manifold. Let N be a real $2n$-dimensional Kaehlerian manifold equipped with a parallel almost complex structure F and a Riemannian metric tensor G which is F-Hermitian, and covered by a system of coordinate neighborhoods $\{U; x^A\}$. Let M be a real hypersurface of N covered by a system of coordinate neighborhoods $\{V; y^h\}$ and immersed isometrically in N by the immersion $i: M \to N$. Throughout the present paper the following convention on the range of indices are used, unless otherwise stated:

$$A, B, \cdots = 1, 2, \cdots, 2n; \qquad i, j, \cdots = 1, 2, \cdots, 2n-1.$$

The summation convention will be used with respect to those system of indices. When the argument is local, M need not be distinguished from $i(M)$. Thus, for simplicity, a point p in M may be identified with the point $i(p)$ and a tangent vector X at p may also be identified with the tangent vector $i_*(X)$ at $i(p)$ via the differential i_* of i. We represent the immersion i locally by $x^A = x^A(y^h)$ and $B_j = (B_j{}^A)$ are also $(2n-1)$-linearly independent local tangent vectors of M, where $B_j{}^A = \partial_j x^A$ and $\partial_j = \partial/\partial y^j$. A unit normal C to M may then be chosen. The induced Riemannian metric g with components g_{ji} on M is given by $g_{ji} = G(B_j, B_i)$ because the immersion is isometric.

For the unit normal C to M, the following representation are obtained in each coordinate neighborhood:

$$(1.1) \qquad FB_i = J_i{}^h B_h + P_i C, \qquad FC = -P^i B_i,$$

where we have put $J_{ji} = G(FB_j, B_i)$ and $P_i = G(FB_i, C)$, P^h being components of a vector field P associated with P_i and $J_{ji} = J_j{}^r g_{ri}$. By the properties of the almost Hermitian structure F, it is clear that J_{ji} is skew-symmetric. A tensor field of type (1,1) with components $J_i{}^h$ will be denoted by J. By the properties of the almost complex structure F, the following relations are then given:

$$J_i{}^r J_r{}^h = -\delta_i{}^h + p_i p^h, \quad p^r J_r{}^h = 0, \quad p_r J_i{}^r = 0, \quad p_i p^i = 1,$$

that is, the aggregate (J, g, P) defines an almost contact metric structure. Denoting by ∇_j the operator of van der Waerden-Bortolotti covariant differentiation formed with g_{ji}, equations of Gauss and Weingarten for M are respectively obtained:

(1.2) $$\nabla_j B_i = h_{ji} C, \qquad \nabla_j C = -h_j{}^r B_r,$$

where h_{ji} are components of a second fundamental from σ, $A=(h_j{}^k)$ which is related by $h_{ji}=h_j{}^r g_{ri}$ being the shape operator derived form C. We notice here that h_{ji} is symmetric. By means of (1.1) and (1.2) the covariant derivatives of the structure tensors are yielded:

(1.3) $$\nabla_j J_{ih} = -h_{ji} p_h + h_{jh} p_i, \qquad \nabla_j p_i = -h_{jr} J_i{}^r.$$

In the sequel, the ambient Kaehlerian manifold N is assumed to be of constant holomorphic sectional curvature c and real dimension $2n$, which is called a complex space form and denoted by $M^n(c)$. Then the curvature tensor K of $M^n(c)$ takes the following form:

$$K_{DCBA} = \frac{c}{4}(G_{DA}G_{CB} - G_{DB}G_{CA} + F_{DA}F_{CB} - F_{DB}F_{CA} - 2F_{DC}F_{BA}).$$

Thus, equations of Gauss and Codazzi for M are respectively obtained:

(1.4) $$R_{kjih} = \frac{c}{4}(g_{kh}g_{ji} - g_{jh}g_{ki} + J_{kh}J_{ji} - J_{jh}J_{ki} - 2J_{kj}J_{ih}) + h_{kh}h_{ji} - h_{jh}h_{ki},$$

(1.5) $$\nabla_k h_{ji} - \nabla_j h_{ki} = \frac{c}{4} A_{kji}, \qquad A_{kji} = p_k J_{ji} - p_j J_{ki} - 2p_i J_{kj},$$

where R_{kjih} are components of the Riemannian curvature tensor R of M. Let S_{ji} be components of the Ricci tensor S of M, then the Gauss equation implies

(1.6) $$S_{ji} = \frac{c}{4}\{(2n+1)g_{ji} - 3p_j p_i\} + h h_{ji} - h_{ji}^2,$$

where h denotes the trace of the shape operator A and $h_{ji}^2 = h_{jr} h_i{}^r$.

2. Cylic-parallel hypersurfaces.

Let M be a real hypersurface of a complex space form $M^n(c)$. The hypersurface M is called *cyclic-parallel* if the cyclic sum of $\nabla\sigma$ vanishes identically, namely

(2.1) $$\nabla_k h_{ji} + \nabla_j h_{ik} + \nabla_i h_{kj} = 0.$$

It was proved in [4] that geodesic hypersurfaces of a complex space form $M^n(c)$, $c \neq 0$, are cyclic-parallel and not parallel. Throughout the present paper we only consider the case where the holomorphic sectional curvature c is not zero.

From now on we suppose that M is of cyclic-parallel. Then we have from (1.5)

$$2\nabla_k h_{ji} = -\nabla_i h_{kj} + \frac{c}{4} A_{kji},$$

or equivalently $3\nabla_k h_{ji}=c/4(A_{kji}-A_{ikj})$. By the second equation of (1.5), it follows that

(2.2) $$\nabla_k h_{ji}=\frac{c}{4}(p_j J_{ik}+p_i J_{jk}).$$

Differentiating this covariantly along M and making use of (1.3), we find

(2.3) $$\nabla_m\nabla_k h_{ji}=\frac{c}{4}\{(\nabla_m p_j)J_{ik}+(\nabla_m p_i)J_{jk}-h_{mi}p_j p_k-h_{mj}p_k p_i+2h_{mk}p_j p_i\}.$$

Since equation (2.2) tells us that $\nabla_k h_j{}^k=0$, the Ricci formula for h_{ji} gives rise to

$$\nabla_k\nabla_j h_i{}^k=S_{jr}h_i{}^r-R_{kjih}h^{kh}.$$

If we substitute (1.4), (1.6) and (2.3) into the last equation and take account of (1.3), we get

(2.4) $$hh_{ji}{}^2=\left\{h_2-\frac{c}{2}(n+1)\right\}h_{ji}+ch_{rs}J_j{}^r J_i{}^s$$
$$+\frac{c}{2}\{(h_{jr}p^r)p_i+(h_{ir}p^r)p_j\}+\frac{c}{4}h(g_{ji}-p_j p_i),$$

where $h_2=h_{ji}h^{ji}$, which yields

(2.5) $$hh_{jr}{}^2p^r=\left(h_2-\frac{c}{2}n\right)h_{jr}p^r+\frac{c}{2}\alpha p_j,$$

where we have have defined $\alpha=h_{rs}p^r p^s$. Thus, it follows that

(2.6) $$h\beta=\left\{h_2-\frac{c}{2}(n-1)\right\}\alpha,\qquad \beta=h_{ji}{}^2p^j p^i.$$

On the other hand, if we substitute (1.4) and (2.3) into the Ricci formula, which is given by

$$\nabla_m\nabla_k h_{ji}-\nabla_k\nabla_m h_{ji}=-R_{mkjr}h_i{}^r-R_{mkir}h_j{}^r,$$

then we have

(2.7) $$h_{ik}{}^2h_{mj}-h_{im}{}^2h_{kj}+h_{jk}{}^2h_{im}-h_{jm}{}^2h_{ik}$$
$$=\frac{c}{4}\{h_{mi}(g_{kj}-p_k p_j)-h_{ki}(g_{mj}-p_m p_j)+h_{jm}(g_{ki}-p_k p_i)-h_{jk}(g_{mi}-p_m p_i)$$
$$+J_{jk}(\nabla_m p_i+\nabla_i p_m)-J_{jm}(\nabla_k p_i+\nabla_i p_k)+J_{ik}(\nabla_m p_j+\nabla_j p_m)$$
$$-J_{im}(\nabla_k p_j+\nabla_j p_k)+2J_{mk}(\nabla_j p_i+\nabla_i p_j)\},$$

where we have used the second equation of (1.3). By transvecting (2.7) with J^{ik} and $p^j p^i p^k$ respectively and making use of the fact that properties of the almost contact metric structure (J, g, P), we can see that

$$J^{sr}(h_{ms}h_{jr}{}^{2}+h_{js}h_{mr}{}^{2}) = \frac{1}{4}(2n+1)c(\nabla_j p_m+\nabla_m p_j)-\frac{1}{4}c\{(p^r\nabla_r p_j)p_m+(p^r\nabla_r p_m)P_j\}\,, \tag{2.8}$$

$$\alpha h_{mr}{}^{2}p^r=\beta h_{mr}p^r. \tag{2.9}$$

Combining (2.5) and (2.6) with (2.9), it follows that $\alpha(h_{jr}p^r-\alpha p_j)=0$ and hence $\alpha(\beta-\alpha^2)=0$.

Let M_1 be a set consisting of points of M at which the function $\beta-\alpha^2$ does not vanish. Suppose that M_1 is not empty. We then have $\alpha=0$ and thus $\beta h_{mr}p^r=0$ because of (2.9). By transvecting $h_s{}^m p^s$, it follows that $\beta^2=0$ and hence β vanishes on M_1. Therefore the assumption of M_1 will produce a contradiction. Accordingly we have $\beta=\alpha^2$ on M, which means that P is the principal curvature vector corresponding to α, that is,

$$h_{jr}p^r=\alpha p_j. \tag{2.10}$$

Applying p^m to (2.8) and summing up m, we obtain

$$p^r\nabla_r p_j=0 \tag{2.11}$$

because of the fact that $c\neq 0$. By means of (2.2), (2.10), (2.11) and the definition of α, we can easily see that α is constant everywhere. Thus, differentiating (2.10) covariantly along M, we find

$$(\nabla_k h_{jr})p^r+h_{jr}\nabla_k p^r=\alpha\nabla_k p_j,$$

which together with (1.3) and (2.2) yield

$$\frac{c}{4}J_{jk}-h_{jr}h_{ks}J^{rs}=\alpha\nabla_k p_j. \tag{2.12}$$

If we take the symmetric part of this, then we obtain $\nabla_k p_j+\nabla_j p_k=0$ provided that $\alpha\neq 0$. But, if $\alpha=0$, then (2.12) implies $h_{jr}h_{is}{}^{2}J^{rs}=-(c/4)\nabla_i p_j$ with the aid of (1.3), which together with (2.8) and (2.11) give $\nabla_j p_m+\nabla_m p_j=0$. Consequently we see in any case that $h_j{}^r J_r{}^k=J_j{}^r h_r{}^k$. Thus we have the following fact:

LEMMA 1. *Let M be a cyclic-parallel real hypersurfaces of $M^n(c)$, $c\neq 0$. Then the shape operator and the induced structure tensor commute each other, that is,*

$$AJ=JA. \tag{2.13}$$

REMARK 1. Chen, Ludden and Montiel [3] proved this lemma for the case where $c<0$. The converse assertion of Lemma 1 is well known. The proof was used the theory of Riemann fibre bundles (cf. [3], [8]). But, we introduce here the other simple proof. The method is similar to that used in the previous paper [5].

From (2.13), it is easy to see that

$$h_{jr}p^r=\alpha p_j \tag{2.14}$$

by means of the properties of the almost contact metric structure. Differentiating (2.14) covariantly and taking account of (1.3), we obtain

$$(\nabla_k h_{jr})p^r-h_{jr}h_{ks}J^{rs}=\alpha_k p_j-\alpha h_{kr}J_j{}^r, \tag{2.15}$$

where $\alpha_k=\nabla_k\alpha$, which together with equations of Codazzi and (2.13) give

$$\frac{c}{2}J_{jk}+2h_{jr}h_s{}^rJ_k{}^s=\alpha_k p_j-\alpha_j p_k+2\alpha h_{jr}J_k{}^r. \tag{2.16}$$

It means that $\alpha_k=Bp_k$ for some function B. It is easy to see that α is constant everywhere. Thus, the last equation reduces to

$$h_{ji}{}^2=\alpha h_{ji}+\frac{c}{4}(g_{ji}-p_jp_i) \tag{2.17}$$

because of (2.13) and the properties of (J, g, P). Accordingly (2.15) becomes

$$(\nabla_k h_{jr})p^r=\frac{c}{4}J_{jk}. \tag{2.18}$$

LEMMA 2. *Let M be a real hypersurface satisfying* (2.13) *of $M^n(c)$, $c\neq 0$. Then M is of cyclic-parallel provided that $\alpha^2+c=0$.*

PROOF. Since we have $\alpha^2+c=0$, the relationships (2.14) and (2.17) tell us that M has at most two constant principal curvatures α and $\alpha/2$. Their multiplicities are denoted respectively by r and $2n-1-r$. Thus, the trace of the shape operator is given by

$$h=\frac{\alpha}{2}(2n-1+r) \tag{2.19}$$

and that of A^2 is given by

$$h_2=\frac{\alpha^2}{4}(2n-1+3r). \tag{2.20}$$

On the other hand, it is seen from (2.17) that $h_2=\alpha h-(\alpha^2/2)(n-1)$. Therefore, the last three equations imply that $r=1$ because of $\alpha^2+c=0$ and $c\neq 0$. Accordingly (2.19) and (2.20) reduces respectively to

$$h=n\alpha,\qquad h_2=\frac{1}{2}(n+1)\alpha^2. \tag{2.21}$$

We also have the followings:

$$h_3=\frac{1}{4}(n+3)\alpha^3,\qquad h_4=\frac{1}{8}(n+7)\alpha^4, \tag{2.22}$$

where h_3 and h_4 denote the trace of A^3 and A^4 respectively. By using (2.21) and (2.22), it is not hard to see that

$$h_{ji}^2=\frac{3}{2}\alpha h_{ji}-\frac{\alpha^2}{2}g_{ji},$$

which together with (2.17) implies that $h_{ji}=(1/2)\alpha(g_{ji}+p_jp_i)$ because of $\alpha\neq 0$. Differentiating this covariantly, we find

$$\nabla_k h_{ji}=\frac{1}{2}\alpha\{(\nabla_k p_j)p_i+(\nabla_k p_i)p_j\}.$$

Therefore, by means of (1.3) and (2.13) we can verify that M is of cyclic-parallel. This completes the proof.

Differentiation (2.17) covariantly and making use of (1.3), we get

$$(\nabla_k h_{jr})h_i{}^r+(\nabla_k h_{ir})h_j{}^r=\alpha\nabla_k h_{ji}+\frac{c}{4}\{(h_{kr}J_j{}^r)p_i+(h_{kr}J_i{}^r)p_j\}, \tag{2.23}$$

from which, taking the skew-symmetric part with respect to indices k and j and utilizing (2.13) and (2.14),

$$h_{jr}\nabla_k h_i{}^r-h_{kr}\nabla_j h_i{}^r=\frac{c}{4}\alpha(p_kJ_{ji}-p_jJ_{ki})+\frac{c}{2}p_i(h_{kr}J_j{}^r).$$

Thus, it follows that

$$h_j{}^r\nabla_k h_{ir}-h_i{}^r\nabla_k h_{jr}=\frac{c}{4}\{p_jh_{ir}J_k{}^r-p_ih_{jr}J_k{}^r+\alpha(p_jJ_{ik}-p_iJ_{jk})\},$$

where we have used (1.5), (2.13) and (2.14). From this and (2.23), it is seen that

$$2h_j{}^r\nabla_k h_{ir}-\alpha\nabla_k h_{ji}=\frac{c}{4}\{-2p_i(h_{jr}J_k{}^r)+\alpha(p_jJ_{ik}-p_iJ_{jk})\}. \tag{2.24}$$

Transforming this by $h_m{}^j$ and using (2.13), (2.17) and (2.18), we obtain

$$\alpha h_j{}^r\nabla_k h_{ir}+\frac{c}{2}\nabla_k h_{ji}=\frac{c}{4}\left\{\left(\alpha^2+\frac{c}{2}\right)J_{ik}p_j-\frac{c}{2}J_{kj}p_i-\alpha p_i(h_{jr}J_k{}^r)\right\}.$$

Combining this with (2.24), it follows that

$$(\alpha^2+c)\left\{\nabla_k h_{ji}-\frac{c}{4}(p_jJ_{ik}+p_iJ_{jk})\right\}=0,$$

which shows that M is of cyclic-parallel because of Lemma 2.

From this fact and Lemma 1 we have

THEOREM 3. *Let M be a real hypersurface of a complex space form $M^n(c)$, $c\neq 0$. Then M is of cyclic-parallel if and only if $AJ=JA$.*

REMARK 2. It is obvious that if M is of cyclic-parallel, then the Ricci tensor is cyclic-parallel because of (1.3), (1.6) and (2.10).

3. Homogeneous hypersurfaces.

It is known that the complete and simply connected complex space form $M^n(c)$ consists of a complex projective space P_nC, a complex Euclidean space C_n or a complex hyperbolic space H_nC, according as $c>0$, $c=0$ or $c<0$. Some standard examples given by [9], [12], [14] of real hypersurfaces $M^n(c)$, $c\neq 0$ whose second fundamental form are cyclic-parallel are introduced. In a complex Euclidean space C^{n+1} equipped with Hermitian form ϕ, the Euclidean metric of C^{n+1} which is identified with R^{2n+2} is given by $\mathrm{Re}\,\phi$. The unit sphere $S^{2n+1}=\{z\in C^{n+1}: \phi(z, z)=1\}$ is denoted.

First of all, examples of real hypersurfaces of P_nC are considered. For any positive number r a hypersurface $N_0(2n, r)$ of S^{2n+1} is defined by

$$N_0(2n, r)=\left\{(z_1, \cdots, z_{n+1})\in S^{2n+1}\subset C^{n+1}: \sum_{j=1}^{n}|z_j|^2=r|z_{n+1}|^2\right\}.$$

For an integer m $(2\leqq m\leqq n-1)$ and a positive number s, a hypersurface $N(2n, m, s)$ of S^{2n+1} is defined by

$$N(2n, m, s)=\left\{(z_1, \cdots, z_{n+1})\in S^{2n+1}\subset C^{n+1}: \sum_{j=1}^{m}|z_j|^2=s\sum_{j=m+1}^{n+1}|z_j|^2\right\}.$$

Then, for the projection π of the Hopf-fibration S^{2n+1} onto P_nC, $M_0(2n-1, r)=\pi(N_0(2n, r))$ and $M(2n-1, m, s)=\pi(N(2n, m, s))$ $(n\geqq 3)$ are examples of real hypersurfaces of P_nC whose shape operator and the induced structure tensor commute each other. It is known [14] that $M_0(2n-1, r)$ and $M(2n-1, m, s)$ are both compact connected real hypersurfaces of P_nC with constant two or three distinct principal curvatures respectively, which are said to be of type A_1 and A_2 respectively. In [13], it is proved that $M_0(2n-1, r)$ and $M(2n-1, m, s)$ are only hypersurfaces of P_nC satisfying $AJ=JA$.

In the next place, the example of real hypersurfaces of H_nC defined by Montiel [11] and Montiel and Romero [12] is introduced. In C^{n+1} with standard basis, a Hermitian form ϕ is defined by

$$\phi(z, w)=-z_0\bar{w}_0+\sum_{k=1}^{n}z_k\bar{w}_k.$$

where $z=(z_0, \cdots, z_n)$ and $w=(w_0, \cdots, w_n)$ are in C^{n+1}. Let H_1^{2n+1} be a real hypersurface of the Minkoski space C_1^{n+1} defined by

$$H_1^{2n+1}=\{z\in C_1^{n+1}: \phi(z, z)=-1\},$$

and let $\bar{G}$ be a semi-Riemannian metric of H_1^{2n+1} induced from the complex Lorentzian metric $\mathrm{Re}\,\phi$ of C_1^{n+1}. Then $(H_1^{2n+1}, \bar{G})$ is the Lorentzian manifold of constant curvature -1, which is called an anti-de Sitter space.

Let r and s be integers with $r+s=n-1$ and $t\in R$ with $0<t<1$. We consider a Lorentzian hypersurface $N_{r+s}(t)$ of H_1^{2n+1} defined by the following:

$$N_{r+s}(t)=\left\{(z_0, \cdots, z_n)\in H_1^{2n+1}:\ t(-|z_0|^2)+\sum_{j=1}^{r}|z_j|^2=-\sum_{k=r+1}^{n}|z_k|^2\right\}$$

and a Lorentzian hypersurface of H_1^{2n+1} is given by

$$N_n=\{(z_0, \cdots, z_n)\in H_1^{2n+1}:\ |z_0-z_1|=1\}.$$

Since it is known that H_1^{2n+1} is a principal S^1-bundle over a complex hyperbolic space with projection $\bar{\pi}: H_1^{2n+1}\to H_nC$, and $N_{r+s}(t)$ and N_n are S^1-invariant, we see that $M_{r+s}(t)=\pi(N_{r+s}(t))$ and $M_n=\pi(N_n)$ are real hypersurfaces of H_nC, where $\pi: N_{r+s}(t)\to M_{r+s}(t)$ and $\pi: N_n\to M_n$ are semi-Riemannian submersions which are compatible with S^1-fibration. It is seen that $M_{r+s}(t)$ and M_n are complete connected real hypersurfaces of H_nC with constant two or three distinct principal curvatures, which are said to be of type A ([9]). In [12], it is proved that $M_{r+s}(t)$ and M_n are only complete hypersurfaces of H_nC satisfying $AJ=JA$. Thus, by combining above facts and Theorem 3, we obtain the following classifications.

THEOREM 4. *$M_0(2n-1, r)$, $M(2n-1, m, s)$, $M_{r+s}(t)$ and M_n are only complete and connected cyclic-parallel real hypersurfaces of $M^n(c)$, $c\neq 0$.*

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