Ann. Funct. Anal. 2 (2011), no. 1, 51–58
ANNALS OF FUNCTIONAL ANALYSIS
ISSN: 2008-8752 (electronic)
URL: www.emis.de/journals/AFA/

EDELSTEIN TYPE FIXED POINT THEOREMS

ERDAL KARAPINAR[1]

Communicated by G. L. Acedo

ABSTRACT. Recently, Suzuki [Nonlinear Anal. 71 (2009), no. 11, 5313–5317.] published a paper on which Edelstein's fixed theorem was generalized. In this manuscript, we give some theorems which are the generalization of the fixed theorem of Suzuki's Theorems and thus Edelstein's result [J. London Math. Soc. 37 (1962), 74-79].

1. Introduction and Preliminaries

A self-mapping T on a metric space X is called contraction if for each $x, y \in X$, there exists a constant $k \in [0, 1)$ such that

$$d(Tx, Ty) \leq d(x, y).$$

Due to Banach [1], we know that every contraction on a complete metric space has a unique fixed point. This theorem, known as the Banach contraction mapping principle, is formulated in his thesis in 1920 and published in 1922. The Banach contraction mapping principle has preserved its importance in Fixed Point Theory. This crucial result of Banach has many applications not only in several branches of mathematics but also in economics.

After Banach, many authors attempt to generalize the Banach contraction mapping principle such as Kannan [6], Reich [11], Hardy and Rogers [5], Ćirić [2] and many others.

Very recently, Suzuki proved the following fixed point theorem:

Theorem 1.1. *(Suzuki [14].) Let (X, d) be a compact metric space and let T be a mapping on X. Assume $\frac{1}{2}d(x, Tx) < d(x, y)$ implies $d(Tx, Ty) < d(x, y)$ for all $x, y \in X$. Then T has a unique fixed point.*

Date: Received: 28 January 2011; Accepted: 23 March 2011.
2010 *Mathematics Subject Classification.* Primary 47H10; Secondary: 54E50, 28B05.
Key words and phrases. Fixed point theorem, contraction.

This result is based on the following two theorems:

Theorem 1.2. *(Edelstein [4].) Let (X,d) be a compact metric space and let T be a mapping on X. Assume $d(Tx,Ty) < d(x,y)$ for all $x,y \in X$ with $x \neq y$. Then T has a unique fixed point.*

Suzuki also suggest the following fixed point theorem:

Theorem 1.3. *(Suzuki [15].) Define a non-increasing function θ from [0,1) onto (1/2,1] by*

$$\theta(r) = \begin{cases} 1 & \text{if } 0 \leq r \leq (\sqrt{5}-1)/2, \\ (1-r)r^{-2} & \text{if } (\sqrt{5}-1)/2 \leq r \leq \ 2^{-1/2}, \\ (1+r)^{-1} & \text{if } 2^{-1/2} \leq r < 1. \end{cases}$$

Then for a metric space (X,d), the following are equivalent:

(1) *X is complete.*
(2) *Every mapping T on X satisfying the following has a fixed point: There exists $r \in [0,1)$ such that $\theta(r)d(x,Tx) \leq d(x,y)$ implies $d(Tx,Ty) \leq rd(x,y)$ for all $x,y \in X$.*

The author [7] suggests some theorems which generalize the result of Suzuki [14] which was also considered by many authors (see e.g. [3, 8, 9, 10, 12]). In this manuscript, main theorem is on the form of Edelstein's fixed point theorem and generalizes the results of both [7] and Suzuki [14].

2. Main Results

We prove the following theorems:

Theorem 2.1. *Let T be a self mapping on a compact metric space (X,d). Assume that*

$$\frac{1}{2}d(x,Tx) < d(x,y) \Rightarrow d(Tx,Ty) < M(x,y) \quad \text{for all } x,y \in X, \tag{2.1}$$

where $M(x,y) = \max\{d(x,y), d(Tx,x), d(y,Ty), \frac{1}{2}d(Tx,y), \frac{1}{2}d(x,Ty)\}$. Then, T has a unique fixed point $z \in X$, that is, $Tz = z$.

Proof. Set $\theta = \inf\{d(x,Tx) : x \in X\}$ and choose a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} d(x_n,Tx_n) = \theta$. Regarding that X is compact, without loss of generality, assume that $\{x_n\}$ and $\{Tx_n\}$ converge to some points z and w in X, respectively.

Claim that θ is equal to zero. Assume $\theta > 0$. Notice that

$$\lim_{n\to\infty} d(x_n,w) = d(z,w) = \lim_{n\to\infty} d(x_n,Tx_n) = \theta.$$

One can choose $k \in \mathbb{N}$ in a way that

$$\frac{2}{3}\theta < d(x_n,w) \quad \text{and} \quad d(x_n,Tx_n) < \frac{4}{3}\theta,$$

for each $n \geq k$. Consequently, $\frac{1}{2}d(x_n,Tx_n) < d(x_n,w)$ for each $n \geq k$. By (2.1), one can get $d(Tx_n,Tw) < M(x_n,w)$ for each $n \geq k$, where $M(x_n,w) = \max\{d(x_n,w), d(Tx_n,x_n), d(w,Tw), \frac{1}{2}d(Tx_n,w), \frac{1}{2}d(x_n,Tw)\}$. Thus we have

$$d(w,Tw) = \lim_{n\to\infty} d(Tx_n,Tw) < \lim_{n\to\infty} M(x_n,w). \tag{2.2}$$

where

$$\begin{aligned}\lim_{n\to\infty} M(x_n, w) &= \max\{d(z, w), d(w, z), d(w, Tw), \tfrac{1}{2}d(w, w), \tfrac{1}{2}d(z, Tw)\} \\ &= \max\{d(z, w), d(w, Tw), \tfrac{1}{2}d(z, Tw)\}\end{aligned}$$

If $\lim_{n\to\infty} M(x_n, w) = d(w, Tw)$ then the expression (2.2) turns into

$$d(w, Tw) = \lim_{n\to\infty} d(Tx_n, Tw) < \lim_{n\to\infty} M(x_n, w) = d(Tw, w).$$

which is impossible.

If $\lim_{n\to\infty} M(x_n, w) = \frac{1}{2}d(Tw, z)$ then the expression (2.2) turns into

$$\begin{aligned}d(w, Tw) = \lim_{n\to\infty} d(Tx_n, Tw) &< \lim_{n\to\infty} M(x_n, w) \\ = \lim_{n\to\infty} \tfrac{1}{2}d(Tw, z) &\leq \tfrac{1}{2}[d(z, w) + d(w, Tw)].\end{aligned}$$

Thus, $d(w, Tw) < d(z, w) = \theta$.

If $\lim_{n\to\infty} M(x_n, w) = d(w, z)$ then the expression (2.2) turns into

$$d(w, Tw) = \lim_{n\to\infty} d(Tx_n, Tw) < \lim_{n\to\infty} M(x_n, w) = d(w, z) = \theta..$$

In any case, (2.2)

$$d(w, Tw) < d(w, z) = \theta.$$

On account of the definition of θ, one can conclude that $d(w, Tw) = \theta$. Notice that we always have the inequality $\frac{1}{2}d(w, Tw) < d(w, Tw)$. Again by applying (2.1), one can obtain

$$d(Tw, T^2w) < M(w, Tw) \tag{2.3}$$

where

$$\begin{aligned}M(w, Tw) &= \max\{d(w, Tw), d(Tw, w), d(Tw, T^2w), \tfrac{1}{2}d(Tw, Tw), \tfrac{1}{2}d(w, T^2w)\} \\ &= \max\{d(w, Tw), d(Tw, T^2w), \tfrac{1}{2}d(w, T^2w)\}\end{aligned}$$

Clearly, the case $M(w, Tw) = d(Tw, T^2w)$ contradicts (2.3). If we consider the case $M(w, Tw) = d(w, Tw)$ by considering (2.3), we get $d(Tw, T^2w) < d(Tw, w) = \theta$. For the last case $M(w, Tw) = \frac{1}{2}d(w, T^2w)$, by (2.3), we have $d(Tw, T^2w) < \frac{1}{2}d(w, T^2w) \leq \frac{1}{2}[d(w, Tw) + d(Tw, T^2w)]$ which is equivalent to $d(Tw, T^2w) < d(Tw, w) = \theta$.

In any case, this is a contradiction with the definition of θ. Hence $\theta = 0$.

We assert that T has a fixed point. To show this, we use the method of Reductio ad absurdum. Suppose T has no fixed point. Since $0 < \frac{1}{2}d(Tx_n, x_n) < d(Tx_n, x_n)$ is true for each n, the inequality $d(Tx_n, T^2x_n) < M(x_n, Tx_n)$ holds for every $n \in \mathbb{N}$, where

$$\begin{aligned}M(x_n, Tx_n) &= \max\{d(x_n, Tx_n), d(Tx_n, x_n), d(Tx_n, T^2x_n), \\ &\quad \tfrac{1}{2}d(Tx_n, Tx_n), \tfrac{1}{2}d(x_n, T^2x_n)\} \\ &= \max\{d(x_n, Tx_n), d(Tx_n, T^2x_n), \tfrac{1}{2}d(x_n, T^2x_n)\}.\end{aligned}$$

Hence, simple calculations yield that

$$d(Tx_n, T^2x_n) < d(Tx_n, x_n) \tag{2.4}$$

holds for each $n \in \mathbb{N}$ and

$$\lim_{n\to\infty} d(z, Tx_n) = d(z, w) = \lim_{n\to\infty} d(Tx_n, x_n) = \theta = 0.$$

Thus, $z = w$. In other words, $\{x_n\}$ and $\{Tx_n\}$ converge to the same point. Due to triangular inequality and considering (2.4), one can obtain that

$$\begin{aligned}\lim_{n\to\infty} d(z, T^2x_n) &\leq \lim_{n\to\infty} [d(z, Tx_n) + d(Tx_n, T^2x_n)] \\ &< \lim_{n\to\infty} [d(z, Tx_n) + d(x_n, Tx_n)] = 2d(z, z) = 2\theta = 0.\end{aligned}$$

Hence, $\{T^2x_n\}$ converges to z, either.

Suppose that

$$d(x_n, z) \leq \frac{1}{2}d(x_n, Tx_n), \quad \text{and} \quad d(Tx_n, z) \leq \frac{1}{2}d(Tx_n, T^2x_n) \tag{2.5}$$

holds. Consequently, applying (2.4), (2.5) and using triangular inequality, we have

$$\begin{aligned}d(x_n, Tx_n) &\leq d(x_n, z) + d(Tx_n, z) \leq \tfrac{1}{2}d(x_n, Tx_n) + \tfrac{1}{2}d(Tx_n, T^2x_n) \\ &< \tfrac{1}{2}d(x_n, Tx_n) + \tfrac{1}{2}d(x_n, Tx_n) = d(x_n, Tx_n)\end{aligned}$$

This is a contradiction. Thus, either

$$d(x_n, z) > \frac{1}{2}d(x_n, Tx_n), \quad \text{or} \quad d(Tx_n, z) > \frac{1}{2}d(Tx_n, T^2x_n)$$

holds for each $n \in \mathbb{N}$. Regarding (2.1), one of them holds:

$$d(Tx_n, Tz) < M(x_n, z), \tag{2.6}$$

where

$$M(x_n, z) = \max\{d(x_n, z), d(Tx_n, x_n), d(z, Tz), \frac{1}{2}d(Tx_n, z), \frac{1}{2}d(x_n, Tz)\}.$$

$$d(T^2x_n, Tz) < M(Tx_n, z), \tag{2.7}$$

where

$$M(Tx_n, z) = \max\{d(Tx_n, z), d(T^2x_n, Tx_n), d(z, Tz), \frac{1}{2}d(T^2x_n, z), \frac{1}{2}d(Tx_n, Tz)\}.$$

It is equivalent to saying that either

(i) There is an infinite subset I of $\mathbb{N}$ so that $d(Tx_n, Tz) < M(x_n, z)$ for all $n \in I$, or

(ii) There is an infinite subset J of $\mathbb{N}$ so that $d(T^2x_n, Tz) < M(Tx_n, z)$ for all $n \in J$,

holds.

Consider the case (2.6):

$$d(z, Tz) = \lim_{n\in I, n\to\infty} d(Tx_n, Tz) < \lim_{n\in I, n\to\infty} M(x_n, z),$$

where

$$M(x_n, z) = \max\{d(x_n, z), d(Tx_n, x_n), d(z, Tz), \frac{1}{2}d(Tx_n, z), \frac{1}{2}d(x_n, Tz)\}.$$

For the case $M(x_n, z) = d(z, Tz)$ we have

$$\begin{aligned}d(z, Tz) &= \lim_{n\in I, n\to\infty} d(Tx_n, Tz) < \lim_{n\in I, n\to\infty} M(x_n, z) \\ &= \lim_{n\in I, n\to\infty} d(z, Tz) = d(z, Tz),\end{aligned}$$

which is a contradiction. For the case $M(x_n, z) = \frac{1}{2}d(Tz, x_n)$ we have

$$\begin{aligned} d(z, Tz) &= \lim_{n\in I, n\to\infty} d(Tx_n, Tz) < \lim_{n\in I, n\to\infty} M(x_n, z) \\ &= \lim_{n\in I, n\to\infty} \tfrac{1}{2}d(Tz, x_n) = \tfrac{1}{2}d(Tz, z), \end{aligned}$$

which is also a contradiction. For the other cases, by simple calculation, one can obtain that

$$d(z, Tz) < d(z, z) = 0.$$

Thus, one can conclude that $Tz = z$.

For the other case (2.7):

$$d(z, Tz) = d(T^2x_n, Tz) < \lim_{n\in J, n\to\infty} \frac{1}{2}M(Tx_n, z) \text{ where}$$

$$M(Tx_n, z) = \max\{d(Tx_n, z), d(T^2x_n, Tx_n), d(z, Tz), \frac{1}{2}d(T^2x_n, z), \frac{1}{2}d(Tx_n, Tz)\}.$$

For the case $M(Tx_n, z) = d(z, Tz)$ we have

$$\begin{aligned} d(z, Tz) &= \lim_{n\in J, n\to\infty} d(T^2x_n, Tz) < \lim_{n\in J, n\to\infty} \tfrac{1}{2}M(Tx_n, z) \\ &= \lim_{n\in J, n\to\infty} d(z, Tz) = d(z, Tz), \end{aligned}$$

which is a contradiction. For the case $M(Tx_n, z) = \frac{1}{2}d(Tz, Tx_n)$ we have

$$\begin{aligned} d(z, Tz) &= \lim_{n\in J, n\to\infty} d(T^2x_n, Tz) < \lim_{n\in J, n\to\infty} \tfrac{1}{2}M(Tx_n, z) \\ &= \lim_{n\in J, n\to\infty} \tfrac{1}{2}d(Tz, Tx_n) = \tfrac{1}{2}d(Tz, z), \end{aligned}$$

which is also a contradiction. For the other cases, by simple calculation, one can obtain that

$$d(z, Tz) < d(z, z) = 0.$$

Thus, (2.6) and (2.7) imply the same conclusion, that is, $Tz = z$. This is a contradiction with assumption. Hence, T has a fixed point. In other words, there exists $z \in X$ such that $Tz = z$.

To show $z \in X$ is the unique fixed point of T, take $y \in X$ where $y \neq z$ and $Ty = y$. Thus, $d(y, z) > 0$ and $0 = \frac{1}{2}d(z, Tz) < d(z, y)$ are satisfied. By (2.1), we have

$$d(z, y) = d(Tz, Ty) < M(z, y)$$

where

$$M(z, y) = \max\{d(z, y), d(Tz, z), d(y, Ty), \frac{1}{2}d(z, Ty), \frac{1}{2}d(Tz, y)\} = d(z, y)$$

which implies that $d(z, y) < d(z, y)$ which is a contradiction. Hence, z is the unique fixed point of T. □

Regarding the very recent paper of Singh and Mishra [13], we state the following theorem:

Theorem 2.2. *Let $S, T : Y \to X$ be such that $S(Y) \subset T(Y)$ and $T(Y)$ is a compact subspace of a metric space (X, d). Assume that*

$$Tx = Ty \Rightarrow Sx = Sy \quad \textit{for all } x, y \in Y,$$

and

$$\frac{1}{2}d(Tx,Sx) < d(Tx,Ty) \Rightarrow d(Sx,Sy) < M(Tx,Ty) \quad \textit{for all } x,y \in Y,$$

where $M(Tx,Ty) = \max\{d(Tx,Ty), d(Sx,Tx), d(Ty,Sy), \frac{1}{2}d(Sx,Ty), \frac{1}{2}d(Tx,Sy)\}$. *Then, S and T have a coincidence point $z \in Y$, that is, $Tz = Sz$. Further, if $Y = X$, then S and T have a unique common fixed point provided that S and T commute at z.*

Proof. Set $F : T(Y) \to T(Y)$ by $F\alpha = S(T^{-1}\alpha)$ for each $\alpha \in T(Y)$. The map F is well-defined. Indeed, take $x \in T^{-1}\alpha$, then $F\alpha = Sx$. Recall that $S(Y) \subset T(Y)$, thus, $F\alpha \subset T(Y)$. Now, take $x, y \in T^{-1}\alpha$ such that $\beta = Sx$ and $\gamma = Sy$. Notice that $Tx = Ty$, thus, $\beta = \gamma$ and F is well-defined.

Take $\alpha, \beta \in T(Y)$ with $\alpha \neq \beta$, so $T^{-1}\alpha \cap T^{-1}\beta = \emptyset$. Assume that $\frac{1}{2}d(\alpha, F\alpha) < d(\alpha, \beta)$ for distinct $\alpha, \beta \in T(Y)$. Thus, for $x \in T^{-1}\alpha$ and $y \in T^{-1}\beta$, one can observe that

$$\frac{1}{2}d(Tx,Sx) = \frac{1}{2}d(\alpha, F\alpha) < d(\alpha,\beta) = d(Tx,Ty).$$

By assumption of the theorem, this inequality implies that $d(Sx,Sy) < M(Tx,Ty)$ and thus $d(F\alpha, F\beta) < M(\alpha,\beta)$ where

$$M(Tx,Ty) = M(\alpha,\beta) = \max\{d(\alpha,\beta), d(F\alpha,\alpha), d(\beta,F\beta), \frac{1}{2}d(F\alpha,\beta), \frac{1}{2}d(\alpha,F\beta)\}.$$

Due to Theorem 2.1, F has a unique fixed point ω, that is $F\omega = \omega$. Thus, for any $z \in T^{-1}\omega$, we have $Sz = F\omega = \omega = Tz$, that is, z is a coincidence point of S and T.

If S and T are commuting at z, then $Sz = Tz$ implies that $SSz = STz = TSz = TTz$ and $S\omega = T\omega$. If $Sz \neq SSz$, then $\frac{1}{2}d(Tz,Sz) = 0 < d(Tz,TTz) = d(Tz,TSz)$ and it implies that

$$d(\omega, S\omega) = d(Sz,SSz) < M(Tz,TSz) \tag{2.8}$$

where

$$\begin{aligned} M(Tz,TSz) &= \max\{d(Tz,TSz), d(Tz,Sz), d(TSz,SSz), \\ &\quad \tfrac{1}{2}d(Tz,SSz), \tfrac{1}{2}d(TSz,Sz)\} \\ &= \max\{d(Tz,TSz) = d(Tz,STz), \tfrac{1}{2}d(Tz,SSz) = \tfrac{1}{2}d(TSz,Sz)\} \\ &= \max\{d(\omega,S\omega), \tfrac{1}{2}d(\omega,S\omega)\} = d(\omega,S\omega) \end{aligned}$$

Thus, the expression (2.8) turns into

$$d(\omega,S\omega) = d(Sz,SSz) < M(Tz,TSz) = d(\omega,S\omega)$$

which is a contradiction. Thus, ω is a common fixed point.

Now, we claim that the common fixed point is a unique. Assume the contrary, that is, there is $\upsilon \neq \omega$ and $S\upsilon = \upsilon = T\upsilon$. Since $\frac{1}{2}d(T\omega,S\omega) = 0 < d(T\omega,T\upsilon)$, we have

$$d(\upsilon,\omega) = d(S\upsilon,S\omega) < M(T\upsilon,T\omega) \tag{2.9}$$

where

$$\begin{aligned} M(T\upsilon,T\omega) &= \max\{d(T\upsilon,T\omega), d(T\upsilon,S\upsilon), d(T\omega,S\omega), \tfrac{1}{2}d(T\upsilon,S\omega), \tfrac{1}{2}d(T\omega,S\upsilon)\} \\ &= d(\upsilon,\omega). \end{aligned}$$

Thus, the expression (2.9) turns into

$$d(\upsilon,\omega)=d(S\upsilon,S\omega)<M(T\upsilon,T\omega)=d(\upsilon,\omega)$$

which is a contradiction. Thus, ω is the unique common fixed point of S and T. □

Regarding the analogy, we omit the proofs of the following theorems:

Theorem 2.3. *Let T be a self mapping on a compact metric space (X,d). Assume that*

$$\frac{1}{2}d(x,Tx)<d(x,y)\Rightarrow d(Tx,Ty)<M(x,y)\quad \text{for all } x,y\in X,$$

where $M(x,y)=\max\{d(x,y),d(Tx,x),d(y,Ty),\frac{1}{2}[d(Tx,y)+d(x,Ty)]\}$. Then, T has a unique fixed point $z\in X$, that is, $Tz=z$.

Theorem 2.4. *Let $S,T:Y\to X$ be such that $S(Y)\subset T(Y)$ and $T(Y)$ is a compact subspace of a metric space (X,d). Assume that*

$$Tx=Ty\Rightarrow Sx=Sy\quad \text{for all } x,y\in Y,$$

and

$$\frac{1}{2}d(Tx,Sx)<d(Tx,Ty)\Rightarrow d(Sx,Sy)<M(Tx,Ty)\quad \text{for all } x,y\in Y,$$

where

$$M(Tx,Ty)=\max\{d(Tx,Ty),d(Sx,Tx),d(Ty,Sy),\frac{1}{2}[d(Sx,Ty)+d(Tx,Sy)]\}.$$

Then, S and T have a coincidence point $z\in Y$, that is, $Tz=Sz$. Further, if $Y=X$, then S and T have a unique common fixed point provided that S and T commutes at z.

Theorem 2.5. *Let T be a self mapping on a compact metric space (X,d). Assume that*

$$\frac{1}{2}d(x,Tx)<d(x,y)\Rightarrow d(Tx,Ty)<M(x,y)\quad \text{for all } x,y\in X,$$

where $M(x,y)\in\{d(x,y),d(Tx,x),d(y,Ty),\frac{1}{2}[d(Tx,y)+d(x,Ty)]\}$. Then, T has a unique fixed point $z\in X$, that is, $Tz=z$.

Theorem 2.6. *Let $S,T:Y\to X$ be such that $S(Y)\subset T(Y)$ and $T(Y)$ is a compact subspace of a metric space (X,d). Assume that*

$$Tx=Ty\Rightarrow Sx=Sy\quad \text{for all } x,y\in Y,$$

and

$$\frac{1}{2}d(Tx,Sx)<d(Tx,Ty)\Rightarrow d(Sx,Sy)<M(Tx,Ty)\quad \text{for all } x,y\in Y,$$

where $M(Tx,Ty)\in\{d(Tx,Ty),d(Sx,Tx),d(Ty,Sy),\frac{1}{2}[d(Sx,Ty)+d(Tx,Sy)]\}$. Then, S and T have a coincidence point $z\in Y$, that is, $Tz=Sz$. Further, if $Y=X$, then S and T have a unique common fixed point provided that S and T commutes at z.

References

[1] S. Banach, *Sur les operations dans les ensembles abstraits et leur application aux equations itegrales*, Fund. Math., **3** (1922), 133–181.

[2] L.B. Ciric, *A generalization of Banach principle*, Proc. Amer. Math. Soc., **45** (1974), 727–730.

[3] S. Dhompongsa and H. Yingtaweesittikul, *Fixed point for multivalued mappings and the metric completeness*, Fixed Point Theory Appl. **2009**, Art. ID 972395, 15 pp.

[4] M. Edelstein, *On fixed and periodic points under contractive mappings*, J. London Math. Soc. **37** (1962), 74-79.

[5] G.E. Hardy and T.D. Rogers, *A generalization of a fixed point theorem of Reich*, Canad. Math. Bull. **16** (1973), 201–206.

[6] R. Kannan, *Some results on fixed points*, Bull. Cal. Math. Soc. **60** (1968), 71–76.

[7] E. Karapınar and K. Tas *Generalized (C)-conditions and related fixed point theorems*, Comput Math. Appl. **61** (2011), no. 11, 3370–3380.

[8] M. Kikkawa and T. Suzuki, *Some similarity between contractions and Kannan mappings*, Fixed Point Theory Appl. **2008**, Art. ID 649749, 8 pp.

[9] M. Kikkawa and T. Suzuki, *Three fixed point theorems for generalized contractions with constants in complete metric spaces*, Nonlinear Anal., **69** (2008), 2942-2949.

[10] G. Mot and A. Petrusel, *Fixed point theory for a new type of contractive multivalued operators*, Nonlinear Anal. **70** (2009), 3371-3377.

[11] S. Reich, *Kannans fixed point theorem*, Boll.Un. Mat. Ital. (4) **4** (1971), 1-11.

[12] Sh. Rezapour, S. M. A. Aleomraninejad and N. Shahzad, *On fixed point generalizations of Suzukis method*, Appl. Math. Lett. (2011), doi:10.1016/j.aml.2010.12.025.

[13] S.L. Singh and S.N. Mishra, *Remarks on Recent Fixed Point Theorems*, Fixed Point Theory Appl. **2010**, Art. ID 452905, 18 pp.

[14] T. Suzuki, *A new type of fixed point theorem in metric spaces*, Nonlinear Anal. **71** (2009), no. 11, 5313–5317.

[15] T. Suzuki, *A generalized Banach contraction principle that characterizes metric completeness*, Proc. Amer. Math. Soc. **136** (2008), 1861-1869.

[1] Department of Mathematics, Atilim University, İncek, Ankara 06836, Turkey.
E-mail address: erdalkarapinar@yahoo.com, ekarapinar@atilim.edu.tr