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Confidence Bounds for Extreme Wind Speed Estimates: A Comparison For the Gumbel - Burr XII Distribution and Classical Extreme Value Distributions

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Abstract. Recently, the Gumbel-Burr XII (GUBXII) distribution has been introduced in Osatohanmwen *et al.* (2017), as a new member from the T-X family of distributions. Several properties of the new distribution were studied by the authors and an application to a real lifetime data set was carried out. In this paper, analysis of extreme wind speed frequency is undertaken. A comparison for the GUBXII distribution, generalized extreme value distribution and the Gumbel distribution in fitting weekly highest wind speed observations collected in Benin City, Nigeria for 200 weeks is performed. Results obtained indicate that the GUBXII distribution outperformed the Gumbel and the generalized extreme value distribution in fitting the extreme wind speeds data while also providing the best confidence bound for wind speed estimates with short return periods.

Key words: wind speeds; AIC; extreme values; maximum likelihood; return period; confidence bound; delta method.
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Résumé. Dans cet article, les extrèmes des vitesses de vent sont étudiés dans la ville du Bénin au Nigéria. Les données concernent une période de 200 semaines. L'étude a été modéliséeen utilisant une famille de lois de probabilité dénommée Gumbel-Burr XII (GUBXII) qui aété introduite dans Osatohanmwen et al. (2017), commeétant un nouveau membre de la famille des distributions T-X, et ensuite avec la distribution généralisée des extrêmes. Une comparaison a été faite.

1. Introduction

Extreme value analysis differs from other approaches of statistical analysis in its aim to quantify the stochastic behavior of a process at usually large or small levels. It is based on the analysis of the maxima or minima of identically distributed sequences of random variables capturing a particular phenomenon over a given time period. Problems on extreme values appeared in the work of Bernoulli back in 1709 for studying the problem of the mean largest distance from origin for n random numbers on a straight line (Johnson *et al.* (1995)).

In a few words, the extreme value theory started with the univariate case, especially with independent data. Given a sequence of independent and identically distributed random variables $(X_n)_{n\geq 0}$ with common cumulative distribution function (*cdf*) and defined on the same probability space $(\Omega, \mathcal{A}, \mathbb{P})$, the max-stability problem consists of finding possible limits in distribution of the sequences of partial maxima

$$M_n = max(X_1, ..., X_n), \; n \geq 1,$$

when appropriately centered and normalized. Precisely, we want to find non-random sequences $(a_n > 0)_{n\geq 1} \subset \mathbb{R}$ and $(b_n)_{n\geq 1} \subset \mathbb{R}$ such that $(M_n - b_n)/a_n$ converges in distribution to a random variable Z as $n \to +\infty$, denoted as

$$\frac{M_n - b_n}{a_n} \rightsquigarrow Z. \;\; (M)$$

This problem was solved around the middle of 20-th century with the contributions of many people, from whom we can cite Gnedenko (1943), Fisher and Tippet (1928), Fréchet (1927), etc. The following result is usually quoted as the Gnedenko (1943) result since he had the chance to close the characterization theorem. If Z is non-degenerate, that is : Z takes at least two different values, Formula (M) can hold only if Z is one of the three types in terms of its *cdf*:

the Fréchet type with parameter $\gamma \in \Gamma_1 = \{x \in \mathbb{R}, \; x > 0\}$:

$$H_\gamma(x) \equiv \phi_\gamma(x) = \exp(-x^{-1/\gamma})1_{(x\geq 0)},$$

the Weibull type with parameter $\gamma \in \Gamma_2 = \{x \in \mathbb{R}, \; x < 0\}$:

$$H_\gamma(x) \equiv \psi_\gamma(x) = \exp((-x)^{-1/\gamma})1_{(x<0)} + 1_{(x\geq 0)}$$

and the Gumbel type with parameter $\gamma \in \Gamma_0 = \{0\}$:

$$H_\gamma(x) \equiv \Lambda(x) = \exp(-e^{-x}), \quad x \in \mathbb{R}.$$

By type of distribution, we mean that any non-degenerate Z in Formula (M) have a *cdf* F_Z which is of the form $F_Z(x) = H_\gamma(Ax+B)$, where $0 < A \in \mathbb{R}$, $B \in \mathbb{R}$, and $\gamma \in \mathbb{R}$. These three types may be gathered in the form of the Generalized Extreme Value (GEV) Distribution

$$G_\gamma(x) = \exp(-(1+\gamma x)^{-1/\gamma}),\ 1+\gamma x \geq 0,\ \gamma \in \mathbb{R}.$$

A modern account of the theory, including statistical estimation, can be found in Galambos (1958), de Haan (1970), de Haan and Ferreira (2006), Embrechts e*t al.* (1997), Resnick (1987), Beirlant *et al.* (2014), etc. This theory is part of the general weak convergence theory which is thoroughly treated in Billingsley (1968) and **?**. Among other reliable sources, we may also cite (Sarkar *et al.* (2011)) and Johnson *et al.* (1995). Actually, this theory is part the larger field of weak convergence which is thoroughly treated in Billingsley (1968) and **?**, and comprehensively intrdoduced in Lo *et al.* (2016).

In this paper we are concerning with the application of such a theory, specifically in wind speed studies. In such a filed, analysis of extreme wind speeds is important for several practical applications such as in estimating wind loads on buildings, making predictions on storms and gusts, engineering design of wind structures and in carrying out draught analysis for general agricultural purposes. Because wind speed is a stochastic variable, its behaviour over time can be captured using specific probability distributions, from which certain properties, pattern and progression of a wind regime is ascertained. For extreme wind speeds analysis, it is usually required that an extreme value distribution is used to fit a sequence of extreme wind speeds collected over time in order to make statistical inference concerning the wind regime of a particular location.

In many special cases, the classical extreme values distributions may not be flexible enough to capture the behavior of an extreme wind speed regime based on some historical data. For example, when the distribution of the extreme wind speeds presents a bimodal density, the classical extreme values distributions break down since they are all unimodal. The need to offer more flexibility to the standard classical extreme values distributions have spurred the development of new extreme value distributions either by adding extra parameter(s) to the standard distributions or by compounding the classical extreme value distributions with other well-known probability distributions. In the light of these realities, Nadarajah and Kotz (2004) developed the beta Gumbel distribution which can be unimodal and bimodal, Nadarajah (2006) proposed the exponentiated Gumbel distribution by adding a shape parameter to the Gumbel distribution in other to add more flexibility to the distribution, Osatohanmwen *et al.* (2017) compounded the Gumbel distribution with the Burr XII distribution by adopting the logit transformation of the cumulative distribution function (CDF) of the Burr XII random variable to obtain the Gumbel-Burr XII (GUBXII) distribution. The density was shown to be both unimodal and bimodal by the authors.

In this paper, the performance of the GUBXII distribution in fitting and estimating extreme wind speeds is compared with that of the GEV and the Gumbel distributions with CDFs given respectively by

$$F_{\text{GEV}}(x) = \exp\left\{-\left[1+\zeta\left(-\frac{x-\mu}{\delta}\right)\right]^{-1/\zeta}\right\}, \tag{1}$$

$$1+\zeta(x-\mu)/\delta > 0, \quad -\infty < \mu, \zeta < \infty, \delta > 0,$$

$$F_{\text{Gumbel}}(x) = \exp\left[-\exp - \left(\frac{x-\mu}{\delta}\right)\right], \tag{2}$$

$$-\infty < x, \mu < \infty, \delta > 0,$$

where The parameters ζ, μ and δ are shape, location and scale parameters respectively. Confidence bounds for extreme wind speed estimates for a given return period are further constructed and compared for the three distributions. The analysis is based on a weekly highest wind speed observations collected in 200 weeks between (2011-2015) in Benin City, South-South, Nigeria.

This paper is organized in six sections. In section 2, we take a look at the GUBXII distribution and discuss the maximum likelihood estimation of its parameters. Extreme wind speed frequency analysis and construction of confidence bounds for extreme wind speed estimates are discussed in sections 3 and 4 respectively. Results from analysis using weekly highest wind speeds observations are presented in section 5, with discussion of results and conclusion, contained in section 6.

2. The Gumbel-Burr XII (GUBXII) Distribution

Osatohanmwen *et al.* (2017) proposed the GUBXII distribution as a new member from the T-X class of distributions developed by Alzaatreh *et al.* (2013). The CDF and probability density function (PDF) of the GUBXII distribution was given respectively by

$$G(x) = \exp\left\{-e^{\epsilon/\alpha}\left[\left(1+\left(\frac{x}{c}\right)^s\right)^\lambda - 1\right]^{-1/\alpha}\right\}. \tag{3}$$

$$\begin{aligned} g(x) =& \frac{s\lambda e^{\epsilon/\alpha}}{\alpha c}\left(\frac{x}{c}\right)^{s-1}\left(1+\left(\frac{x}{c}\right)^s\right)^{\lambda-1}\left[\left(1+\left(\frac{x}{c}\right)^s\right)^\lambda - 1\right]^{-1-1/\alpha} \times \\ & \exp\left\{-e^{\epsilon/\alpha}\left[\left(1+\left(\frac{x}{c}\right)^s\right)^\lambda - 1\right]^{-1/\alpha}\right\}, \end{aligned} \tag{4}$$

$$x > 0, -\infty < \epsilon < \infty, \alpha, c, s, \lambda > 0,$$

where the parameters ϵ,α,λ, and s are shape parameters and c a scale parameter. Several properties of the distribution such as its hazard, moments, relationship with some continuous univariate distributions, quantiles, Shannon entropy, mode, skewness and kurtosis were studied, as well as application of the distribution. The distribution was shown to be highly flexible, having both unimodal and bimodal density at some certain parameters values.

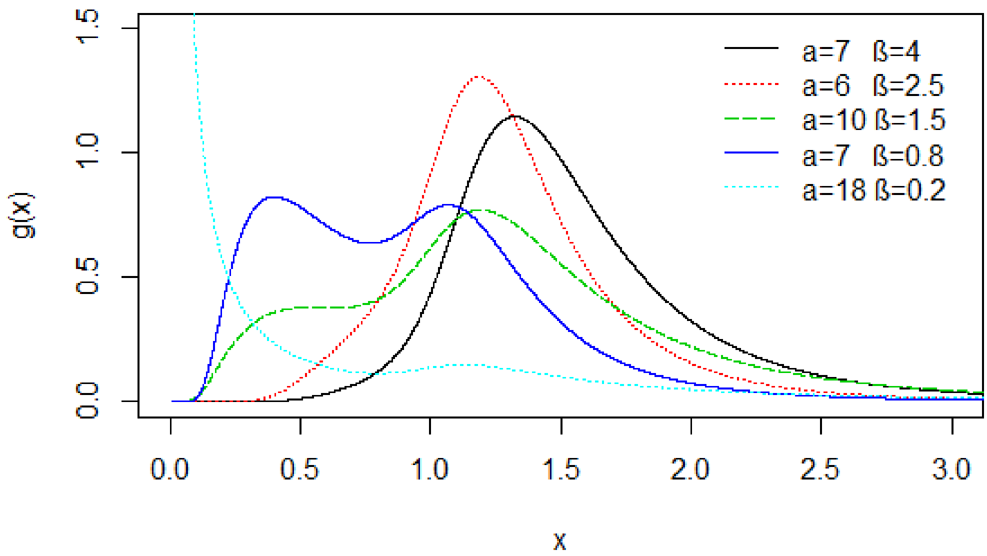

Figure 1: GUBXII density ($c = 1, \lambda = 4, s = 8$)

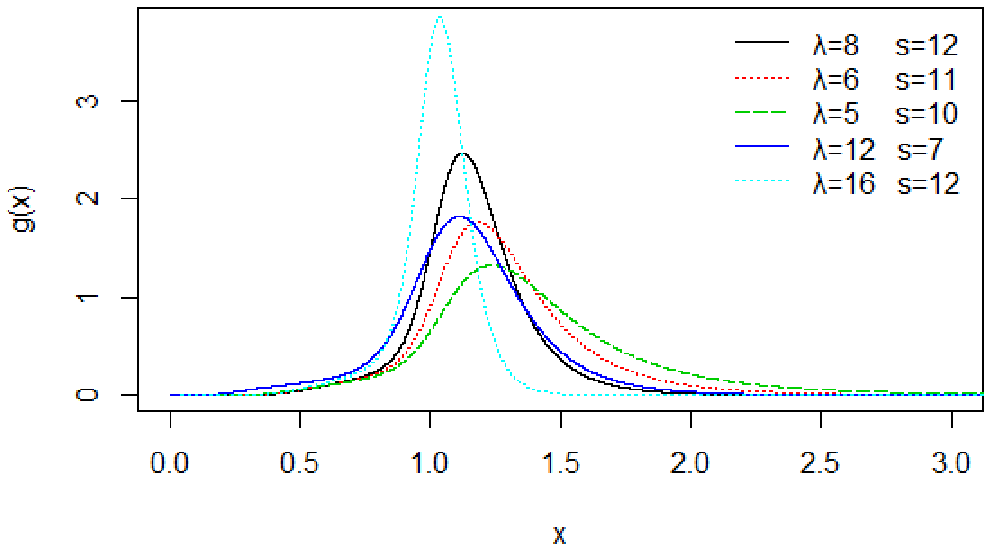

Figure 2: GUBXII density ($c = 1, \alpha = 10, \beta = 3$)

2.1. Maximum likelihood estimation of the parameters of the GUBXII distribution

For a random independent sample $x_1, x_2, \ldots, x_n$ of size n, the log-likelihood function of the 5-Parameter GUBXII distribution is given by

$$\begin{aligned} L = \sum_{i=1}^{n} \ln\left(g(x_i)\right) &= n\left(\ln\lambda + \ln s + \ln\beta - \ln\alpha - s\ln c\right) + (s-1)\sum_{i=1}^{n} \ln x_i \\ &+ (\lambda+1)\sum_{i=1}^{n} \ln\left(1 + (x_i/c)^s\right) - (1 + 1/\alpha)\sum_{i=1}^{n} \ln\left(\left(1 + (x_i/c)^s\right)^{\lambda} - 1\right) \\ &- \beta\sum_{i=1}^{n}\left(\left(1 + (x_i/c)^s\right)^{\lambda} - 1\right)^{-1/\alpha}, \end{aligned} \tag{5}$$

where $\beta = \mathrm{e}^{\epsilon/\alpha}$. Let $\Theta = (\alpha, \beta, c, \lambda, s)^T$ be the unknown parameter vector, the associated score function is given by

$$\mathbf{U}(\Theta) = \left(\frac{\partial L}{\partial \alpha}, \frac{\partial L}{\partial \beta}, \frac{\partial L}{\partial c}, \frac{\partial L}{\partial \lambda}, \frac{\partial L}{\partial s}\right)^T,$$

where $\frac{\partial L}{\partial \alpha}, \frac{\partial L}{\partial \beta}, \frac{\partial L}{\partial c}, \frac{\partial L}{\partial \lambda}$ and $\frac{\partial L}{\partial s}$ are the partial derivatives of the log-likelihood function w.r.t. to each parameter.

The maximum likelihood estimate of Θ can be obtained by solving the non-linear systems of equations, $\mathbf{U}(\Theta) = 0$. Since the resulting systems of equations are not in closed form, the solutions can be found numerically using some specialized numerical optimization method such as the Quasi-Newton method. The BFGS iterative method which is implemented in the R software is an example of an iterative scheme based on the Quasi-Newton method. The parameters c, s and λ are from the Burr XII distribution, the maximum likelihood estimates of c, s and λ for the Burr XII distribution can be used as initial guess to begin the iteration (i.e., c_0, s_0 and λ_0). Also α and ϵ are from the Gumbel distribution hence, the moments estimates of these parameters can be used as initial guess for the iterations too. In particular, the random sample $x_1, x_2, \ldots, x_n$ is transformed to a sample from the Gumbel distribution by $t_i = \ln\left(\left(1 + (x_i/c_0)^{s_0}\right)^{\lambda_0} - 1\right)$, where t_i is a Gumbel variate. Thus, the initial estimates for β and α are $\beta_0 = \mathrm{e}^{\epsilon_0/\alpha_0}$ with $\alpha_0 = S_T\sqrt{6/\pi}$ and $\epsilon_0 = \bar{T} - \gamma\alpha_0$,where ϵ_0 and α_0 are the moments estimates of ϵ and α (Johnson *et al.* (1995)). $\bar{T}$ and S_T are the mean and standard deviation of $(t_1, t_2, \ldots, t_n)$ respectively, $\gamma = 0.5772$ is the Euler constant (Osatohanmwen *et al.* (2017)).

The Fisher information matrix (FIM) of the GUBXII distribution is the 5×5 symmetric matrix given by

$$\mathbf{I}(\Theta) = -E_\Theta \begin{pmatrix} I_{\alpha\alpha} & I_{\alpha\beta} & I_{\alpha c} & I_{\alpha\lambda} & I_{\alpha s} \\ I_{\beta\alpha} & I_{\beta\beta} & I_{\beta c} & I_{\beta\lambda} & I_{\beta s} \\ I_{c\alpha} & I_{c\beta} & I_{cc} & I_{c\lambda} & I_{cs} \\ I_{\lambda\alpha} & I_{\lambda\beta} & I_{\lambda c} & I_{\lambda\lambda} & I_{\lambda s} \\ I_{s\alpha} & I_{s\beta} & I_{sc} & I_{s\lambda} & I_{ss} \end{pmatrix},$$

where the elements $I_{i,j}(\Theta) = \left[\dfrac{\partial^2 L}{\partial\Theta_i \partial\Theta_j}\right]$.Thus, the elements of the FIM can be obtained by considering the second order partial derivatives of the log-likelihood function w.r.t. to the parameters.These second order partial derivatives can be easily obtained using some symbolic computing software like MATHEMATICA. The total FIM, $\mathbf{I}(\Theta)$, can be approximated by

$$\mathbf{J}\left(\hat{\Theta}\right) \approx \left[-\frac{\partial^2 L}{\partial\Theta_i \partial\Theta_j}\bigg|_{\Theta=\hat{\Theta}}\right]_{5\times 5}.$$

For real data, $\mathbf{J}\left(\hat{\Theta}\right)$is obtained after the maximum likelihood estimate of $\hat{\Theta}$ is gotten, which implies the convergence of the iterative numerical procedure involved in finding such estimate.

2.2. Asymptotic confidence intervals for the maximum likelihood estimates of the GUBXII parameters

Suppose $\hat{\Theta}$ is the maximum likelihood estimate of Θ. Under the usual regularity conditions and that the parameters are in the interior of the parameter space, but not on the boundary, we have: $\sqrt{n}\left(\hat{\Theta} - \Theta\right) \xrightarrow{d} N_5\left(\mathbf{0}, \mathbf{I}^{-1}(\Theta)\right)$, where $\mathbf{I}^{-1}(\Theta)$ is the inverse of the expected FIM, which also corresponds to the variance-covariance matrix of the parameters. The asymptotic behavior is still valid if $\mathbf{I}^{-1}(\Theta)$ is replaced by the inverse of the observed information matrix evaluated at Θ, that is $\mathbf{J}\left(\hat{\Theta}\right)$. The multivariate normal distribution with mean vector $\mathbf{0} = (0,0,0,0,0)^T$ and covariance matrix $\mathbf{I}^{-1}(\Theta)$ can be used to construct confidence intervals for the GUBXII parameters. The approximate $100(1-\theta)\%$ two-sided confidence interval for the parameters $\alpha, \beta, c, \lambda$ and s are given by

$$\hat{\alpha} \pm Z_{\theta/2}\sqrt{\mathbf{I}^{-1}_{\alpha\alpha}\left(\hat{\Theta}\right)}, \quad \hat{\beta} \pm Z_{\theta/2}\sqrt{\mathbf{I}^{-1}_{\beta\beta}\left(\hat{\Theta}\right)}, \quad \hat{c} \pm Z_{\theta/2}\sqrt{\mathbf{I}^{-1}_{cc}\left(\hat{\Theta}\right)},$$

$$\hat{\lambda} \pm Z_{\theta/2}\sqrt{\mathbf{I}^{-1}_{\lambda\lambda}\left(\hat{\Theta}\right)}, \quad \hat{s} \pm Z_{\theta/2}\sqrt{\mathbf{I}^{-1}_{ss}\left(\hat{\Theta}\right)},$$

respectively, where $\mathbf{I}^{-1}_{\alpha\alpha}\left(\hat{\Theta}\right)$, $\mathbf{I}^{-1}_{\beta\beta}\left(\hat{\Theta}\right)$, $\mathbf{I}^{-1}_{cc}\left(\hat{\Theta}\right)$, $\mathbf{I}^{-1}_{\lambda\lambda}\left(\hat{\Theta}\right)$, $\mathbf{I}^{-1}_{ss}\left(\hat{\Theta}\right)$ are diagonal elements of $\mathbf{I}^{-1}\left(\hat{\Theta}\right)$ and $Z_{\theta/2}$ is the upper $(\theta/2)^{th}$ percentile of a standard normal distribution.

For the GEV and Gumbel distributions, similar results hold for the maximum likelihood estimates of their parameters.

3. Extreme Wind Speed Frequency Analysis

An extreme wind speed event is said to have occurred if the wind speed random variable X with CDF F, is greater than or equal to a particular *threshold* x_T i.e., if $X \geq x_T$. If the event $X \geq x_T$ occurred now, the time it will take for it to happen again is called the "Recurrence Interval". The expected value of the recurrence interval is the return period T of the extreme wind speed event $X \geq x_T$. This is the average number of time (e.g., days, weeks, years) in which the extreme wind speed event $X \geq x_T$returns, which also describe the chance of occurrence of the event. The probability ϕ of the occurrence of the extreme wind speed event $X \geq x_T$ is related to the return period T by

$$\phi = P\left(X \geq x_T\right) = \frac{1}{T}. \tag{6}$$

Thus, the probability of occurrence of the extreme wind speed event $X \geq x_T$ is the inverse of the return period T. Therefore the $T-$ duration return period event is $X \geq x_T$ and it occurs on average once in T duration. From (7) it follows that the extreme wind speed x_T for a given return period T can be obtained by solving the equation

$$1 - T\left(1 - F\left(x_T\right)\right) = 0. \tag{7}$$

By taking F to be the CDF of the GUBXII, GEV and Gumbel distributions, the solution of (8) for the respective distribution are given respectively by

$$x_T = c\left[\left(\exp\left(\epsilon - \alpha\ln\left(-\ln\left(1-\frac{1}{T}\right)\right)\right)+1\right)^{1/\lambda} - 1\right]^{1/s}, \tag{8}$$

$$x_T = \mu + \frac{\delta\left(1-\left(\ln\left(1-\frac{1}{T}\right)\right)^{-\zeta}\right)}{\zeta}, \tag{9}$$

$$x_T = \mu - \delta \ln\left(-\ln\left(1 - \frac{1}{T}\right)\right). \tag{10}$$

4. Confidence Bounds for Extreme Wind Speed Estimates

We have established that under suitable regularity condition that, Θ, which is a vector containing the parameters of a given extreme value distribution is asymptotically normal. In some special cases, one may be interested in the estimation of a function of Θ. The Taylor's formula comes handy in such situation because it holds that an estimate of a function, say $h = h(\Theta)$ is simply found by $h\left(\hat{\Theta}\right)$, where $\hat{\Theta}$ is the maximum likelihood estimator of Θ. In particular, the return period T can be viewed as a function giving us a tool to construct approximate confidence bounds for the $T-$ duration return period extreme value x_T, since x_T is a function of the parameters of the extreme value distribution used for the analysis as shown by (8), (9) and (10). This procedure is known as the*delta method.* It follows that the $1-\theta$ confidence bound for x_T is given as

$$x_T = \left[\hat{x}_T \pm Z_{\theta/2}\sigma\right], \tag{11}$$

with

$$\sigma^2 = \triangledown x_T\left(\hat{\Theta}\right)^T \mathbf{I}^{-1}\left(\hat{\Theta}\right) \triangledown x_T\left(\hat{\Theta}\right),$$

where $\mathbf{I}^{-1}\left(\hat{\Theta}\right)$ is the $k \times k$ variance-covariance matrix evaluated at $\hat{\Theta}$, and

$$\triangledown x_T\left(\hat{\Theta}\right) = \left[\left.\frac{\partial x_T}{\partial \Theta}\right|_{\Theta=\hat{\Theta}}\right]_{k\times 1},$$

k, being the number of parameters in the extreme value distribution used for the analysis.

5. Analysis and Results

Weekly highest wind speed observations obtained from the recording station of the National Center for Energy and Environment (NCEE), Energy Commission of Nigeria (ECN) was used for the analysis (see, www.ncee.org.ng). The maximum likelihood fits of the GUBXII, GEV and Gumbel distributions to the data are presented in Table 1. The variance-covariance matrix $\mathbf{I}^{-1}\left(\hat{\Theta}\right)$, evaluated at the parameter estimates of each distribution is also reported. The density plot and Q-Q plots of the fitted distributions is given by Figure 3(a-d). Estimates of x_T (in m/s) using the three distributions for a given return period and the corresponding 95% confidence bounds for x_T are contained in Table 2.

Table 1: Maximum likelihood fits of weekly highest wind speeds

Distribution	Gumbel	GEV	GUBXII
Parameter estimates	$\hat{\delta}$=0.5264 (0.0294) $\hat{\mu}$=1.2532 (0.0391)	$\hat{\delta}$=0.5069 (0.0299) $\hat{\mu}$=1.2258 (0.0479) $\hat{\zeta}$=0.0892 (0.0400)	$\hat{\alpha}$=14.7493 (1.2053) $\hat{\epsilon}$=-2.0129 (1.3194) $\hat{c}$=1.4122 (0.0032) $\hat{\lambda}$=2.3143 (0.3033) $\hat{s}$=25.5480 (0.0032)
AIC	386.1084	383.2876	381.4605

(Standard error of estimates in parenthesis)

$$\mathbf{I}^{-1}_{\text{GUBXII}}\left(\hat{\Theta}\right) = \begin{pmatrix} 1.45270 & 0.90890 & 0.27250 & 0.00071 & 0.00063 \\ 0.90890 & 1.74070 & 0.21900 & 0.00021 & -0.00002 \\ 0.27250 & 0.21900 & 0.09200 & 0.00012 & 0.00013 \\ 0.00071 & 0.00021 & 0.00012 & 0.00001 & 0.0000004 \\ 0.00063 & -0.00002 & 0.00013 & 0.0000004 & 0.00001 \end{pmatrix}$$

$$\mathbf{I}^{-1}_{\text{GEV}}\left(\hat{\Theta}\right) = \begin{pmatrix} 0.00090 & -0.00021 & 0.00056 \\ -0.00021 & 0.00230 & -0.00058 \\ 0.00056 & -0.00058 & 0.00160 \end{pmatrix}$$

$$\mathbf{I}^{-1}_{\text{Gumbel}}\left(\hat{\Theta}\right) = \begin{pmatrix} 0.00087 & 0.00035 \\ 0.00035 & 0.00150 \end{pmatrix}$$

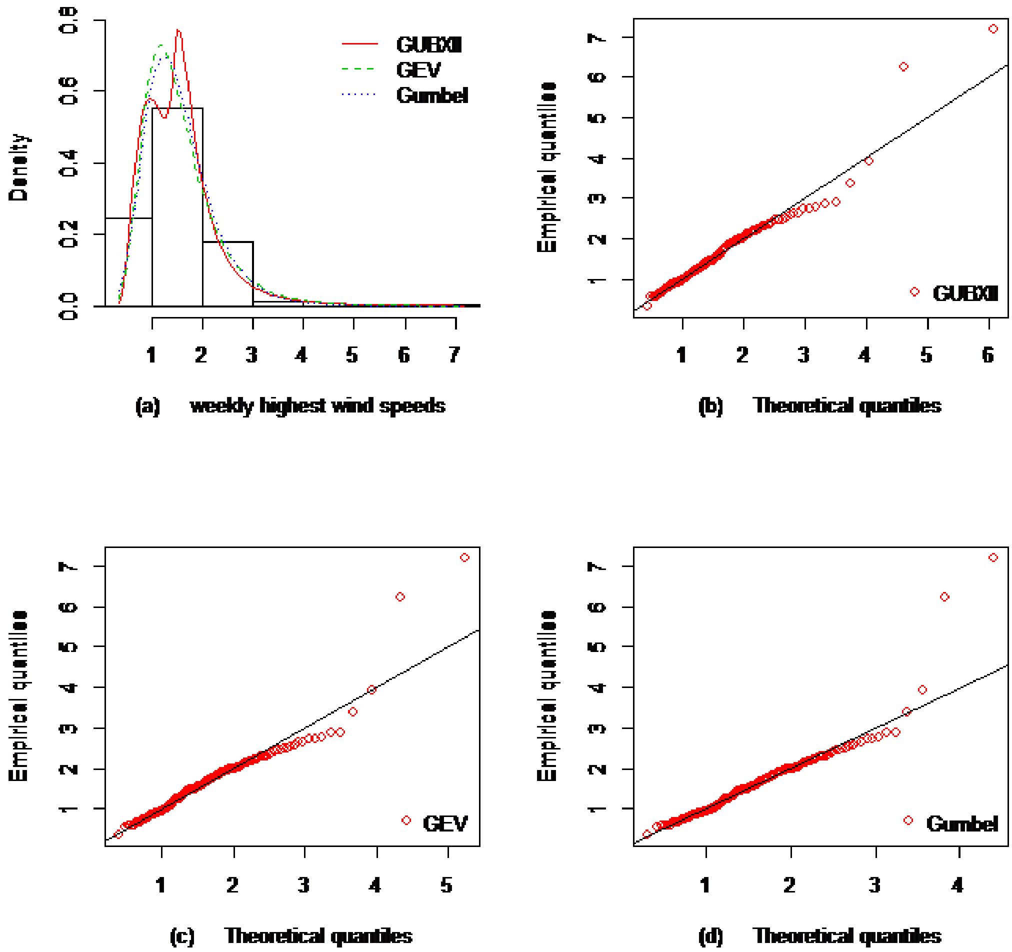

Figure 3 (a-d): Densities and Q-Q plots of fitted distributions

Table 2: Extreme wind speed estimates (in m/s) for given return periods and corresponding confidence bounds

	Return periods with 95% confidence bounds			
	$T = 5(x_5)$	$T = 20(x_{20})$	$T = 100(x_{100})$	$T = 200(x_{200})$
GUBXII	1.9843	2.8635	4.3001	5.1150
	(1.8565,2.1121)	(2.5240,3.2030)	(3.35006,5.0996)	(4.0133,6.2167)
GEV	2.0393	2.94797	4.1088	4.6571
	(1.8945,2.1841)	(2.6377,3.2617)	(3.4093,4.8083)	(3.7130,5.6012)
Gumbel	2.0419	2.8158	3.6738	4.0400
	(1.9104,2.1734)	(2.6078,3.9719)	(3.3757,3.9719)	(3.7027,4.3773)

6. Discussion of Results and Conclusion

Results obtained from fitting the data using the three distributions as contained in Table 1 clearly showed that the three distributions reported very good fits to the data, as shown from the density and Q-Q plots. The fit of the GUBXII distribution to the data is observed to be best, given that it possessed the smallest AIC value.

Results obtained from the extreme wind speed frequency analysis, as reported in Table 2, reveal that as the return period T increases, the estimated $T-$ Week extreme wind speeds also increase for the three distributions used. For $T = 5$, the $T-$ Week extreme wind speed estimate using the Gumbel distribution was highest, followed by that of the GEV distribution, and then the GUBXII distribution. Also, the confidence bound for the GUBXII distribution was shortest. For $T = 20$, the$T-$ Week extreme wind speed estimate using the GEV distribution was highest, followed by that of the GUBXII distribution, and then Gumbel distribution. The confidence bounds obtained using the GEV distribution is also observed to be shortest. For $T = 100$,and $T = 200$, the $T-$ Week extreme wind speed estimate using the GUBXII is highest, and the confidence bounds of the Gumbel distribution are shortest.

In conclusion, we observe that the GUBXII distribution can be very effective when used in construction of confidence bounds for extreme wind speed estimates with short return periods. This implies that for wind regimes with frequent extreme wind speeds, the GUBXII distribution can be effectively used to give good predictions. On the other hand, the Gumbel distribution is observed to be best for longer return period extreme wind speeds predictions.

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