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COMMON COUPLED FIXED POINT THEOREMS IN d-COMPLETE TOPOLOGICAL SPACES

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ABSTRACT. In this paper, we give two unique common coupled fixed point theorems for mappings satisfying a generalized condition in d-complete topological spaces.

1. INTRODUCTION AND PRELIMINARIES

In 1992, Hicks [5] introduced the notion of d-complete topological spaces as a generalization of complete metric spaces.

Definition 1.1. (See [5]) Let (X, τ) be a topological space and $d : X \times X \to [0, \infty)$ satisfy

(i) $d(x, y) = 0$ if and only if $x = y$,
(ii) for any sequence $\{x_n\}$ in X,

$$\sum_{n=1}^{\infty} d(x_n, x_{n+1}) < \infty \Rightarrow \{x_n\} \text{ is convergent in } (X, \tau).$$

Then the triplet (X, τ, d) is called a d-complete topological space.

For details on d-complete topological spaces, we refer to Iseki [9] and Kasahara [10, 11, 12]. Hicks [5] and Hicks and Rhoades [6, 7] proved several fixed point theorems in d- complete topological spaces. Hicks and Saliga [8] and Saliga [18] obtained fixed point theorems for non-self maps in d-complete topological spaces.

In 2006, Bhaskar and Lakshmikantham [2] introduced the notion of a coupled fixed point in partially ordered metric spaces, also discussed some problems of

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the uniqueness of a coupled fixed point and applied their results to the problems of the existence and uniqueness of a solution for the periodic boundary value problems.

Later several authors proved coupled fixed and common coupled fixed point theorems in partial ordered metric spaces, partially ordered cone metric spaces and cone metric spaces for two maps (see e.g. [1, 3, 4],[13]-[17],[19]-[22].)

In this paper, we prove a common coupled fixed point theorem for four mappings in d-complete topological spaces.

Definition 1.2. (See [2]) Let X be a nonempty set. An element $(x, y) \in X \times X$ is called a coupled fixed point of the mapping $F : X \times X \to X$ if $x = F(x, y)$ and $y = F(y, x)$.

Definition 1.3. (See [1]) Let X be a nonempty set. An element $(x, y) \in X \times X$ is called

(i) a coupled coincidence point of $F : X \times X \to X$ and $g : X \to X$ if $gx = F(x, y)$ and $gy = F(y, x)$.

(ii) a common coupled fixed point of $F : X \times X \to X$ and $g : X \to X$ if $x = gx = F(x, y)$ and $y = gy = F(y, x)$.

Definition 1.4. (See [1]) Let X be a nonempty set. The mappings $F : X \times X \to X$ and $g : X \to X$ are called weakly-compatible if $g(F(x, y)) = F(gx, gy)$ whenever $gx = F(x, y)$ and $gy = F(y, x)$ for some $(x, y) \in X \times X$.

2. An implicit relation

Let Φ_7 be the family of all continuous mappings $\phi : \mathbb{R}^7_+ \to \mathbb{R}$ satisfying the following conditions :

(ϕ_1) there exists $0 \le h_1 < 1$ such that $u, v, w, p \ge 0$ with
  (ϕ_a) $\phi(u, v, w, u, v, p, 0) \le 0$ or,
  (ϕ_b) $\phi(u, v, w, v, u, 0, p) \le 0$
  implies $u \le h_1 \max\{v, w\}$,

(ϕ_2) there exists $0 \le h_2 < 1$ such that $u, v \ge 0$ with

$$\phi(u, u, v, 0, 0, u, u) \le 0 \Rightarrow u \le h_2 v.$$

Example 2.1.

$$\phi(t_1, t_2, t_3, t_4, t_5, t_6, t_7) = t_1 - \frac{\alpha}{2}(t_2 + t_3) - \beta(t_4 + t_5),$$

where $\alpha, \beta \ge 0$ with $\alpha + 2\beta < 1$.
If $u = 0$, then (ϕ_1) and(ϕ_2) are satisfied. Assume that $u > 0$.

$$\begin{aligned} \phi(u, v, w, u, v, p, 0) \le 0 \quad & \Rightarrow u - \tfrac{\alpha}{2}(v + w) - \beta(v + u) \le 0 \\ & \Rightarrow u \le \tfrac{\alpha}{2}(v + w) + \beta(v + u) \\ & \Rightarrow (1 - \beta)u \le \alpha \max\{v, w\} + \beta \max\{v, w\} \\ & \Rightarrow u \le h_1 \max\{v, w\}, \end{aligned}$$

where $h_1 = \frac{\alpha+\beta}{\alpha-\beta} < 1$.

Similarly $\phi(u, v, w, v, u, 0, p) \le 0 \Rightarrow u \le h_1 \max\{v, w\}$. $\phi(u, u, v, 0, 0, u, u) \le 0 \Rightarrow u \le h_2 v, where\ h_2 = \frac{\alpha/2}{1-\alpha/2} < 1$.

Example 2.2.

$$\phi(t_1,t_2,t_3,t_4,t_5,t_6,t_7) = t_1 - \alpha\max\{t_2,t_3,t_4,t_5\} - \beta\min\{t_6,t_7\},$$

where $\alpha,\beta \geq 0$ with $\alpha+\beta<1$.

Example 2.3.

$$\phi(t_1,t_2,t_3,t_4,t_5,t_6,t_7) = t_1 - \frac{\alpha}{2}(t_2+t_3) - L\min\{t_4,t_5,t_6,t_7\},$$

where $0 \leq \alpha < 1$ and $L \geq 0$.

Example 2.4.

$$\phi(t_1,t_2,t_3,t_4,t_5,t_6,t_7) = t_1 - \alpha t_2 - \beta t_3 - L\min\{t_4,t_5,t_6,t_7\},$$

where $\alpha,\beta \geq 0$ such that $\alpha+\beta<1$ and $L \geq 0$.

Example 2.5.

$$\phi(t_1,t_2,t_3,t_4,t_5,t_6,t_7) = t_1^2 - \frac{\alpha}{2}(t_2^2+t_3^2) - \beta t_4t_5 - \gamma\ t_6t_7,$$

where $\alpha,\beta,\gamma \geq 0$ with $\alpha+\beta<1$ and $\alpha+\gamma<1$.

Example 2.6.

$$\phi(t_1,t_2,t_3,t_4,t_5,t_6,t_7) = t_1^2 - \frac{\alpha}{2}(t_2^2+t_3^2) - \beta\frac{t_4^2+t_5^2}{t_6+t_7+1},$$

where $\alpha,\beta \geq 0$ with $\alpha+2\beta<1$.

3. Main results

Theorem 3.1. . *Let (X,τ) be a Hausdorff topological space. Let $F,G : X\times X \to X$ and $f,g : X \to X$ be mappings satisfying*

$$\phi\begin{pmatrix} d(F(x,y),G(u,v)),d(fx,gu),d(gv,fy),d(F(x,y),fx), \\ d(G(u,v),gu),d(F(x,y),gu),d(G(u,v),fx) \end{pmatrix} \leq 0 \tag{3.1}$$

for all $x,y,u,v \in X$, where $\phi \in \Phi_7$,

- (a) *$F(X\times X) \subseteq g(X), G(X\times X) \subseteq f(X)$,*
- (b) *one of $(f(X),\tau,d)$ and $(g(X),\tau,d)$ is d-complete,*
- (c) *the pairs (F,f) and (G,g) are weakly compatible,*
- (d) *$d(x,y) = d(y,x)$ for all $x,y \in X$ and*
- (e) *for each $y \in X, d(x_n,y) \to d(x,y)$, whenever $\{x_n\} \subseteq X, x \in X$ such that $x_n \to x$.*

Then there exists a unique $(\alpha,\beta) \in X\times X$ such that
$f\alpha = g\alpha = \alpha = F(\alpha,\beta) = G(\alpha,\beta)$ and $f\beta = g\beta = \beta = F(\beta,\alpha) = G(\beta,\alpha)$.

Proof. Let x_0 and y_0 be in X. Since $F(X\times X) \subseteq g(X)$, we can choose $x_1,y_1 \in X$ such that

$$gx_1 = F(x_0,y_0) \text{ and } gy_1 = F(y_0,x_0).$$

Since $G(X\times X) \subseteq f(X)$, we can choose $x_2,y_2 \in X$ such that

$$fx_2 = G(x_1,y_1) \text{ and } fy_2 = G(y_1,x_1).$$

Continuing this process, we can construct the sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\begin{aligned} gx_{2n+1} &= F(x_{2n}, y_{2n}) = z_{2n+1}, \\ gy_{2n+1} &= F(y_{2n}, x_{2n}) = p_{2n+1}, \\ fx_{2n+2} &= G(x_{2n+1}, y_{2n+1}) = z_{2n+2}, \\ fy_{2n+2} &= G(y_{2n+1}, x_{2n+1}) = p_{2n+2}. \end{aligned}$$

Putting $x = x_{2n}, y = y_{2n}, u = x_{2n+1}, v = y_{2n+1}$ in (3.1), we get

$$\phi\left(\begin{array}{c} d(z_{2n+1}, z_{2n+2}), d(z_{2n}, z_{2n+1}), d(p_{2n+1}, p_{2n}), d(z_{2n+1}, z_{2n}), \\ d(z_{2n+2}, z_{2n+1}), 0, d(z_{2n+2}, z_{2n}) \end{array}\right) \leq 0$$

From (ϕ_b), we have

$$d(z_{2n+1}, z_{2n+2}) \leq h_1 \max\{d(z_{2n}, z_{2n+1}), d(p_{2n+1}, p_{2n})\}$$

where $0 \leq h_1 < 1$. Also putting $x = y_{2n}, y = x_{2n}, u = y_{2n+1}, v = x_{2n+1}$ in (3.1) and using (ϕ_b), we get

$$d(p_{2n+1}, p_{2n+2}) \leq h_1 \max\{d(z_{2n}, z_{2n+1}), d(p_{2n+1}, p_{2n})\}.$$

Thus

$$\max\{d(z_{2n+1}, z_{2n+2}), d(p_{2n+1}, p_{2n+2})\} \leq h_1 \max\{d(z_{2n}, z_{2n+1}), d(p_{2n+1}, p_{2n})\}.$$

Similarly, using (3.1) and (ϕ_a), we can show that

$$\max\{d(z_{2n}, z_{2n+1}), d(p_{2n+1}, p_{2n})\} \leq h_1 \max\{d(z_{2n}, z_{2n-1}), d(p_{2n-1}, p_{2n})\}.$$

Hence

$$\max\{d(z_n, z_{n+1}), d(p_n, p_{n+1})\} \leq h_1 \max\{d(z_{n-1}, z_n), d(p_{n-1}, p_n)\}.$$

Inductively we have

$$\max\{d(z_n, z_{n+1}), d(p_n, p_{n+1})\} \leq h_1^n \max\{d(z_0, z_1), d(p_0, p_1)\}.$$

Since $\sum_{n=1}^{\infty} h_1^n$ is convergent, it follows that $\sum\limits_{n=1}^{\infty} d(z_n, z_{n+1})$ and $\sum_{n=1}^{\infty} d(p_n, p_{n+1})$ are convergent. Hence $d(z_n, z_{n+1}) \to 0$ and $d(p_n, p_{n+1}) \to 0$ as $n \to \infty$.

Suppose $(f(X), \tau, d)$ is d-complete. Then $\{z_{2n+2}\} = \{fx_{2n+2}\} \subseteq f(X)$ and $\{p_{2n+2}\} = \{fy_{2n+2}\} \subseteq f(X)$ converge to α and β respectively for some $\alpha, \beta \in f(X)$. Hence there exist x and y in X such that $\alpha = fx$ and $\beta = fy$. Also the subsequences $\{z_{2n+1}\}$ and $\{p_{2n+1}\}$ converge to α and β, respectively.

Putting $x = x, y = y, u = x_{2n+1}, v = y_{2n+1}$ in (3.1), we get

$$\phi\left(\begin{array}{c} d(F(x,y), z_{2n+2}), d(fx, z_{2n+1}), d(p_{2n+1}, fy), d(F(x,y), fx), \\ d(z_{2n+2}, z_{2n+1}), d(F(x,y), z_{2n+1}), d(z_{2n+2}, fx) \end{array}\right) \leq 0$$

Letting $n \to \infty$, we get

$$\phi(d(F(x,y), fx), 0, 0, d(F(x,y), fx), 0, d(F(x,y), fx), 0) \leq 0.$$

From (ϕ_a), we have $F(x,y) = fx = \alpha$. Replacing x, y, u, v with y, x, y_{2n+1}, x_{2n+1} respectively in (3.1) and letting $n \to \infty$ and by using (ϕ_a), we get

$$F(y,x) = fy = \beta.$$

Since the pair (F, f) is weakly compatible, we have

$$f\alpha = f(F(x, y)) = F(fx, fy) = F(\alpha, \beta) \text{ and}$$
$$f\beta = f(F(y, x)) = F(fy, fx) = F(\beta, \alpha).$$

Putting $x = \alpha, y = \beta, u = x_{2n+1}, v = y_{2n+1}$ in (3.1), we get

$$\phi\begin{pmatrix} d(f\alpha, z_{2n+2}), d(f\alpha, z_{2n+1}), d(p_{2n+1}, f\beta), 0, \\ d(z_{2n+2}, z_{2n+1}), d(f\alpha, z_{2n+1}), d(z_{2n+2}, f\alpha) \end{pmatrix} \leq 0$$

Letting $n \to \infty$, we get

$$\phi(d(f\alpha, \alpha), d(f\alpha, \alpha), d(f\beta, \beta), 0, 0, d(f\alpha, \alpha), d(f\alpha, \alpha)) \leq 0.$$

From (ϕ_2), we have $d(f\alpha, \alpha) \leq h_2 d(f\beta, \beta)$, where $0 \leq h_2 < 1$.
Putting $x = \beta, y = \alpha, u = y_{2n+1}, v = x_{2n+1}$ in (3.1) and letting $n \to \infty$ and then using (ϕ_2), we get

$$d(f\beta, \beta) \leq h_2 d(f\alpha, \alpha).$$

Hence $f\alpha = \alpha$ and $f\beta = \beta$. Thus

$$\alpha = f\alpha = F(\alpha, \beta) \text{ and } \beta = f\beta = F(\beta, \alpha). \tag{3.2}$$

Since $F(X \times X) \subseteq g(X)$, there exist $\gamma, \delta \in X$ such that

$$g\gamma = F(\alpha, \beta) = f\alpha = \alpha \text{ and } g\delta = F(\beta, \alpha) = f\beta = \beta.$$

Putting $x = \alpha, y = \beta, u = \gamma, v = \delta$ in (3.1),we get

$$\phi(d(g\gamma, G(\gamma, \delta)), 0, 0, 0, d(G(\gamma, \delta), g\gamma), 0, d(G(\gamma, \delta), g\gamma)) \leq 0.$$

From (ϕ_b), we have $g\gamma = G(\gamma, \delta)$.
Putting $x = \beta, y = \alpha, u = \delta, v = \gamma$ in (3.1) and using (3.2) and (ϕ_b), we have $g\delta = G(\delta, \gamma)$. Since the pair (G, g) is weakly compatible, we have

$$g\alpha = g(g\gamma) = g(G(\gamma, \delta)) = G(g\gamma, g\delta) = G(\alpha, \beta) \text{ and}$$
$$g\beta = g(g\delta) = g(G(\delta, \gamma)) = G(g\delta, g\gamma) = G(\beta, \alpha).$$

Putting $x = x_{2n}, y = y_{2n}, u = \alpha, v = \beta$ in (3.1) and letting $n \to \infty$ and then using (ϕ_2), we get

$$d(\alpha, g\alpha) \leq h_2 d(g\beta, \beta).$$

Similarly, by putting $x = y_{2n}, y = x_{2n}, u = \beta, v = \alpha$ in (3.1) and letting $n \to \infty$ and then using (ϕ_2), we get

$$d(g\beta, \beta) \leq h_2 d(\alpha, g\alpha).$$

Hence $g\alpha = \alpha$ and $g\beta = \beta$.

$$\alpha = g\alpha = G(\alpha, \beta) \text{ and } \beta = g\beta = G(\beta, \alpha) \tag{3.3}$$

From (3.2) and (3.3), we have

$$\begin{aligned} f\alpha = g\alpha = \alpha = F(\alpha, \beta) = G(\alpha, \beta) \text{ and} \\ f\beta = g\beta = \beta = F(\beta, \alpha) = G(\beta, \alpha). \end{aligned} \tag{3.4}$$

Suppose there exists $(\alpha_1, \beta_1) \in X \times X$ such that

$$f\alpha_1 = g\alpha_1 = \alpha_1 = F(\alpha_1, \beta_1) = G(\alpha_1, \beta_1) \text{ and}$$
$$f\beta_1 = g\beta_1 = \beta_1 = F(\beta_1, \alpha_1) = G(\beta_1, \alpha_1).$$

By putting $x=\alpha_1, y=\beta_1, u=\alpha, v=\beta$ and $x=\beta_1, y=\alpha_1, u=\beta, v=\alpha$ in (3.1) and using (ϕ_2), one can show that $\alpha_1=\alpha$ and $\beta_1=\beta$. Thus (α,β) is the unique pair satisfying (3.4). □

The following example illustrates Theorem 3.1.

Example 3.2. Let $X=[0,1]$ and $d(x,y)=|x^2-y^2|$ for all $x,y\in X$.
Define $F(x,y)=\frac{1}{2}=G(x,y)$ and

$$fx=\begin{cases}\frac{1}{2}, & if\ x\in[0,\frac{1}{2}],\\ x, & if\ x\in(\frac{1}{2},1],\end{cases},\quad gx=\begin{cases}\frac{1}{2}, & if\ x\in[0,\frac{1}{2}],\\ 1, & if\ x\in(\frac{1}{2},1].\end{cases}$$

Now, we prove a unique common coupled fixed point theorem for a pair of Jungck type maps without using symmetry of d.

Theorem 3.3. *. Let (X,τ) be a Hausdorff topological space. Let $F: X\times X\to X$ and $f: X\to X$ be mappings satisfying*

$$\phi\left(\begin{array}{c} d(F(x,y),F(u,v)), d(fx,fu), d(fy,fv), d(fu,F(u,v)),\\ d(fx,F(x,y)), d(fx,F(u,v)), d(F(x,y),fu)\end{array}\right)\le 0 \qquad (3.5)$$

for all $x,y,u,v\in X$, where $\phi\in\Phi_7$ without (ϕ_b),

(a) $F(X\times X)\subseteq f(X)$,
(b) $(f(X),\tau,d)$ is d-complete,
(c) the pair (F,f) is weakly compatible,
(d) for each $y\in X, d(x_n,y)\to d(x,y)$, whenever $\{x_n\}\subseteq X, x\in X$ such that $x_n\to x$.

Then the mappings F and f have a unique common coupled fixed point.

Proof. Let x_0 and y_0 be in X.
Since $F(X\times X)\subseteq f(X)$, we can choose $x_1,y_1\in X$ such that

$$fx_1=F(x_0,y_0) \text{ and } fy_1=F(y_0,x_0).$$

Continuing this process, we can construct two sequences $\{x_n\}$ and $\{y_n\}$ in X such that (for $n=0,1,2,\cdots$)

$$fx_{n+1}=F(x_n,y_n)=z_n \text{ and } fy_{n+1}=F(y_n,x_n)=p_n.$$

Putting $x=x_n, y=y_n, u=x_{n+1}, v=y_{n+1}$ in (3.5), we get

$$\phi\left(\begin{array}{c} d(z_n,z_{n+1}), d(z_{n-1},z_n), d(p_{n-1},p_n), d(z_n,z_{n+1}),\\ d(z_{n-1},z_n), d(z_{n-1},z_{n+1}), 0\end{array}\right)\le 0$$

From (ϕ_a), there exists $h_1\in[0,1)$ such that

$$d(z_n,z_{n+1})\le h_1\max\{d(z_{n-1},z_n), d(p_{n-1},p_n)\}.$$

Similarly, by putting $y=x_n, x=y_n, v=x_{n+1}, u=y_{n+1}$ in (3.5) and using (ϕ_a), we get

$$d(p_n,p_{n+1})\le h_1\max\{d(z_{n-1},z_n), d(p_{n-1},p_n)\}.$$

Thus

$$\begin{aligned}\max\{d(z_n, z_{n+1}), d(p_n, p_{n+1})\} &\leq h_1 \max\{d(z_{n-1}, z_n), d(p_{n-1}, p_n)\}\\ &\leq h_1^2 \max\{d(z_{n-2}, z_{n-1}), d(p_{n-2}, p_{n-1})\}\\ &.\\ &.\\ &.\\ &\leq h_1^n \max\{d(z_0, z_1), d(p_0, p_1)\}.\end{aligned}$$

Since $\sum\limits_{n=1}^{\infty} h_1^n$ is convergent, it follows that $\sum_{n=1}^{\infty} d(z_n, z_{n+1})$ and $\sum\limits_{n=1}^{\infty} d(p_n, p_{n+1})$ are convergent. Hence $d(z_n, z_{n+1}) \to 0$ and $d(p_n, p_{n+1}) \to 0$ as $n \to \infty$.

Suppose $(f(X), \tau, d)$ is d-complete. Then there exist α and β in $f(X)$ such that $\{z_n\}$ and $\{p_n\}$ converge to α and β, respectively. Hence there exist $x, y \in X$ such that $\alpha = fx$ and $\beta = fy$.
Replacing x, y, u, v with x_n, y_n, x, y respectively in (3.5), we get

$$\phi\left(\begin{array}{c} d(z_n, F(x,y)), d(z_{n-1}, fx), d(p_{n-1}, fy), d(fx, F(x,y)), \\ d(z_{n-1}, z_n), d(z_{n-1}, F(x,y)), d(z_n, fx) \end{array}\right) \leq 0$$

Letting $n \to \infty$ and using (d), we get

$$\phi(d(fx, F(x,y)), 0, 0, d(fx, F(x,y)), 0, d(fx, F(x,y)), 0) \leq 0.$$

Now, from (ϕ_a), we have $F(x,y) = fx$.
Similarly, replacing x, y, u, v with y_n, x_n, y, x in (3.5) and letting $n \to \infty$ and using (d), (ϕ_a), we can show that $F(y,x) = fy$.
Since (F, f) is a weakly compatible pair, we have

$$f\alpha = ffx = f(F(x,y)) = F(fx, fy) = F(\alpha, \beta) \text{ and}$$

$$f\beta = ffy = f(F(y,x)) = F(fy, fx) = F(\beta, \alpha).$$

Putting $x = x_n, y = y_n, u = \alpha, v = \beta$ and $x = y_n, y = x_n, u = \beta, v = \alpha$ in (3.5) and letting $n \to \infty$ and using (ϕ_2), we can show that $f\alpha = \alpha$ and $f\beta = \beta$.
Thus

$$\alpha = f\alpha = F(\alpha, \beta) \text{ and } \beta = f\beta = F(\beta, \alpha). \tag{3.6}$$

Using (3.5) and (ϕ_2), one can show that (α, β) is the unique pair in $X \times X$ satisfying (3.6). ☐

The following example illustrates Theorem 3.3.

Example 3.4. Let $X = \{0, 1\}$ and $d(x,y) = |x^2 - y|$ for all $x, y \in X$. Define $F(x,y) = 1$ and $f0 = 0$ and $f1 = 1$.

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References

1. M. Abbas, M. Alikhan and S. Radenović, *Common coupled fixed point theorems in cone metric spaces for W-compatible mappings*, Appl. Math. Comput. **217** (2010), no. 1, 195–202.
2. T.G. Bhaskar and V. Lakshmikantham, *Fixed point theorems in partially ordered cone metric spaces and applications*, Nonlinear Anal. **65** (2006), no. 7, 825–832.
3. B.S. Choudhury and A. Kundu, *A coupled coincidence point result in partially ordered metric spaces for compatible mappings*, Nonlinear Anal. **73** (2010), no. 8, 2524–2531.
4. H.-S. Ding and L. Li, *Coupled fixed point theorems in partial ordered cone metric spaces*, Filomat **25** (2011), no. 2, 137–149.
5. T.L. Hicks, *Fixed point theorems for d-complete topological spaces-I*, Internat. J. Math. Math. Sci. **15** (1992), no. 3, 435–439.
6. T.L. Hicks and B.E. Rhoades,*Fixed point theorems for d-complete topological spaces-II*, Math. Japon. **37** (1992), no. 5, 847–853.
7. T.L.Hicks and B.E. Rhoades, *Fixed points for pairs of mappings in d-complete topological spaces*, Internat. J. Math. Math. Sci. **16** (1993), 259–266.
8. T.L. Hicks and L.M. Saliga, *Fixed point theorems for non-self maps-I*, Internat. J. Math. Math. Sci. **17** (1994), 713–716.
9. K. Iséki, *An approach to fixed point theorems*, Math. Sem. Notes Kobe Univ. **3** (1975), no. 2, paper no. XXX, 11 pp.
10. S. Kasahara, *On some generalizations of Banach Contraction theorem*, Math. Sem. Notes Kobe Univ. **3** (1975), no. 2, paper no. XXIII, 10 pp.
11. S. Kasahara, *Some fixed point and coincidence theorems in L-Spaces*, Math. Sem. Notes Kobe Univ. **3** (1975), no. 2, paper no. XXVIII, 7 pp.
12. S. Kasahara, *Iff fixed point iterations in L-space*, Math. Sem. Notes Kobe Univ. **4** (1976), no. 2, 205–210.
13. E. Karapınar, *Couple fixed point theorems for nonlinear contractions in cone metric spaces*, Comp. Math. Anal. **59** (2010),3656–3668.
14. E. Karapınar,*Couple fixed point on cone metric spaces*, Gazi Univ. J. Sci. **24** (2011), no. 1, 51–58.
15. V. Lakshmikantham and Lj. Ciric, *Coupled fixed point theorems for nonlinear contractions in partial ordered metric space*, Nonlinear Anal. **70** (2009),4341–4349.
16. N.V. Luong and N.X. Thuan, *Coupled fixed point theorems in partially ordered metric spaces*, Bull. Math. Anal. Appl. **2** (2010), no. 4, 16–24.
17. N. Van Luong and N.X. Thuan, *Coupled fixed points in partially ordered metric spaces and application*, Nonlinear Anal. **74** (2011), 983–992.
18. L.M. Saliga,*Fixed point theorems for non-self maps in d-complete topological spaces*, Internat. J. Math. Math. Sci.**19** (1996), no. 1, 103–110.
19. F. Sabetghadam, H.P. Masiha and A.H. Sanatpour, *Some coupled fixed point theorems in cone metric spaces*, Fixed Point Theory Appl. **2009**, Art. ID 125426, 8 pp.
20. B. Samet, *Coupled fixed point theorems for a generalized Meir-Keeler Contraction in partially ordered metric spaces*, Nonlinear Anal. **72** (2010), 4508–4517.
21. W. Shatanawi,*Partially ordered cone metric spaces and coupled fixed point results*, Comput. Math. Appl. **60** (2010), no. 8, 2508–2515.
22. W. Shatanawi, *Some common coupled fixed point results in cone metric spaces*, Int. J. Math. Anal. (Ruse) **4** (2010), no. 45-48, 2381–2388.

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