Survivor rates of new entrants into an undergraduate degree programme under clearance uncertainty

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Abstract. . Considering the clearance exercise in a Nigerian university setting, this study attempts to answer the question of the following sort: 'What is the survivor rate of candidates offered (provisional) admission into a course of study in the face of clearance uncertainty?' We represent the problem as a system of differential equations within the framework of a network model with constant risk and then solve it using the Laplace transform. Rather than relying on steady-states fractional flow rates, we provide a transient solution to the problem. The survivor function for each entry level derived for the problem sheds some light on situations where a decision is to be made on two (or more) preferred courses and thus evades possible trauma in the application process.

Résumé :
Cette étude prend place dans la problématique de l'admission des étudiants dans les universités nigériannes. Elle se propose de répondre aux questions semblables à celle-ci : quel est le taux de survie (final) des candidats provisoirement admis à un programme dans une situation d'admission finale incertaine? Nous modélisons ce problème grâce à un système d'équations différentielles dans le cadre d'un modèle de réseau à risque constant. La transformation de Laplace est notre outil pour résoudre le problème théorique.

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1. Introduction

Starting with the work of [Gani (1963)] mathematical models have been used as an
important tool to shed light on the educational process. The art of applying mathematical
techniques to a wide variety of problems in education has gained prominence in the literature
(for instance, [Jimenez and Salas-Velasco (2000)]; [Mancebon and Muniz (2008)];[Nicholls
(2009)];[Ekhosuehi and Osagiede (2013)];[Johnes (2015)]; [Ekhosuehi (2016)]). These studies
have shown that research in education is a worthwhile venture to educational planners,
managers and consumers of education. Our present study is centred on the admission stage
of the educational process in the context of the university setting in Nigeria. However,
the survivor function derived in this paper may be extended to other degree awarding
institutions in Nigeria such as: Nigerian Defence Academy, Nigerian Police Academy, and
some colleges of: education, technology, agriculture, etc. Without loss of generality, we
use the term *university* and *institution* interchangeably. More specifically, we consider the
clearance of a cohort of candidates who are offered (provisional) admission into a course
of study in the university. A cohort of candidates admitted in the same session is usually
identified by their matriculation (or registration) number. For example, in the University
of Benin with which the authors are acquainted, the first two digits of the matriculation
number indicate the year of entry.

Candidates are normally admitted into either the first year (100 Level) or the second year
(200 Level) of an undergraduate degree programme in the university. For a candidate to be
considered for admission into the first year of any course of study (course for short) in a
university full-time programme, the candidate must sit for the Unified Tertiary Matricula-
tion Examination (UTME) and obtain a reasonable score. The UTME is conducted by the
Joint Admission and Matriculation Board (JAMB). Similarly, candidates for direct entry
admission must apply through JAMB to the university. The university has the prerogative
to admit candidates into a course based on a certain cut-off mark (i.e., merit) and other
approved criteria (such as catchment area, educationally less developed area and other
selection criteria applicable to the institution) in consultation with JAMB, provided the
capacity requirement for the course is not exceeded. The capacity requirement refers to the
maximum number of students that will not cause undue strain on facilities in the university.

The practice of admission in most Nigerian universities is that it is done in batches. The
release of a list of candidates for (provisional) admission is usually followed by a clearance
exercise. The clearance exercise is carried out in the first year of entry to determine the
eligibility of candidates for the course for which provisional admission has been offered.
During this exercise, the candidate may be accepted as a student into the course of study
or rejected. The rejection of a candidate may be due to the failure of the candidate to
present the required documents for clearance (e.g., those awaiting results), deficiency in
the minimum entry requirement into the course of study, non-payment of a non-refundable
acceptance fee, voluntary withdrawal, etc. A candidate may also be relocated to another
course if the candidate does not meet the admission requirements for the course that
admission was initially offered, but satisfies that of the latter. The relocation of a candidate
to another course is treated here as a rejection from the course that admission was initially
offered. The survivor function derived in this study is centred on the course that admission

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was initially offered.

Considering the challenges candidates offered (provisional) admission into universities in Nigeria encounter, one common feature is the notion of uncertainty on whether they would be accepted or rejected. This arises from the uncertainty of the clearance exercise. This study is apt in the sense that it attempts to offer a theoretical explanation on the survivor rates of candidates who undergo the clearance exercise. We derive a system of differential equations from a network of transitions between any two states of the system, viz. the external environment (state 0), 100 Level (state 1) and 200 Level (state 2). This study contributes to the existing literature by providing a useful framework to assist candidates on the choice of courses as this would enable them evade possible trauma in the application process.

2. Theoretical framework

Consider a cohort of candidates of size $N(0)$ that are offered (provisional) admission into a course of study in the university (system hereafter) at the same session who should go through a clearance exercise. The candidates may enter the system at either 100 Level (state 1) or 200 Level (state 2). Let $N_i(t)$ be the number of candidates in State i at moment t, $i = 0, 1, 2$ and t is an instant of time (in days) of a session. The network (N, A) for the system with $N = \{0, 1, 2\}$ and $A = \{(0, 1), (1, 0), (0, 2), (2, 0)\}$ is described by the transition diagram in Figure 1.
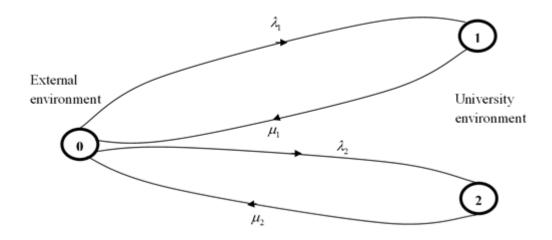

Fig. 1. Transition diagram for the admission process

The three circles represent the states that a candidate may be and the arrows indicate the changes of states which can occur. The symbol λ_i is the propensity of candidates to be

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cleared into state i and μ_i is the propensity of candidates to leave state i in the year of entry
for whatever reasons. The constancy of λ_i and μ_i implies that the transition propensities
do not depend on the time instance of the session. This would mean, for example, that the
time a candidate presents himself/herself for clearance during the period of the exercise
would be of no value in predicting the outcome of the exercise. The propensities λ_i and μ_i
are computed as $\lambda_i = \dfrac{\eta_i}{C_i}$ and $\mu_i = \dfrac{\vartheta_i}{C_i}$, where η_i is the number of candidates cleared into
state i over time,ϑ_i is the number of candidates who leave state i in the year of entry over
time, and C_i is the number of candidates offered (provisional) admission into state i over
time.

The proportion of candidates who remain in the system (i.e., the survivor function), $G(t)$,
is given as

$$G(t) = G_1(t) + G_2(t) \tag{1}$$

where $G_1(t) = \frac{N_1(t)}{N(0)}$ is the survivor function for state 1, $G_2(t) = \frac{N_2(t)}{N(0)}$ is the survivor
function for state 2, and $0 \leq G(t) \leq 1$. The relation $0 \leq G(t) \leq 1$ is used as a proxy for the
uncertainty driven by the clearance exercise. If $G(t) = 1$, then it is certain that everyone
offered provisional admission would be eventually cleared, otherwise $0 \leq G(t) < 1$.

The transition diagram in Figure 1 leads us to formulate the following differential equations
for the system

$$\frac{dN_0(t)}{dt} = -(\lambda_1 + \lambda_2)N_0(t) + \mu_1 N_1(t) + \mu_2 N_2(t) \tag{2}$$

$$\frac{dN_1(t)}{dt} = \lambda_1 N_0(t) - \mu_1 N_1(t) \tag{3}$$

$$\frac{dN_2(t)}{dt} = \lambda_2 N_0(t) = \mu_2 N_2(t) \tag{4}$$

Since every candidate offered (provisional) admission into the system must come from a
state external to the university, $N_0(0) = N(0), N_1(0) = 0$ and $N_2(0) = 0$. Rather than de-
veloping a discrete-time model for the system, we formulate the problem in continuous-time.
This is because admission is done in batches and the clearance to ascertain the eligibility
of candidates for a course is carried out continuously throughout the session.

We represent equations 2, 3 and 4 in a matrix-vector form as

$$\frac{d\mathbf{N}(t)}{dt} = \Gamma\mathbf{N}(t) \tag{5}$$

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where

$$\frac{d\mathbf{N}(t)}{dt} = \begin{bmatrix} \frac{dN_0(t)}{dt} \\ \frac{dN_1(t)}{dt} \\ \frac{dN_2(t)}{dt} \end{bmatrix},$$

$$\Gamma = \begin{bmatrix} (\lambda_1 + \lambda_2) & \mu_1 & \mu_2 \\ \lambda_1 & -\mu & 0 \\ \lambda_2 & 0 & -\mu_2 \end{bmatrix}$$

and

$$\mathbf{N}(t) = \begin{bmatrix} N_0(t) \\ N_1(t) \\ N_2(t) \end{bmatrix},$$

so that the initial structure of the system is

$$\mathbf{N}(0) = \begin{bmatrix} N(0) \\ 0 \\ 0 \end{bmatrix}.$$

Let $\mathbf{M}(s)$ be the Laplace transform of $\mathbf{N}(t)$. That is

$$\mathbf{M}(s) = \int_0^\infty \exp(-st)\mathbf{N(t)}dt \tag{6}$$

By integration by parts, equation 6 simplifies to

$$\mathbf{M(s)} = \left[s\mathbf{I} - \Gamma\right]^{-1}\mathbf{N}(0) \tag{7}$$

where the reciprocal matrix $\left[s\mathbf{I} - \Gamma\right]^{-1}$ is given as

$$\left[s\mathbf{I} - \Gamma\right]^{-1} = \frac{1}{s(s-\alpha)(s-\beta)} \times$$

$$\begin{bmatrix} (s+\mu_1)(s+\mu_2) & \mu_1(s+\mu_2) & \mu_2(s+\mu_1) \\ \lambda_1(s+\mu_2) & (s+\lambda_1+\lambda_2)(s+\mu_2) - \mu_2\lambda_2 & \mu_2\lambda_1 \\ \lambda_2(s+\mu_1) & \mu_1\lambda_2 & (s+\lambda_1+\lambda_2)(s+\mu_1) - \mu_1\lambda_1 \end{bmatrix},$$

with

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$$\alpha = -\frac{1}{2} \times$$

$$\left(\lambda_1 + \lambda_2 + \mu_1 + \mu_2\right.$$

$$\left. + \sqrt{(\lambda_1 + \lambda_2 + \mu_1 + \mu_2)^2 - 4(\lambda_1\mu_2 + \lambda_2\mu_1 + \mu_1\mu_2)}\right) < 0$$

and

$$\beta = -\frac{1}{2} \times$$

$$\left(\lambda_1 + \lambda_2 + \mu_1 + \mu_2\right.$$

$$\left. - \sqrt{(\lambda_1 + \lambda_2 + \mu_1 + \mu_2)^2 - 4(\lambda_1\mu_2 + \lambda_2\mu_1 + \mu_1\mu_2)}\right) < 0.$$

Resolving this reciprocal matrix into partial fractions, we get

$$\left[s\mathbf{I} - \mathbf{Q}\right]^{-1} = \frac{1}{s}\mathbf{A} + \frac{1}{(s-\alpha)}\mathbf{B} + \frac{1}{(s-\beta)}\mathbf{C} \tag{8}$$

where

$$\mathbf{A} = \frac{1}{\alpha\beta} \begin{bmatrix} \mu_1\mu_2 & \mu_1\mu_2 & \mu_2\mu_1 \\ \lambda_1\mu_2 & \lambda_1\mu_2 & \mu_2\lambda_1 \\ \lambda_2\mu_1 & \mu_1\lambda_2 & \lambda_2\mu_1 \end{bmatrix},$$

and

$$\mathbf{B} = \frac{1}{\alpha(\alpha-\beta)} \times$$

$$\begin{bmatrix} (\alpha+\mu_1)(\alpha+\mu_2) & \mu_1(\alpha+\mu_2) & \mu_2(\alpha+\mu_1) \\ \lambda_1(\alpha+\mu_2) & (\alpha+\lambda_1+\lambda_2)(\alpha+\mu_2)-\mu_2\lambda_2 & \mu_2\lambda_1 \\ \lambda_2(\alpha+\mu_1) & \mu_1\lambda_2 & (\alpha+\lambda_1+\lambda_2)(\alpha+\mu_1)-\mu_1\lambda_1 \end{bmatrix}$$

and

$$\mathbf{C} = \frac{1}{\beta(\beta-\alpha)} \times$$

$$\begin{bmatrix} (\beta+\mu_1)(\beta+\mu_2) & \mu_1(\beta+\mu_2) & \mu_2(\beta+\mu_1) \\ \lambda_1(\beta+\mu_2) & (\beta+\lambda_1+\lambda_2)(\beta+\mu_2)-\mu_2\lambda_2 & \mu_2\lambda_1 \\ \lambda_2(\beta+\mu_1) & \mu_1\lambda_2 & (\beta+\lambda_1+\lambda_2)(\beta+\mu_1)-\mu_1\lambda_1 \end{bmatrix}.$$

Equation 8 is equivalent to writing

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$$\left[s\mathbf{I} - \Gamma\right]^{-1} = \int_0^\infty \exp\left(-st\right)\mathbf{A}dt$$
$$+ \int_0^\infty \exp\left(-(s-\alpha)t\right)\mathbf{B}dt$$
$$+ \int_0^\infty \exp\left(-(s-\beta)t\right)\mathbf{C}dt$$

Therefore

$$\mathbf{M}(s) = \left[s\mathbf{I} - \Gamma\right]^{-1}\mathbf{N}(0)$$
$$= \int_0^\infty \exp\left(-st\right)\Big[\mathbf{A} + \exp\left(\alpha t\right)\mathbf{B} + \exp\left(\beta t\right)\mathbf{C}\Big]\mathbf{N}(0)dt.$$

Hence we have

$$\mathbf{N}(t) = \Big[\mathbf{A} + \exp\left(\alpha t\right)\mathbf{B} + \exp\left(\beta t\right)\mathbf{C}\Big]\mathbf{N}(0) \tag{9}$$

From equation 9, we obtain the survivor functions $G_1(t) = \frac{N_1(t)}{N(0)}$ and $G_2(t) = \frac{N_2(t)}{N(0)}$ as

$$G_1(t) = \frac{\lambda_1\mu_2}{\alpha\beta} + \frac{\lambda_1(\alpha+\mu_2)}{\alpha(\alpha-\beta)}\exp\left(\alpha t\right)$$
$$+ \frac{\lambda_1(\beta+\mu_2)}{\beta(\beta-\alpha)}\exp\left(\beta t\right), \tag{10}$$

$$G_2(t) = \frac{\lambda_2\mu_1}{\alpha\beta} + \frac{\mu_2(\alpha+\mu_1)}{\alpha(\alpha-\beta)}\exp\left(\alpha t\right)$$
$$+ \frac{\mu_2(\beta+\mu_1)}{\beta(\beta-\alpha)}\exp\left(\beta t\right), \tag{11}$$

respectively. Based on these survivor functions for each level, we have

$$\lim_{t\to\infty} G(t) = \frac{\lambda_1\mu_2 + \lambda_2\mu_1}{\alpha\beta}$$

The choice of a course by a candidate may be guided by the magnitude of $G_1(t)$ or $G_2(t)$ depending on whether the candidate is applying for admission into 100 Level or 200 Level, respectively. A UTME candidate, for example, who is indecisive on two preferred courses, say $C1$ and $C2$, may select $C2$ if, on the average, the survivor rate for the course $C2$ is higher than that of course $C1$.

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3. Illustrative example

To get an insight into the behaviour of the survivor functions $G_1(t)$ and $G_2(t)$,the simulation of observations on the number of candidates cleared and the number of candidates who leave the system in the year of entry is a helpful tool. We randomly generate ten replicates for each set of observations using the function *rand* in the MATLAB environment. The MATLAB command "rand" generates uniform random numbers. This command in MATLAB returns a list of simulated sequence of observations which are used to compute the propensities, λ_i and μ_i. In calculating λ_i and μ_i, we set the number of candidates offered provisional admission to be 100 and 20 for the 100 Level and the 200 Level, respectively. We assume that the number of candidates cleared is partly constant and then access the simulated sequence of observations and the propensities with

```
clc
n=10;  ad=[70+rand(1,n)*30];
rj=100-ad;  v1=mean(ad)/100,
u1=mean(rj)/100,

ad2=[10+rand(1,n)*10];
rj2=20-ad2;  v2=mean(ad2)/20,
u2=mean(rj2)/20,
```

However, one should not attach undue importance to this simulation as the purpose of this illustration is to give an insight on the behaviour of the survivor functions. From the simulated observations, we obtain the plot in Figure 2, which displays the survivor rates for the 100 Level and the 200 Level. As shown, it is easy to see that the sum of $G_1(t)$ and $G_2(t)$ lies within the bounds 0 and 1, and that the survivor rates are increasing monotonically with time (in days). This is what is expected as in the real life situation the clearance of candidates into any course does not reduce the number already cleared but gradually adds to it until the desired capacity of the course is attained. This monotone pattern may be attributed to the time spent on verification of documents, the time required to certify and collate all relevant documents for the clearance, the traffic encountered in the payment of acceptance fees, etc. so much so that the clearance exercise is not instantaneous.

4. Concluding remarks

This paper characterises in a theoretical way the survivor rates of candidate who has been offered (provisional) admission into a course of study when they undergo the clearance exercise. It turns out that the survivor function derived in this study, which utilises a network model with constant risk as its theoretical underpinning, is a monotonically increasing function. The survivor function may be used as a decision making tool on the choice of courses. Nonetheless, this approach is not meant to replace the candidate's intuition, but to guide the candidate on decisions based on the survivor rates. The results obtained from the simulation suggest that the survivor function is a descriptive realisation of the clearance process. However, since it has not been validated with real life data, it is still only a possible explanation. To validate the survivor function, it would be necessary to implement policy changes de-
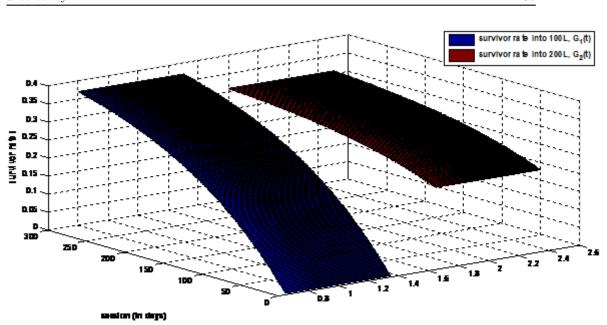
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Fig. 2. Survivor rates for the simulated observations.

signed to affect the propensities λ_i or μ_i and demonstrate that the monotonically increasing
pattern do occur. This is not yet done.

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