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SPHERES, SYMMETRIC PRODUCTS, AND QUOTIENT OF HYPERSPACES OF CONTINUA

By

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Abstract. A *continuum* means a nonempty, compact and connected metric space. Given a continuum X, the symbols $F_n(X)$ and $C_1(X)$ denotes the hyperspace of all subsets of X with at most n points and the hyperspace of subcontinua of X, respectively. If $n > 1$, we consider the quotient spaces $SF_1^n(X) = F_n(X)/F_1(X)$ and $C_1(X)/F_1(X)$ obtained by shrinking $F_1(X)$ to a point in $F_n(X)$ and $C_1(X)$, respectively. In this paper, we study the continua X such that $SF_1^n(X)$ is homeomorphic to $C_1(X)/F_1(X)$ and we analyze when the spaces $F_n(X)$ and $SF_1^n(X)$ are homeomorphic to some sphere.

1. Introduction

A *continuum* means a nonempty, compact and connected metric space. The symbols $\mathbf{N}$ and $\mathbf{R}$ will denote the set of all natural numbers and real numbers, respectively. Also I will be the unit interval $[0, 1]$. Consider the following hyperspaces of a continuum X:

$$2^X = \{A \subset X : A \text{ is closed and nonempty}\}, \quad \text{for } n \in \mathbf{N}$$

$$C_n(X) = \{A \in 2^X : A \text{ has at most } n \text{ components}\},$$

$$F_n(X) = \{A \in 2^X : A \text{ has at most } n \text{ points}\}.$$

These hyperspaces are considered with the Vietoris topology (see [16, Theorem 0.11, p. 9]). The hyperspace $F_n(X)$ is also known as the n^{th}-*symmetric product of* X. Symmetric products were introduced by K. Borsuk and S. Ulam in [2], they proved that, if $n = 1, 2, 3$, $F_n(I)$ is homeomorphic to I^n, for $n \geq 4$, $F_n(I)$ is not

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homeomorphic to any subset of $\mathbf{R}^n$ and $F_2(S^1)$ is homeomorphic to Möbius Strip, where S^1 is the 1-sphere. In [14], R. Molski proved that $F_2(I^2)$ is homeomorphic to the 4-cell and for $n \geq 3$ neither $F_n(I^2)$ nor $F_2(I^n)$ is homeomorphic to any subset of $\mathbf{R}^{2n}$. In [3], R. Bott corrected Borsuk's statement (see [1]) that $F_3(S^1)$ is homeomorphic to $S^1 \times S^2$ by showing that, actually $F_3(S^1)$ is homeomorphic to S^3, where S^n denotes the n-sphere. In this direction, in this paper we prove the following theorem:

THEOREM 4.3. *Let X be a continuum. The following statements are true*:

(1) (*Triviality*) *If $n = 1$, then $F_n(X)$ is homeomorphic to S^m if and only if X is homeomorphic to S^m*,
(2) *$F_n(X)$ is homeomorphic to S^m for some $m \leq n$ if and only if either $n = 3$ or $n = 1$, and $X = S^1$.*

Furthermore, in 1979 S. B. Nadler, Jr. introduced the *hyperspace suspension of a continuum X* as the quotient space $C_1(X)/F_1(X)$, [17], in that paper the author studied the fixed point property of this quotient spaces. For $m, n \in \mathbf{N}$ with $m < n$ and a continuum X, we consider the quotient space $F_n(X)/F_m(X)$ that we will denote by $SF_m^n(X)$ obtained by shrinking $F_m(X)$ to a point in $F_n(X)$, with the quotient topology (see [6]). It is well known that $C_1(I)/F_1(I)$ and $SF_1^2(I)$ are 2-cells (see [13, In proof of Corollary 3.10, p. 129] and [6, Example 3.1]), $C_1(S^1)/F_1(S^1)$ is homeomorphic to S^2 (see [13, In proof of Corollary 3.10, p. 129]), but $SF_1^2(S^1)$ is the Real Projective Plane (see [6, Example 3.1]). In view of this, it is easily suspected that the spaces X for which $C_1(X)/F_1(X)$ is homeomorphic to $F_n(X)/F_1(X)$ are very limited. In fact, in this paper we show the following results:

THEOREM 3.4. *Let X be a finite-dimensional and arcwise connected continuum. Then $C_1(X)/F_1(X)$ is homeomorphic to $SF_1^2(X)$ if and only if X is homeomorphic to* $[0, 1]$.

THEOREM 3.6. *If Y is an arcwise connected continuum and $n \geq 3$, then $C_1(Y)/F_1(Y)$ is not homeomorphic to $SF_1^n(X)$, for every finite dimensional continuum X.*

Since $SF_1^2(S^1)$ is the Real Projective Plane (see [6, Example 3.1]), $SF_1^2(T_m)$ is homeomorphic to $F_2(T_m)$ (see [6, Example 3.3]) and $SF_1^n(Q)$ is homeomorphic to

Q for each $n \in \mathbf{N}$ (see [6, Example 3.1]), where T_m is a simple m-od and Q is the Hilbert Cube. As a consequence of the results obtained in this paper, we obtain the following

Corollary 4.8. *If X is a continuum and $n \geq 2$, then $SF_1^n(X)$ is not homeomorphic to S^m, for each $2 \leq m \leq n$.*

Finally, the following questions remain open.

Question 3.7. *Can we omit the arcwise connectedness hypothesis in Theorems 3.4 and 3.6?*

Question 4.9. *Does there exist a continuum X and $n \geq 2$ such that $F_n(X)$ is homeomorphic to S^m for some $m \geq 4$?*

Question 4.10. *Does there exist a continuum X and $m, n \geq 2$ such that $SF_m^n(X)$ is homeomorphic to S^m for some $m \in \mathbf{N}$?*

2. Definitions and Preliminaries

Given a continuum Z and a subset A of Z, $\mathrm{cl}_Z(A)$, $\mathrm{int}_Z(A)$, $\mathrm{Bd}(A)$ denotes the closure, interior and boundary of A in Z, respectively. A *subcontinuum* of a space Z is a continuum contained in Z. The symbol $|A|$ denotes the cardinality of A and $\mathrm{cone}(Z)$ denotes the quotient space $Z \times [0,1]/Z \times \{1\}$. Let $z \in Z$ and β be a cardinal number, we say that z has *order less than or equal to β in Z*, written $ord(z, Z) \leq \beta$, provided that for each open subset $U \subset Z$ such that $z \in U$, there exists V an open subset of Z such that $z \in V \subset U$ and $|\mathrm{Bd}(V)| \leq \beta$.

An n-od ($n \in \mathbf{N}$ and $n \geq 3$) is a continuum X which contains a subcontinuum Y such that the complement of Y in X is the union of n nonempty mutually separated sets (if Y is a singleton and the components of $X \setminus Y$ are arcs, we say that X is a *simple n-od*). A simple 3-od, will be called a *simple triod*. An *arc* is any space homeomorphic to I. A *free arc* in a continuum X is an arc $\alpha \subset X$ such that $\mathrm{int}_X(\alpha) \neq \emptyset$.

Given a finite collection, $U_1, \ldots, U_m$, of subsets of X, $\langle U_1, \ldots, U_m \rangle_n$, denote the following subset of $F_n(X)$

$$\left\{ A \in F_n(X) : A \subset \bigcup_{i=1}^{m} U_i \text{ and } A \cap U_i \neq \emptyset \text{ for each } i = 1, \ldots, m \right\}.$$

If each U_i is an open subset of X, it is known that the family of all subsets of the form $\langle U_1, \dots, U_m \rangle_n$, is a basis for the topology of $F_n(X)$ called the *Vietoris topology* (see [16, Theorem 0.11, p. 9]).

Given a continuum X, $\rho^X_{m,n} : F_n(X) \to SF^n_m(X)$ denotes the natural quotient function. Also, let $F^n_m(X)$ denotes the point $\rho^X_{m,n}(F_m(X))$.

Remark 2.1. Using an appropriate restriction of $\rho^X_{m,n}$, it is clear that $SF^n_m(X) \setminus \{F^n_m(X)\}$ is homeomorphic to $F_n(X) \setminus F_m(X)$.

In this paper, *dimension* means inductive dimension as defined in [16, (0.44), p. 21]. The symbol dim will be used to denote dimension. If $\dim(X) \in \mathbf{N} \cup \{-1, 0\}$ we will writte $\dim(X) < \infty$ and $\dim(X) = \infty$ in other case. By [9, p. 20], for every continuum X, $\dim(X) \geq 1$.

The following result is a particular case of [9, Corollary 1, p. 32].

Theorem 2.2. *Let X be a continuum and $n \in \mathbf{N} \cup \{0\}$. If $X = Y \cup Z$, Y is closed in X, $\dim(Y) \leq n$ and $\dim(Z) \leq n$, then $\dim(X) \leq n$.*

Corollary 2.3. *If X is a continuum, $n \in \mathbf{N}$, Y is a subcontinuum of X, $\dim(X) = n$ and $\dim(Y) < n$, then $\dim(X \setminus Y) = n$.*

Proof. Is clear that $\dim(X \setminus Y) \leq n$. If $\dim(X \setminus Y) < n$, then $\dim(X \setminus Y) \leq n - 1$. By Theorem 2.2, $\dim(X) \leq n - 1$, this is a contradiction. □

3. $SF^n_1(X)$ Homeomorphic to $C_1(X)/F_1(X)$

Proposition 3.1. *If X is a finite-dimensional continuum, then $F_n(X)$ and $SF^n_m(X)$ are finite-dimensional continua.*

Proof. By [8, proof of Lemma 3.1, p. 253], $\dim(F_n(X)) \leq n \cdot \dim(X)$, thus $F_n(X)$ is a finite-dimensional continuum. On the other hand, since $\dim(F_n(X) \setminus F_m(X)) \leq \dim(F_n(X))$ and $F_n(X) \setminus F_m(X)$ is homeomorphic to $SF^n_m(X) \setminus \{F^n_m(X)\}$, $\dim(SF^n_m(X) \setminus \{F^n_m(X)\}) \leq n \cdot \dim(X)$. Thus, by Corollary 2.3, $\dim(SF^n_m(X))$ is finite. □

Proposition 3.2. *Let X be a 1-dimensional continuum and $n \geq 2$, then $\dim(F_n(X)) = \dim(SF^n_1(X))$ and $\dim(C_1(X)) = \dim(C_1(X)/F_1(X))$.*

Proof. Notice that $\dim(F_1(X)) = 1$. By Corollary 2.3, $\dim(F_n(X)) = \dim(F_n(X)\backslash F_1(X))$. We conclude

$$\dim(SF_1^n(X)\backslash\{F_1^n(X)\}) = \dim(F_n(X)).$$

By Theorem 2.2, $\dim(SF_1^n(X)) = \dim(F_n(X))$. In a similar method we can show that $\dim(C_1(X)) = \dim(C_1(X)/F_1(X))$. □

Lemma 3.3. *If an arcwise connected continuum X has hyperspace $C_1(X)$ of dimension at most 2, then X is homeomorphic to either S^1 or I.*

Proof. By [10, Theorem 70.1, p. 337] X does not contain simple triods. Therefore, $ord(x, X) \leq 2$ for every $x \in X$ because X is arcwise connected. Thus, by [18, Proposition 9.5, p. 142], X is an arc or X is homeomorphic to S^1. □

Theorem 3.4. *Let X be a finite-dimensional and arcwise connected continuum. Then $C_1(X)/F_1(X)$ is homeomorphic to $SF_1^2(X)$ if and only if X is homeomorphic to $[0, 1]$.*

Proof. If X is an arc, both $C_1(X)$ and $SF_1^2(X)$ are 2-cells. Conversely, suppose that $C_1(X)/F_1(X)$ is homeomorphic to $SF_1^2(X)$. By Proposition 3.1, $\dim(SF_1^2(X)) < \infty$, thus $\dim(C_1(X)/F_1(X)) < \infty$. Hence, $\dim(C_1(X)) < \infty$. By [11, Theorem 2.1], we have $\dim(X) = 1$. So, $\dim(SF_1^2(X)) \leq 2$ and $\dim(C_1(X)) \leq 2$. By Lemma 3.3 X is an arc or X is homeomorphic to S^1. But, $C_1(S^1)/F_1(S^1)$ is the 2-sphere and $SF_1^2(S^1)$ is homeomorphic to the real projective plane. We conclude that X most be an arc. □

Lemma 3.5. *If X is a continuum and $n \geq 3$, then $F_n(X)$ and $SF_1^n(X)$ does not contains 2-dimesional subsets with nonempty interior.*

Proof. Suppose that there exist a 2-dimensional subset $\mathcal{D}$ of $F_n(X)$ with nonempty interior. Let $\mathcal{U} = \langle U_1, \ldots, U_n \rangle_n$ be an open subset of $F_n(X)$ such that $\mathcal{U} \subset \mathcal{D}$. By the denseness of $\{A \in F_n(X) : |A| = n\}$ in $F_n(X)$ (see [7, In the proof of Lemma 3.1]) there is $A \in (F_n(X)\backslash F_{n-1}(X)) \cap \mathcal{U}$. Since $|A| = n$ we can assume that $U_i \cap U_j = \emptyset$ and $A \cap U_i \neq \emptyset$ for every $i, j \in \{1, 2, \ldots, n\}$. Under this conditions we can take $C_1, C_2, \ldots, C_n$ nondegenerate subcontinua of X such that $C_i \subset U_i$ for each i. Notice that $\langle C_1, \ldots, C_n \rangle_n$ is homeomorphic to $C_1 \times \cdots \times C_n$.

So, $\mathcal{U}$ contains a homeomorphic subset to $C_1 \times \cdots \times C_n$. Hence $\dim(\mathcal{U}) \geq 3$. This is a contradiction. □

THEOREM 3.6. *If Y is an arcwise connected continuum and $n \geq 3$, then $C_1(Y)/F_1(Y)$ is not homeomorphic to $SF_1^n(X)$, for every finite dimensional continuum X.*

PROOF. Suppose that there is a finite dimensional continuum X, such that $C_1(Y)/F_1(Y)$ is homeomorphic to $SF_1^n(X)$. By Proposition 3.1, $\dim(C_1(Y)/F_1(Y)) < \infty$. Thus, $\dim(C_1(Y)) < \infty$. By [11, Theorem 2.1], $\dim(Y) = 1$. Let $m = \dim(C_1(Y))$. By [10, Theorem 70.1, p. 337] and using arcwise connectedness of Y, this continuum does not contain simple $(m+1)$-ods. By [12, Theorem 11, p. 179], Y must contain a free arc, which implies that $C_1(Y)/F_1(Y)$ contains a 2-dimensional subset with nonempty interior, but this contradicts Lemma 3.5. So, the theorem is true. □

QUESTION 3.7. *Can we omit the arcwise connectedness hypothesis in Theorems* 3.4 *and* 3.6?

4. Continua X such that $F_n(X)$ and/or $SF_1^n(X)$ are n-spheres

THEOREM 4.1. *If X is a continuum, then for each $n \geq 2$, neither $F_n(X)$ nor $SF_1^n(X)$ is homeomorphic to S^2.*

PROOF. Let X be a continuum such that $F_n(X)$ is homeomorphic to S^2 for some $n \geq 2$. Then, $F_n(X)$ is locally connected. By [8, Lemma 2.2, p. 252] X is locally connected. Since, $\dim(S^2) = 2$ then $\dim(X) = 1$ and $n = 2$. By [6, Lemma 5.9], X cannot contain simple m-ods, for each $m \geq 3$. Therefore, by [18, Proposition 9.5, p. 142], X must be an arc or a simple closed curve. But, $F_2(I)$ is a 2-cell and $F_2(S^1)$ is a Möbius Strip, which contradicts the assumption $F_n(X)$ homeomorphic to S^2.

Now, to the case $SF_1^n(X)$. Let X be a continuum and suppose that $SF_1^n(X)$ is homeomorphic to S^2. Thus, X is locally connected. Since $C_1(S^1)/F_1(S^1)$ is homoemorphic to S^2 (see [13, In proof of Corollary 3.10, p. 129]), by Theorem 3.6, we have $n = 2$. It is clear that $\dim(X)$ must be equal to 1. By [6, Example 3.3] and [6, Lemma 5.9], X cannot contain simple m-ods, for each $m \geq 3$. So, by [18, Proposition 9.5, p. 142], X is an arc or a simple closed curve. By [6, Example 3.1], in both cases $SF_1^2(X)$ is not homeomorphic to S^2. □

Theorem 4.2. *Let X be a continuum. If $n \geq 2$ and $n \neq 3$, then neither $F_n(X)$ nor $SF_1^n(X)$ is not homeomorphic to S^m, for each $2 \leq m \leq n$.*

Proof. The conclusion for $n = 2$ follows from Theorem 4.1.

Let $n > 3$ and suppose that $F_n(X)$ (or $SF_1^n(X)$) is homeomorphic to S^m for some $2 \leq m \leq n$. Then, $F_n(X)$ is locally connected. By [8, Lemma 2.2, p. 252] X is locally connected. Thus, X is arcwise connected. Let α be an arc in X and $x, y \in \alpha$, $x \neq y$. So, there is a system of neighborhoods γ of $\{x, y\}$ in $F_n(X)$ (of $\rho_{n,1}^X(\{x, y\})$ in $SF_1^n(X)$, respectively) such that for every $V \in \gamma$, V cannot be embedded in $\mathbf{R}^n$ (see [2]). But, each point in S^m have a system of neighborhoods, each one of which is embedded in $\mathbf{R}^n$, this is a contradiction. □

Theorem 4.3. *Let X be a continuum. The following statements are true*:

1. (*Triviality*) *If $n = 1$, then $F_n(X)$ is homeomorphic to S^m if and only if X is homeomorphic to S^m*,
2. *$F_n(X)$ is homeomorphic to S^m for some $m \leq n$ if and only if either $n = 3$ or $n = 1$, and $X = S^1$.*

Proof. (1) is true, because $F_1(X)$ is homeomorphic to X. The sufficiency of (2) is true by [3] and (1).

For the necessity of (2), suppose that $F_n(X)$ is homeomorphic to S^m for some $m \leq n$. By Theorem 4.2, $n = 1$ or $n = 3$. If $n = 1$, since $F_1(X)$ is homeomorphic to X, then X is homeomorphic to S^1. If $n = 3$, by [5, Corollary 5.9], X is homeomorphic to S^1. □

Since each continuum Z is a compact, metric space, cone(Z) is homeomorphic to the so-called geometric cone over Z (see [18, Exercise 3.28, p. 47]). So, the following remark is easy to be seen.

Remark 4.4. If Z is a continuum and $n \geq 2$, cone(Z) can be embedded in $\mathbf{R}^n$ if and only if Z can be embedded in $\mathbf{R}^{n-1}$.

Lemma 4.5. *If T_3 is a simple triod, then $F_3(T_3)$ and $SF_1^3(T_3)$ can not be embedded in $\mathbf{R}^3$.*

Proof. Let v_1, v_2 and v_3 the end points of T_3. Let

$$Z = \{A \in F_3(T_3) : A \cap \{v_1, v_2, v_3\} \neq \emptyset\}.$$

Since cone(Z) is homeomorphic to $F_3(T_3)$ (see [4]). In order to prove that $F_3(T_3)$ can not be embedded in $\mathbf{R}^3$ we only need to show Z can not be embedded in $\mathbf{R}^2$. Let v be the vertex of T_3. Is easy to construct a system of neighborhoods γ of the point $\{v_1, v\}$ such that for each $V \in \gamma$, V contain a homeomorphic copy of $T_3 \times I$, but by [4, Lemma 3.1, p. 58], each one of them can not be embedded in $\mathbf{R}^2$. In a similar method, we can show that $SF_1^3(T_3)$ can not be embedded in $\mathbf{R}^3$. □

LEMMA 4.6. *If X is homeomorphic to I or S^1, then $F_3(X)$ is not homeomorhic to $SF_1^3(X)$.*

PROOF. First suppose that X is homeomorphic to I. By [2, Theorem 6, p. 880], there exists a homeomorphism $k : F_3(X) \to D$ where

$$D = \{(x, y, z) \in \mathbf{R}^3 : x^2 + y^2 + z^2 \leq 1\}$$

and $k(F_1(X))$ is the linear segment that joint the points $(0, 0, 1)$ and $(0, 0, -1)$. So, $SF_1^3(X)$ is not homeomorphic to I^3, and then $F_3(X)$ and $SF_1^3(X)$ are not homeomorphics.

Now, if X homeomorphic to S^1, suppose that there is a homeomorphism $h : SF_1^3(X) \to F_3(X)$. Let $p = h(F_1^3(X))$. By Remark 2.1, $S^3 \backslash \{p\}$ is homeomorphic to $F_3(X) \backslash F_1(X)$. On the other hand, $S^3 \backslash \{p\}$ is homeomorphic to $\mathbf{R}^3$. Moreover by [15, Theorem 2] there is a homeomorphism between $F_3(X)$ and S^3 such that the image of $F_1(X)$ is a trefoil knot T in S^3. Thus, $\mathbf{R}^3$ and $S^3 \backslash T$ are homeomorphic. But, its first fundamental groups $\pi_1(\mathbf{R}^3)$ and $\pi_1(S^3 \backslash T)$ are not isomorphic, which is a contradiction. □

THEOREM 4.7. *If X is a continuum, then $SF_1^3(X)$ is not homeomorphic to S^3.*

PROOF. Suppose that X is a continuum and $SF_1^3(X)$ is homeomorphic to S^3. So, X is locally connected. By Lemma 4.5, X cannot contain simple triods, because each point in S^3 has a system of neighborhoods, γ, such that for each $V \in \gamma$, V can be embedded in $\mathbf{R}^3$. So, X must be an arc or a simple closed curve. This contradicts Lemma 4.6. □

By Theorems 4.2 and 4.7 we obtain the following corollary.

COROLLARY 4.8. *If X is a continuum and $n \geq 2$, then $SF_1^n(X)$ is not homeomorphic to S^m, for each $2 \leq m \leq n$.*

To finish this paper, we pose the following questions.

QUESTION 4.9. *Does there exist a continuum X and $n \geq 2$ such that $F_n(X)$ is homeomorphic to S^m for some $m \geq 4$?*

QUESTION 4.10. *Does there exist a continuum X and $m, n \geq 2$ such that $SF_m^n(X)$ is homeomorphic to S^m for some $m \in \mathbf{N}$?*

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