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The inverse limit of the Burnside ring for a family of subgroups of a finite group

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Abstract. Let G be a finite nontrivial group and $A(G)$ the Burnside ring of G. Let $\mathcal{F}$ be a set of subgroups of G which is closed under taking subgroups and taking conjugations by elements in G. Then let $\mathfrak{F}$ denote the category whose objects are elements in $\mathcal{F}$ and whose morphisms are triples (H, g, K) such that H, $K \in \mathcal{F}$ and $g \in G$ with $gHg^{-1} \subset K$. Taking the inverse limit of $A(H)$, where $H \in \mathcal{F}$, we obtain the ring $A(\mathfrak{F})$ and the restriction homomorphism $\mathrm{res}^G_{\mathcal{F}} : A(G) \to A(\mathfrak{F})$. We study this restriction homomorphism.

Key words: Burnside ring, restriction homomorphism, inverse limit.

1. Introduction

Let G be a finite group. Let $\mathcal{S}(G)$ denote the set of all subgroups of G and $\mathfrak{S}$ the subgroup category whose objects are all subgroups of G and whose morphisms are all triples (H, g, K) such that H, $K \in \mathcal{S}(G)$ and $g \in G$ with $gHg^{-1} \subset K$. Here the source object of (H, g, K) is H, the target object of (H, g, K) is K, and for morphisms (H, a, K) and (K, b, L) in $\mathfrak{S}$, the composition $(K, b, L) \circ (H, a, K)$ in $\mathfrak{S}$ is defined to be (H, ba, L). We remark that morphisms (H, g, K) in $\mathfrak{S}$ are not maps. Let $\mathfrak{A}$ denote the category of abelian groups whose objects are all abelian groups and whose morphisms are all (group) homomorphisms. Let $A(G)$ denote the Burnside ring of G, i.e. the Grothendieck group of the category of finite G-sets. For $\alpha = [X] - [Y] \in A(G)$ and $H \in \mathcal{S}(G)$, the integer $\chi_H(\alpha)$ is defined to be $|X^H| - |Y^H|$, where X and Y are finite G-sets, and $|X^H|$ stands for the number of elements in the H-fixed point set X^H of X. Let $A = (A_*, A^*) : \mathfrak{S} \to \mathfrak{A}$ denote the Burnside ring functor, where A_* and A^* are covariant and contravariant functors respectively. That is, $A = (A_*, A^*)$ is a Mackey functor in the sense of [2] and $A(H)$ $(= A_*(H) = A^*(H))$

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is the Burnside ring of H for each $H \in \mathcal{S}(G)$. Moreover $A = (A_*, A^*)$ can be regarded as a Green ring functor in the sense of [2]. Let $\mathcal{F}$ be a subset of $\mathcal{S}(G)$ such that $\mathcal{F}$ is closed under taking subgroups and taking conjugations by elements in G. Let $\mathfrak{F}$ denote the full subcategry of $\mathfrak{S}$ such that $\mathrm{Obj}(\mathfrak{F}) = \mathcal{F}$. Then we obtain the inverse limit $A(\mathfrak{F}) = \varprojlim_{\mathfrak{F}} A(-)$ in the sense of [1, p. 243], i.e. $A(\mathfrak{F})$ consists of all elements (x_H) of $\prod_{H\in\mathcal{F}} A(H)$, where $x_H \in A(H)$, such that $A^*((H, g, K))(x_K) = x_H$ for all H, $K \in \mathcal{F}$, and $g \in G$ with $gHg^{-1} \subset K$. The restriction homomorphisms $\mathrm{res}^G_H : A(G) \to A(H)$ yield the homomorphism $\mathrm{res}^G_{\mathcal{F}} : A(G) \to \prod_{H\in\mathcal{F}} A(H)$ and we readily see $\mathrm{Im}(\mathrm{res}^G_{\mathcal{F}}) \subset A(\mathfrak{F})$.

Finite G-CW complexes X and Y are called *χ-equivalent* if $\chi(X^H) = \chi(Y^H)$ for all $H \in \mathcal{S}(G)$, where $\chi(X^H)$ stands for the Euler characteristic of the H-fixed point set X^H of X. Let $\Omega(G)$ denote the set of χ-equivalence classes of finite G-CW complexes. By assigning to an element $[X] - [Y] \in A(G)$ the element $[Z] \in \Omega(G)$ such that $\chi(Z^H) = |X^H| - |Y^H|$ for all $H \in \mathcal{S}(G)$, we obtain a map $A(G) \to \Omega(G)$, where X and Y are finite G-sets and Z is a finite G-CW complex. This map $A(G) \to \Omega(G)$ is a bijection, see e.g. [5], [8]. Therefore we identify $\Omega(G)$ with $A(G)$ via the map. Let $\mathcal{M} = (M_H)_{H\in\mathcal{F}}$ be a tuple consisting of compact (smooth) H-manifolds M_H. For each $H \in \mathcal{F}$ we have the element $[M_H]$ in $\Omega(H) = A(H)$ determined by M_H, and hence $([M_H])_{H\in\mathcal{F}}$ lies in $\prod_{H\in\mathcal{F}} A(H)$. If there exists a G-manifold M_G such that $\mathrm{res}^G_H M_G$ is H-diffeomorphic to M_H for all $H \in \mathcal{F}$, then the element $([M_H])_{H\in\mathcal{F}}$ belongs to $\mathrm{Im}(\mathrm{res}^G_{\mathcal{F}})$ $(\subset A(\mathfrak{F}))$. Thus the coset $\sigma(\mathcal{M})$ including $([M_H])_{H\in\mathcal{F}}$ in $(\prod_{H\in\mathcal{F}} A(H))/\mathrm{Im}(\mathrm{res}^G_{\mathcal{F}})$ can be regarded as an obstruction to extend $\mathcal{M}$ to 'a G-manifold'. Set $A(G)|_{\mathcal{F}} = \mathrm{Im}(\mathrm{res}^G_{\mathcal{F}})$ and observe the exact sequence

$$A(\mathfrak{F})/A(G)|_{\mathcal{F}} \hookrightarrow \Big(\prod_{H\in\mathcal{F}} A(H)\Big)/A(G)|_{\mathcal{F}} \twoheadrightarrow \Big(\prod_{H\in\mathcal{F}} A(H)\Big)/A(\mathfrak{F}).$$

In the theory of the Burnside ring, see e.g. [5], it is well-known that $\prod_{H\in\mathcal{F}} A(H)$ is a free $\mathbb{Z}$-module and it is readily seen that $(\prod_{H\in\mathcal{F}} A(H))/A(\mathfrak{F})$ is also a free $\mathbb{Z}$-module, where $\mathbb{Z}$ is the ring of integers.

Proposition 1.1 *Let G be a nontrivial finite group of order n. Then $nA(\mathfrak{F})$ is contained in $A(G)|_{\mathcal{F}}$.*

This proposition immediately follows from Lemmas 3.2 and 3.3. Thus $A(\mathfrak{F})/A(G)|_{\mathcal{F}}$ is a finite abelian group.

The next result also follows from the theory of the Burnside ring.

Proposition 1.2 *The exact sequence*

$$0 \longrightarrow \mathrm{Ker}(\mathrm{res}^G_{\mathcal{F}}) \longrightarrow A(G) \xrightarrow{\mathrm{res}^G_{\mathcal{F}}} A(G)|_{\mathcal{F}} \longrightarrow 0$$

splits as $\mathbb{Z}$-modules and the $\mathbb{Z}$-rank of $A(G)|_{\mathcal{F}}$ (resp. $\mathrm{Ker}(\mathrm{res}^G_{\mathcal{F}})$) is equal to the number of G-conjugacy classes of subgroups in $\mathcal{F}$ (resp. $\mathcal{S}(G) \smallsetminus \mathcal{F}$).

For the convenience of readers, we will give a proof in Section 3.

For a finite nontrivial group G, let $\mathcal{F}_G$ and $\mathfrak{F}_G$ denote the set $\mathcal{S}(G) \smallsetminus \{G\}$ and the full subcategory of $\mathfrak{S}$ such that $\mathrm{Obj}(\mathfrak{F}_G) = \mathcal{F}_G$, respectively. Let k_G be the integer defined in R. Oliver [9, Lemma 8], i.e. the product of primes p such that G possesses a normal subgroup with index p. If G is a nontrivial perfect group then k_G is equal to 1.

Proposition 1.3 *Let G be a finite nontrivial group, $\mathcal{F} = \mathcal{F}_G$, and $\mathfrak{F} = \mathfrak{F}_G$. Then $\mathrm{Ker}(\mathrm{res}^G_{\mathcal{F}})$ is generated by a unique element $\gamma \in A(G)$ such that $\chi_G(\gamma) = k_G$.*

Our main result in the paper is

Theorem 1.4 *Let G be a finite nontrivial nilpotent group, $\mathcal{F} = \mathcal{F}_G$, and $\mathfrak{F} = \mathfrak{F}_G$. Then $A(G)|_{\mathcal{F}}$ coincides with $A(\mathfrak{F})$ if and only if G is a cyclic group of which the order is a prime or a product of distinct primes.*

We will prove Proposition 1.3 in Section 3 and Theorem 1.4 in Section 4.

2. Examples of $A(G)|_{\mathcal{F}}$ and $A(\mathfrak{F})$

For the Burnside ring functor $A = (A_*, A^*) : \mathfrak{S} \to \mathfrak{A}$ and a morphism (H, g, K) in $\mathfrak{S}$, we use $(H, g, K)_*$ and $(H, g, K)^*$ instead of $A_*((H, g, K))$ and $A^*((H, g, K))$, respectively. Furthermore, $(H, e, K)_*$ and $(H, e, K)^*$, where e is the identity element of G, are denoted by ind^K_H and res^K_H. For a finite ordered set F, let $F_{\max}$ denote the set of all maximal elements in F.

Let S be a set of subgroups of G and M a set of morphisms in $\mathfrak{S}$, i.e. $M \subset \mathrm{Mor}(\mathfrak{S})$. Then we define the inverse limit $A(S, M)$ by

$$
\begin{aligned}
A(S, M) = \{(x_K)_{K\in S} \mid x_K \in A(K) \text{ for } K \in S,\\
f^* x_K = g^* x_L \text{ whenever } K,\, L \in S,\, f = (H, a, K) \in M,\\
g = (H, b, L) \in M \text{ for some } H \in \mathcal{S}(G),\, a,\, b \in G\}.
\end{aligned}
$$

Let $\mathcal{F}$ and $\mathfrak{F}$ be those in Section 1. In the case where S is a set of complete representatives of conjugacy classes of groups in $\mathcal{F}_{\max}$, it is clear that the canonical projection $A(\mathfrak{F}) \to A(S, \mathrm{Mor}(\mathfrak{S}))$ is an isomorphism. In addition, we have the restriction homomorphism $\mathrm{res}^G_S : A(G) \to A(S, \mathrm{Mor}(\mathfrak{S}))$ and the diagram

$$
\begin{array}{ccc}
 & A(G) & \\
\swarrow^{\mathrm{res}^G_{\mathcal{F}}} & & \searrow^{\mathrm{res}^G_S} \\
A(\mathfrak{F}) & \xrightarrow[\mathrm{proj}]{\cong} & A(S, \mathrm{Mor}(\mathfrak{S}))
\end{array}
$$

commutes. Thus we can study $A(\mathfrak{F})$ and $A(G)|_{\mathcal{F}}$ via

$$
A(\mathfrak{F})' = A(S, \mathrm{Mor}(\mathfrak{S})) \quad \text{and} \quad A(G)|_S = \mathrm{Im}[\mathrm{res}^G_S : A(G) \to A(\mathfrak{F})'],
$$

respectively.

In the rest of this section, let $\mathcal{F}$, $\mathfrak{F}$, and S be $\mathcal{F}_G$, $\mathfrak{F}_G$, and a set of complete representatives of conjugacy classes of groups in $\mathcal{F}_{\max}$, respectively.

Proposition 2.1 *Let p be a prime and G a group of order p. Then $A(G)|_{\mathcal{F}}$ coincides with $A(\mathfrak{F})$.*

One can readily prove this proposition.

Let E denote the unit group, i.e. $E = \{e\}$. For an integer $m \geq 1$, let C_m be a cyclic group of order m.

Proposition 2.2 *Let p be a prime and G an elementary abelian p-group of order p^2, i.e. $G \cong C_p \times C_p$. Then $A(\mathfrak{F})/A(G)|_{\mathcal{F}}$ is isomorphic to $\mathbb{Z}_p$ as modules.*

Proof. Let u and v be elements of order p in G generating G, i.e. $G = \langle u, v\rangle$. Set $C^{(0)} = \langle v\rangle$ and $C^{(k)} = \langle uv^k\rangle$ for $k = 1, 2, \dots, p$. Then $S = \{C^{(k)} \mid k = 0, 1, \dots, p\}$ and

$$A(\mathfrak{F})' = \{(a_0[C^{(0)}/C^{(0)}] + b_0[C^{(0)}/E],$$
$$(a_0 + p(b_0 - b_1))[C^{(1)}/C^{(1)}] + b_1[C^{(1)}/E], \ldots\ldots,$$
$$(a_0 + p(b_0 - b_p))[C^{(p)}/C^{(p)}] + b_p[C^{(p)}/E]) \mid a_0,\ b_i \in \mathbb{Z}\}. \quad (2.1)$$

For $w = x[G/G] + \sum_{k=0}^{p} y_k[G/C^{(k)}] + z[G/E]$, we have

$$\mathrm{res}^G_{C^{(k)}} w = (x + py_k)[C^{(k)}/C^{(k)}] + \left(\sum_{i=0}^{p} y_i - y_k + pz\right)[C^{(k)}/E]. \quad (2.2)$$

Since

$$\sum_{k=0}^{p}\left(\sum_{i=0}^{p} y_i - y_k + pz\right) = p\left(\sum_{i=0}^{p} y_i + (p+1)z\right), \quad (2.3)$$

we obtain $A(\mathfrak{F})'/A(G)|_S \cong \mathbb{Z}_p$. □

Proposition 2.3 *Let p be a prime and G an elementary abelian p-group of order p^n with $n \geq 2$. Then there exists an element $w = (w_K)_{K\in\mathcal{F}}$ in $A(\mathfrak{F})$ satisfying $w_K = [K/E] \in A(K)$ for all $K \in \mathcal{F}_{\max}$, where $\mathcal{F}_{\max}$ is the set of subgroups of G with index p. In addition this element w does not lie in $A(G)|_{\mathcal{F}}$.*

Proof. Let $H \in \mathcal{F}$ and $K \in \mathcal{F}_{\max}$ such that $H \subset K$. Then we have $\mathrm{res}^K_H[K/E] = |K/H|[H/E]$. This implies that $([K/E])_{K\in\mathcal{F}_{\max}}$ determines the well-defined element $w \in A(\mathfrak{F})$ as in the proposition.

Let $L \in \mathcal{F}$. For $K \in \mathcal{F}_{\max}$,

$$\mathrm{res}^G_K[G/L] = \begin{cases} p[K/L] & (K \supset L) \\ [K/(L\cap K)] & (K \not\supset L). \end{cases} \quad (2.4)$$

Assume an element $x \in A(G)$ satisfies $\mathrm{res}^G_{\mathcal{F}}(x) = w$. Then x has the form

$$x \equiv \sum_{L\in\mathcal{L}} a_L[G/L] + b[G/E] \quad \mathrm{mod}\ \langle [G/H] \mid H \in \mathcal{S}(G),\ |H| \geq p^2\rangle_{\mathbb{Z}}$$

for some a_L, $b \in \mathbb{Z}$, where $\mathcal{L}$ is the set of all subgroups of G of order p. For $K \in \mathcal{F}_{\max}$, we have

$$\mathrm{res}^G_K x \equiv \sum_{L\in\mathcal{L}_K} a_L[K/E] \qquad \bmod pA(K) + \langle [K/H] \mid H \in \mathcal{S}(K),\ |H| \geq p\rangle_{\mathbb{Z}}, \quad (2.5)$$

where $\mathcal{L}_K = \{L \in \mathcal{L} \mid L \not\subset K\}$. Since $|\mathcal{L}| = (p^n - 1)/(p-1)$, $|\mathcal{L}_K| = p^{n-1}$, and $|\mathcal{F}_{\max}| = (p^n-1)/(p-1)$, we have

$$\sum_{K\in\mathcal{F}_{\max}} \sum_{L\in\mathcal{L}_K} a_L = p^{n-1} \sum_{L\in\mathcal{L}} a_L. \qquad (2.6)$$

On the other hand, since $\mathrm{res}^G_K x = [K/E]$, we get

$$\sum_{L\in\mathcal{L}_K} a_L \equiv 1 \mod p,$$

i.e. $\sum_{L\in\mathcal{L}_K} a_L = 1 + pm_K$ for some $m_K \in \mathbb{Z}$. Thus we have

$$\sum_{K\in\mathcal{F}_{\max}} \sum_{L\in\mathcal{L}_K} a_L = \sum_{K\in\mathcal{F}_{\max}} (1 + pm_K) = \frac{p^n - 1}{p-1} \cdot (1 + pm_K) \equiv 1 \mod p, \qquad (2.7)$$

which contradicts (2.6). Thus w does not belong to $A(G)|_{\mathcal{F}}$. □

Proposition 2.4 *Let p and q be distinct primes. If G is a nontrivial extension of C_q by C_p, i.e. $C_p \lhd G$, $G/C_p = C_q$ and $G \ncong C_{pq}$, then $A(G)|_{\mathcal{F}}$ coincides with $A(\mathfrak{F})$.*

Proof. Note that $\mathcal{S}(G) = \{G, C_p, gC_qg^{-1}, E \mid g \in C_p\}$. For $y \in A(C_p)$ with the form $y = a_1[C_p/C_p] + a_2[C_p/E]$, we have

$$\mathrm{res}^{C_p}_E y = (a_1 + a_2p)[E/E], \qquad (2.8)$$

and for $z \in A(C_q)$ with the form $z = b_1[C_q/C_q] + b_2[C_q/E]$, we have

$$\mathrm{res}^{C_q}_E z = (b_1 + b_2q)[E/E]. \qquad (2.9)$$

Then for $S = \{C_p, C_q\}$, we have

$$\begin{aligned} A(\mathfrak{F})' = \big\{ \big(& a_1[C_p/C_p] + a_2[C_p/E], \\ & (a_1 + pa_2 - b_2q)[C_q/C_q] + b_2[C_q/E] \big) \in A(C_p) \times A(C_q) \big\}, \end{aligned} \tag{2.10}$$

where a_1, a_2, and b_2 range over $\mathbb{Z}$. For $x \in A(G)$ with the form

$$x = c_1[G/G] + c_2[G/C_p] + c_3[G/C_q] + c_4[G/E],$$

we have

$$\begin{aligned} \mathrm{res}^G_{C_p} x &= (c_1 + c_2q)[C_p/C_p] + (c_3 + c_4q)[C_p/E], \\ \mathrm{res}^G_{C_q} x &= (c_1 + c_3)[C_q/C_q] + \left(c_2 + \frac{c_3(p-1)}{q} + c_4p \right)[C_q/E]. \end{aligned} \tag{2.11}$$

Thus $\mathrm{res}^G_S : A(G) \to A(\mathfrak{F})'$ is surjective. □

Proposition 2.5 *Let p be a prime, m a natural number, and G a cyclic group of order p^m. Then $A(\mathfrak{F})/A(G)|_{\mathcal{F}}$ is isomorphic to ${\mathbb{Z}_p}^{\oplus m-1}$ as modules.*

Proof. Let $\{e\} = H_1 < H_2 < \cdots < H_m < H_{m+1} = G$ be the subgroups of G. Set $K = H_m$. Then $S = \{K\}$ and $A(\mathfrak{F})' = A(K)$. Each element $x \in A(G)$ has the form

$$x = a_1[G/H_1] + \cdots + a_m[G/H_m] + a_{m+1}[G/H_{m+1}]$$

with integers $a_1, \ldots, a_{m+1}$. For the x, we have

$$\mathrm{res}^G_K x = p(a_1[K/H_1] + \cdots + a_{m-1}[K/H_{m-1}]) + (a_mp + a_{m+1})[K/H_m]. \tag{2.12}$$

Thus we get $A(\mathfrak{F})'/A(G)|_S \cong {\mathbb{Z}_p}^{\oplus m-1}$. □

Proposition 2.6 *Let p and q be distinct primes, and G a cyclic group of order pq. Then $A(G)|_{\mathcal{F}}$ coincides with $A(\mathfrak{F})$.*

Proof. Let P and Q be Sylow p- and q-subgroups of G, respectively. Since the maximal proper subgroups of G are P and Q, we have $S = \{P, Q\}$ and

$$\begin{aligned} A(\mathfrak{F})' = \{ & (a_1[P/E] + a_2[P/P],\ b_1[Q/E] + (a_1p + a_2 - b_1q)[Q/Q]) \\ & \in A(P) \times A(Q) \mid a_1, a_2, b_1 \in \mathbb{Z} \}. \end{aligned} \tag{2.13}$$

For $x = \sum_{H \leq G} c_H[G/H] \in A(G)$, we have

$$(\mathrm{res}^G_P x, \mathrm{res}^G_Q x) = \big((c_E q + c_Q)[P/E] + (c_P q + c_G)[P/P], \\ (c_E p + c_P)[Q/E] + (c_Q p + c_G)[Q/Q]\big). \tag{2.14}$$

These equalities imply $A(\mathfrak{F})' = A(G)|_S$. □

Proposition 2.7 *Let p and q be distinct primes and G a cyclic group of order p^2q. Then the quotient of $A(\mathfrak{F})/A(G)|_{\mathcal{F}}$ is isomorphic to $\mathbb{Z}_p$ as modules.*

Proof. Regard $K = C_{pq}$, $P = C_{p^2}$, $Q = C_q$, $L = C_p$, and $E = \{e\}$ as groups in $\mathcal{S}(G)$. Since the maximal proper subgroups of G are K and P, we have $S = \{K, P\}$ and

$$A(\mathfrak{F})' = \{(b_1[K/E] + (c_1 p - b_1 q)[K/Q] + b_2[K/L] + b_3[K/K], \\ c_1[P/E] + c_2[P/L] + (b_3 + b_2 q - c_2 p)[P/P]) \in A(K) \times A(P)\} \tag{2.15}$$

where b_1, b_2, b_3, c_1, c_2 range over $\mathbb{Z}$. For $x = \sum_{H \leq G} a_H[G/H] \in A(G)$, we have

$$\mathrm{res}^G_K x = a_E p[K/E] + a_Q p[K/Q] + (a_P + a_L p)[K/L] + (a_G + a_K p)[K/K], \\ \mathrm{res}^G_P x = (a_Q + a_E q)[P/E] + (a_K + a_L q)[P/L] + (a_G + a_P q)[P/P]. \tag{2.16}$$

These equalities show $A(\mathfrak{F})'/A(G)|_S \cong \mathbb{Z}_p$. □

Proposition 2.8 *For $G = A_4$, the alternating group on four letters, $A(G)|_{\mathcal{F}}$ coincides with $A(\mathfrak{F})$.*

Proof. We regard G as $D_4 \rtimes C_3$, where D_4 is a dihedral group of order 4. Then $\mathcal{F} = (D_4) \cup (C_3) \cup (C_2) \cup (E)$ and $\mathcal{S}(D_4) = \{D_4, C_2, C_2', C_2'', E\}$, where C_2, C_2' and C_2'' are distinct subgroups of order 2. For

$$x = x_1[G/G] + x_2[G/D_4] + x_3[G/C_3] + x_4[G/C_2] + x_5[G/E] \in A(G),$$

we have

$$\begin{aligned}\mathrm{res}^G_{D_4} x &= (x_1+3x_2)[D_4/D_4] + x_4([D_4/C_2]+[D_4/C_2']+[D_4/C_2''])\\ &\quad + (x_3+3x_5)[D_4/E],\\ \mathrm{res}^G_{C_3} x &= (x_1+x_3)[C_3/C_3] + (x_2+x_3+2x_4+4x_5)[C_3/E].\end{aligned}$$

Set $S=\{D_4, C_3\}$. Then we have

$$\begin{aligned}A(\mathfrak{F})' = \{(y,z) \mid \alpha,\beta,\gamma,\delta\in\mathbb{Z},\ u=\alpha+6\beta+4\gamma-3\delta\};\\ y = \alpha[D_4/D_4]+\beta([D_4/C_2]+[D_4/C_2']+[D_4/C_2''])+\gamma[D_4/E]\\ \in A(D_4),\ \text{and}\\ z = u[C_3/C_3]+\delta[C_3/E]\in A(C_3).\end{aligned}$$

Here we remark that $\mathrm{res}^{D_4}_E y = \mathrm{res}^{C_3}_E z$. Using these equalities, we can readily see the equality $A(\mathfrak{F})' = A(G)|_S$. □

3. Basic observation of $A(G)|_{\mathcal{F}}$ and $A(\mathfrak{F})$

For each subgroup H of G, we have the homomorphism $\chi_H : A(G)\to\mathbb{Z}$ defined by $\chi_H([X]-[Y]) = |X^H| - |Y^H|$ for finite G-sets X and Y. Let $(\prod_{H\in\mathcal{S}(G)}\mathbb{Z})^G$ denote the G-conjugation invariant subset of $\prod_{H\in\mathcal{S}(G)}\mathbb{Z}$. We get the homomorphism $\sqcap\chi : A(G)\to(\prod_{H\in\mathcal{S}(G)}\mathbb{Z})^G$ by assigning $(\chi_H(x))_{H\in\mathcal{S}(G)}$ to $x\in A(G)$. We recall the next two lemmas, see e.g. [5, I (2.18), I Proposition 2, IV (5.1)–(5.7)], [8, (2.2), (5.1)–(5.3)].

Lemma 3.1 *The homomorphism $\sqcap\chi : A(G)\to(\prod_{H\in\mathcal{S}(G)}\mathbb{Z})^G$ is injective.*

Lemma 3.2 (Burnside Congruence) *An element $(y_H)_{H\in\mathcal{S}(G)}\in(\prod_{H\in\mathcal{S}(G)}\mathbb{Z})^G$ lies in the image of $\sqcap\chi : A(G)\to(\prod_{H\in\mathcal{S}(G)}\mathbb{Z})^G$ if and only if*

$$\sum_{s\in WH} y_K \equiv 0 \mod |WH|$$

for all $H\in\mathcal{S}(G)$, where $WH = N_G(H)/H$ and K is the subgroup of $N_G(H)$ such that $K\supset H$ and $K/H=\langle s\rangle\le WH$.

For $L\in\mathcal{F}$, we denote by φ_L the composition

$$A(\mathfrak{F}) \xrightarrow{incl} \prod_{H \in \mathcal{F}} A(H) \xrightarrow{proj} A(L) \xrightarrow{\chi_L} \mathbb{Z}.$$

Lemma 3.3 *The homomorphism*

$$\varphi_{\mathcal{F}} = \prod_{(H) \subset \mathcal{F}} \varphi_H : A(\mathfrak{F}) \longrightarrow \prod_{(H) \subset \mathcal{F}} \mathbb{Z}$$

is injective.

Proof. Let $x = (x_H)_{H \in \mathcal{F}}$, where $x_H \in A(H)$, be an element of $A(\mathfrak{F})$ such that $\varphi_{\mathcal{F}}(x) = 0$, i.e. $\chi_L(x_L) = 0$ for all $L \in \mathcal{F}$. For $H \in \mathcal{F}$ and $L \leq H$, we have

$$\chi_L(x_H) = \chi_L(\mathrm{res}^H_L x_H) = \chi_L(x_L) = 0.$$

By Lemma 3.1, we get $x_H = 0$ in $A(H)$. This implies $x = 0$ in $A(\mathfrak{F})$. Thus $\varphi_{\mathcal{F}}$ is injective. □

We are ready for proving Proposition 1.2.

Proof of Proposition 1.2. Let a and b denote the numbers of G-conjugacy classes of elements in $\mathcal{F}$ and $\mathcal{S}(G) \smallsetminus \mathcal{F}$, respectively. The Burnside ring $A(G)$ is a free $\mathbb{Z}$-module, and hence $\mathrm{Ker}(\mathrm{res}^G_{\mathcal{F}})$ and $A(G)|_{\mathcal{F}}$ both are free $\mathbb{Z}$-modules. The module $A(G)$ has the $\mathbb{Z}$-basis $\{[G/H] \mid (H) \subset \mathcal{S}(G)\}$, where (H) is the G-conjugacy class of $H \in \mathcal{S}(G)$. It is clear that $\mathrm{rank}\, A(G) = a+b$.

Since $\varphi_{\mathcal{F}}$ is injective and $A(G)|_{\mathcal{F}} \subset A(\mathfrak{F})$, we get

$$\mathrm{rank}\, A(G)|_{\mathcal{F}} \leq \mathrm{rank}\, A(\mathfrak{F}) \leq a. \tag{3.1}$$

The injectivity of $\sqcap\chi$ and $\varphi_{\mathcal{F}}$ imply that the homomorphism

$$\prod_{(K) \subset \mathcal{S}(G) \smallsetminus \mathcal{F}} \chi_K : \mathrm{Ker}(\mathrm{res}^G_{\mathcal{F}}) \to \prod_{(K) \subset \mathcal{S}(G) \smallsetminus \mathcal{F}} \mathbb{Z}$$

is injective. Thus we get

$$\mathrm{rank}\, \mathrm{Ker}(\mathrm{res}^G_{\mathcal{F}}) \leq b. \tag{3.2}$$

Putting these together, we have

$$a + b = \operatorname{rank} A(G) = \operatorname{rank} A(G)|_{\mathcal{F}} + \operatorname{rank} \operatorname{Ker}(\operatorname{res}^G_{\mathcal{F}}) \leq a + b, \tag{3.3}$$

which implies $\operatorname{rank} A(G)|_{\mathcal{F}} = a$ and $\operatorname{rank} \operatorname{Ker}(\operatorname{res}^G_{\mathcal{F}}) = b$. □

The proof above implies the next fact.

Proposition 3.4 *The $\mathbb{Z}$-rank of $A(\mathfrak{F})$ is equal to the number of G-conjugacy classes of subgroups belonging to $\mathcal{F}$.*

The next lemma is essentially due to [9, Lemma 8]. We remark that in the case where G is an elementary abelian p-group for a prime p, the lemma can be proved by explicit calculation, and in the case where G is a nontrivial perfect group, the lemma immediately follows from Lemma 3.2.

Lemma 3.5 *Let G be a finite nontrivial group. Then there exists $\gamma \in A(G)$ such that $\chi_G(\gamma) = k_G$ and $\operatorname{res}^G_H \gamma = 0$ for all $H < G$.*

Proof. Let $\psi : \mathcal{S}(G) \to \mathbb{Z}$ be the function uniquely defined by the conditions

$$\psi(G) = k_G, \quad \text{and} \quad \sum_{K \supset H} \psi(K) = 0 \quad \text{for all } H < G. \tag{3.4}$$

R. Oliver [9, Lemma 8] proved that $|N_G(H)/H|$ divides $\psi(H)$ for any $H \in \mathcal{S}(G)$. By the definition in [9, p. 159], ψ is an integral resolving function. By the arguments used in [9, Proof of Theorem 1, p. 161, line 20–p. 162, line 2], there exists a finite G-CW complex X such that

$$\chi(X^G) = 1 + \psi(G), \quad \text{and} \quad \chi(X^H) = 1 \quad \text{for all } H < G, \tag{3.5}$$

where $\chi(X^H)$ is the Euler characteristic of X^H. Let γ be the element of $A(G)$ satisfying

$$\chi_H(\gamma) = \chi(X^H) - 1 \quad \text{for all } H \in \mathcal{S}(G), \tag{3.6}$$

see [8, p. 129, (1.1)]. Then $\chi_G(\gamma) = k_G$ and $\chi_H(\gamma) = 0$ for all $H < G$. □

We obtain Proposition 1.3 from the lemma above as follows.

Proof of Proposition 1.3. Let $\gamma \in A(G)$ be the element stated in Lemma 3.5. It is clear that $\gamma \in \operatorname{Ker}(\operatorname{res}^G_{\mathcal{F}})$. Let α be an element in

$\mathrm{Ker}(\mathrm{res}^G_{\mathcal{F}})$. If p is a prime and N is a normal subgroup of G with index p, then $\chi_G(\alpha) \equiv \chi_N(\alpha) = 0 \mod p$. This implies that $\chi_G(\alpha)$ is divisible by k_G. By Lemma 3.1, $\alpha = m\gamma$ for some integer m. □

Proposition 3.6 *Let p be a prime, G a nontrivial abelian group of p-power order, and n a natural number prime to p. Then there exists an element $x \in A(G)$ such that $\chi_G(x) = 1$ and $\mathrm{res}^G_H x \in nA(H)$ for all $H < G$.*

Proof. By Lemma 3.5, we have an element $\gamma \in A(G)$ such that $\chi_G(\gamma) = p$ and $\mathrm{res}^G_H \gamma = 0$ for all $H < G$. There exist integers a and b satisfying $ap + bn = 1$. Set $x = a\gamma + bn[G/G]$. Then $\chi_G(x) = ap + bn = 1$ and $\mathrm{res}^G_H x = n(b[H/H])$ for all $H < G$. □

Let N be a normal subgroup of G, L a subgroup of G containing N, and X a finite L-set. Then the N-fixed point set X^N and the complement $X \smallsetminus X^N$ are L-sets, and X^N can be regarded as an L/N-set. For $x = [X] - [Y] \in A(L)$, let x^N denote the element $[X^N] - [Y^N]$ in $A(L/N)$. Then we obtain a homomorphism

$$\mathrm{fix}^N_L : A(L) \to A(L/N);\ x \longmapsto x^N.$$

For a finite group G and a prime p, let $G^{\{p\}}$ denote the smallest normal subgroup of G such that $G/G^{\{p\}}$ is of p-power order.

Proposition 3.7 *Let P be a cyclic group of order p^2 or an elementary abelian p-group of order $\geq p^2$, let G be the cartesian product $P \times P_1 \times \cdots \times P_m$ such that for each $i = 1, \ldots, m$, P_i is a nontrivial elementary abelian p_i-group, and let $\mathcal{F} = \mathcal{F}_G$ and $\mathfrak{F} = \mathfrak{F}_G$. Then $A(G)|_{\mathcal{F}} \neq A(\mathfrak{F})$.*

Proof. In the case that $G = P$, the conclusion follows from Propositions 2.5 and 2.3. Thus we may suppose $m \geq 1$. Let $\mathcal{G} = \mathcal{F}_P$, i.e. $\mathcal{G} = \mathcal{S}(P) \smallsetminus \{P\}$, and $\mathfrak{G} = \mathfrak{F}_P$, hence $\mathrm{Obj}(\mathfrak{G}) = \mathcal{G}$. For each $i = 1, \ldots m$, by Proposition 3.6, we can take an element $u_i \in A[P_i]$ satisfying

$$\chi_{P_i}(u_i) = 1, \text{ and } \mathrm{res}^{P_i}_K u_i \in |P|A(K) \quad \text{for all } K < P_i. \tag{3.7}$$

Let $w = (w_K)_{K \in \mathcal{G}} \in A(\mathfrak{G})$ be the element such that

$$w_K = [K/E] \quad \text{for all } K \in \mathcal{G}_{\max}.$$

We set $u = u_1 \cdots u_m \in A(G^{\{p\}})$ and

$$v_{KG^{\{p\}}} = w_K u \in A(KG^{\{p\}}) \quad (K \in \mathcal{G}_{\max}\ ).$$

Let $\mathcal{H} = \mathcal{S}(G^{\{p\}}) \smallsetminus \{G^{\{p\}}\}$. Then for $S \in \mathcal{H}_{\max}$, the element

$$\mathrm{res}^{KG^{\{p\}}}_{KS} v_{KG^{\{p\}}} = w_K \big(\mathrm{res}^{G^{\{p\}}}_{S} u\big)$$

lies in $|P|A(KS)$. By Lemma 3.2, there exists an element $v_{PS} \in A(PS)$ such that

$$\mathrm{res}^{PS}_{KS} v_{PS} = \mathrm{res}^{KG^{\{p\}}}_{KS} v_{KG^{\{p\}}}.$$

Thus the datum $((v_{PS})_{S\in\mathcal{H}_{\max}},\ (v_{KG^{\{p\}}})_{K\in\mathcal{G}_{\max}})$ determines an element $v = (v_K)_{K\in\mathcal{F}} \in A(\mathfrak{F})$.

For $K \le L$ and $y = \sum_{(H)\subset\mathcal{S}(L)} a_H[L/H] \in A(L)$, let $d(y, L/K)$ denote the coefficient a_H of $[L/K]$.

Assume that there exists an element $x \in A(G)$ such that $\mathrm{res}^G_{\mathcal{F}} x = v$. Then we readily obtain

$$d(\mathrm{res}^G_{KG^{\{p\}}} x, KG^{\{p\}}/G^{\{p\}}) = d(v_{KG^{\{p\}}}, KG^{\{p\}}/G^{\{p\}}),$$
$$d(\mathrm{res}^G_{KG^{\{p\}}} x, KG^{\{p\}}/G^{\{p\}}) = d(\mathrm{res}^P_K(x^{G^{\{p\}}}), K/E),$$

and

$$d(v_{KG^{\{p\}}}, KG^{\{p\}}/G^{\{p\}}) = d(v^{G^{\{p\}}}_{KG^{\{p\}}}, K/E) = d(w_K, K/E) = 1$$

from (3.7). By the arguments proving (2.6), we get

$$\sum_{K\in\mathcal{G}_{\max}} d(\mathrm{res}^P_K x^{G^{\{p\}}}, K/E) \ \text{ is divisible by } p. \tag{3.8}$$

However, since $|\mathcal{G}_{\max}| \equiv 1 \mod p$, the arguments proving (2.7) show

$$\sum_{K\in\mathcal{G}_{\max}} d(w_K, K/E) \equiv 1 \mod p. \tag{3.9}$$

The property (3.8) contradicts the property (3.9), and hence v does not belong to $A(G)|_{\mathcal{F}}$. $\square$

4. Observation of $A(G/N)|_{\overline{\mathcal{F}}}$ and $A(\overline{\mathfrak{F}})$

Throughout this section, let $\mathcal{F} = \mathcal{F}_G$ and $\mathfrak{F} = \mathfrak{F}_G$. Let N be a proper normal subgroup of G, $Q = G/N$, $\pi : G \to Q$ the projection, $\overline{\mathcal{F}} = \mathcal{F}_Q$, and $\overline{\mathfrak{F}} = \mathfrak{F}_Q$. Then the projection π induces the homomorphism $\pi^* : A(Q) \to A(G)$;

$$\pi^*([Q/H]) = [G/\pi^{-1}(H)] \qquad (H \in \mathcal{S}(Q)).$$

We readily see that $\mathrm{fix}_G^N \circ \pi^*$ is the identity map on $A(Q)$. For $w = (w_K)_{K\in\mathcal{F}}$, consider the associated datum

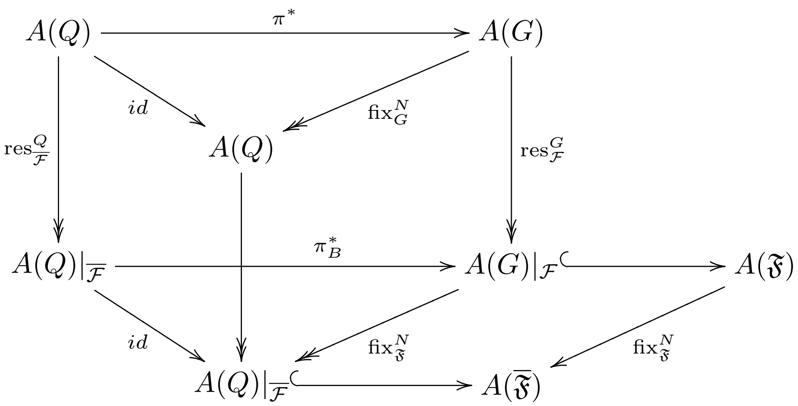
$$({w_K}^N)_{K\in\mathcal{F},\ K\supset N}.$$

This yields the homomorphism $\mathrm{fix}_{\mathfrak{F}}^N : A(\mathfrak{F}) \to A(\overline{\mathfrak{F}})$.

For $x = (x_H)_{H\in\overline{\mathcal{F}}} \in A(Q)|_{\overline{\mathcal{F}}}$, take an element $y \in A(Q)$ such that $\mathrm{res}_{\overline{\mathcal{F}}}^Q(y) = x$ and consider the element $z = (z_K)_{K\in\mathcal{F}} = \mathrm{res}_{\mathcal{F}}^G(\pi^* y)$ in $A(G)|_{\mathcal{F}}$. For any $K \in \mathcal{F}$ with $K \supset N$, ${z_K}^N$ is equal to z_K in $A(K)$ ($\supset A(K/N)$ via $\pi|_K^*$). Since $A(Q)|_{\overline{\mathcal{F}}}$ is a $\mathbb{Z}$-free module, we can get a homomorphism

$$\pi_B^* : A(Q)|_{\overline{\mathcal{F}}} \to A(G)|_{\mathcal{F}}$$

such that $\pi_B^*(x) = \mathrm{res}_{\mathcal{F}}^G(\pi^* y)$ for $x \in A(Q)|_{\overline{\mathcal{F}}}$ and some $y \in A(Q)$ with $\mathrm{res}_{\overline{\mathcal{F}}}^Q(y) = x$. Then the diagram



(4.1)

commutes.

In the rest of this section, let L be a nontrivial subgroup of G, $\mathcal{G} = \mathcal{F}_L$, and $\mathfrak{G} = \mathfrak{F}_L$.

Proposition 4.1 *Let p be a prime and L a nontrivial subgroup of G such that $G = L \times C_p$, and p is prime to the order of L. If $A(L)|_{\mathcal{G}}$ coincides with $A(\mathfrak{G})$ then $A(G)|_{\mathcal{F}}$ coincides with $A(\mathfrak{F})$.*

Proof. We may regard $C_p \subset G$ and $G = L \cdot C_p$. Set $B(G) = A(\mathfrak{F})$ and $Q = G/C_p$. Let $\pi : G \to Q$ be the projection. Since Q is isomorphic to L, the restriction homomorphism $\mathrm{res}^Q_{\overline{\mathcal{F}}} : A(Q) \to B(Q) = A(\overline{\mathfrak{F}})$ is surjective, i.e. $A(Q)|_{\overline{\mathcal{F}}} = A(\overline{\mathfrak{F}})$. Since $B(L) = A(\mathfrak{G})$ is a free $\mathbb{Z}$-module and $\mathrm{res}^G_{\mathcal{G}}$: $A(L) \to B(L)$ is surjective, there is a homomorphism $\iota_B : B(L) \to B(G)$ such that the diagram

$$\begin{array}{ccc} A(L) & \xrightarrow{\mathrm{ind}^G_L} & A(G) \\ {\scriptstyle \mathrm{res}^L_{\mathcal{G}}}\big\downarrow & & \big\downarrow{\scriptstyle \mathrm{res}^G_{\mathcal{F}}} \\ B(L) & \xrightarrow[\iota_B]{} & B(G) \end{array} \tag{4.2}$$

commutes. For a subgroup K of L, define the homomorphism

$$f_K : A(K) \to A(K \cdot C_p) \times A(K)$$

by

$$f_K([K/H]) = ([(K \cdot C_p)/H], p[K/H]) \quad \text{for } H \leq K.$$

Then we obtain the homomorphism

$$\sqcap f_\star = \prod_{K<L} f_K : \prod_{K<L} A(K) \to \prod_{K<L} A(K \cdot C_p) \times \prod_{K<L} A(K).$$

We remark that the diagram

$$\begin{array}{ccccc}
B(L) & & \xrightarrow{\iota_B} & & B(G) \\
\big\downarrow & & & & \big\downarrow \\
\prod_{K<L} A(K) & & & & \prod_{T<G} A(T) \\
& \searrow^{\sqcap f_\star} & & \swarrow^{\text{proj}} & \\
& & \prod_{K<L} A(K \cdot C_p) \times \prod_{K<L} A(K) & &
\end{array} \tag{4.3}$$

commutes.

Decompose $\mathcal{F}$ to $\mathcal{F} = \mathcal{F}_1 \amalg \mathcal{F}_2 \amalg \{L\}$, where

$$\mathcal{F}_1 = \{K \in \mathcal{F} \mid K \supset C_p\},$$
$$\mathcal{F}_2 = \{K \in \mathcal{F} \mid K \not\supset C_p,\ K \neq L\}.$$

Let $x = ((x_K)_{K\in\mathcal{F}_1}, (x_K)_{K\in\mathcal{F}_2}, x_L) \in B(G)$. Since $K \cdot C_p \in \mathcal{F}_1$ for any $K \in \mathcal{F}_2$, the element x is determined by the datum $((x_K)_{K\in\mathcal{F}_1}, x_L)$. Define $u = (u_H)_{H\in\overline{\mathcal{F}}} \in B(Q)$ by $u_{\pi(K)} = \mathrm{fix}_K^{C_p} x_K$ for $K \in \mathcal{F}_1$. Set $y = (y_K)_{K\in\mathcal{F}} = x - \pi_B^*(u)$. For $K \in \mathcal{F}_1$, since ${y_K}^{C_p} = 0$, y_K has the form

$$y_K = \sum_{H\in\mathcal{S}(K)\cap\mathcal{F}_2} b_H[K/H].$$

Let $K^{\{p\}}$ be the normal subgroup of K with index p. Define $v = (v_{K^{\{p\}}})_{K\in\mathcal{F}_1}$ by

$$v_{K^{\{p\}}} = \sum_{H\in\mathcal{S}(K)\cap\mathcal{F}_2} b_H[K^{\{p\}}/H].$$

Then v belongs to $B(L)$. Note that x has the form

$$x = \pi_B^*(u) + \iota_B(v) + w \tag{4.4}$$

with $w = (w_K)_{K\in\mathcal{F}} \in B(G)$ such that $w_K = 0$ for all $K \neq L$. Since $u \in A(Q)|_{\overline{\mathcal{F}}}$ and $v \in A(L)|_{\mathcal{G}}$, where $\overline{\mathcal{F}} = \mathcal{S}(Q) \smallsetminus \{Q\}$ and $\mathcal{G} = \mathcal{S}(L) \smallsetminus \{L\}$, $\pi_B^*(u)$ and $\iota_B(v)$ both belong to $A(G)|_{\mathcal{F}}$, cf. the commutative diagrams (4.1) and (4.2). Let $\tau : G \to L$ be the canonical projection. Set $z = \tau^*(w_L)$. Then $\mathrm{res}_L^G(z) = w_L$ and $\mathrm{res}_K^G(z) = 0$ for all $K \in \mathcal{S}(G) \smallsetminus \{G, L\}$. Thus

$\mathrm{res}^G_{\mathcal{F}} : A(G) \to B(G)$ is surjective. □

Corollary 4.2 *Let G be a nontrivial cyclic group of which the order is a prime or a product of distinct primes. Then $A(G)|_{\mathcal{F}}$ coincides with $A(\mathfrak{F})$.*

Proof. We obtain the corollary from Propositions 2.1 and 4.1. □

Proposition 4.3 *Let G be a nontrivial finite group, N a proper normal subgroup of G, and $Q = G/N$. If all maximal proper subgroups of G contain N and $A(G)|_{\mathcal{F}}$ coincides with $A(\mathfrak{F})$ then $A(Q)|_{\overline{\mathcal{F}}}$ coincides with $A(\overline{\mathfrak{F}})$, where $\overline{\mathcal{F}} = \mathcal{F}_Q$ and $\overline{\mathfrak{F}} = \mathfrak{F}_Q$.*

Proof. In this situation, the projection $\pi : G \to Q$ induces the homomorphism $\pi^*_{\mathfrak{F}} : A(\overline{\mathfrak{F}}) \to A(\mathfrak{F})$ such that $\mathrm{fix}^N_{\mathfrak{F}} \circ \pi^*_{\mathfrak{F}}$ is the identity map on $A(\overline{\mathfrak{F}})$. Since $A(G)|_{\mathcal{F}} = A(\mathfrak{F})$, we get $A(Q)|_{\overline{\mathcal{F}}} = A(\overline{\mathfrak{F}})$. □

Now we give the proof of Theorem 1.4.

Proof of Theorem 1.4. By Corollary 4.2, it suffices to prove that if $A(G)|_{\mathcal{F}} = A(\mathfrak{F})$ then G is a cyclic group of which the order is a prime or a product of distinct primes. Assume that G is a minimal nilpotent group with respect to the order such that $A(G)|_{\mathcal{F}} = A(\mathfrak{F})$ but G is not a cyclic group of which the order is a prime or a product of distinct primes. Write G as the product $P_1 \times \cdots \times P_m$ of Sylow p_i-subgroups P_i. Let N_i denote the intersection of all maximal proper subgroups of P_i and set $N = N_1 \cdots N_m$. First set $Q = G/N$. By Proposition 4.3, we obtain $A(Q)|_{\overline{\mathcal{F}}} = A(\overline{\mathfrak{F}})$ from $A(G)|_{\mathcal{F}} = A(\mathfrak{F})$. It is readily seen that Q is a product of elementary abelian p_i-groups. Thus by Proposition 3.7, Q is a cyclic group of order $p_1 \cdot \cdots \cdot p_m$. This implies that each P_i admits a unique maximal proper subgroup N_i. If N_i is nontrivial then there exists a subgroup $C^{(i)}$ of order p_i such that $C^{(i)} \subset N_i \cap Z_i$, where Z_i is the center of P_i. Now set $Q = G/C_i$. Using Proposition 4.3, we obtain $A(Q)|_{\overline{\mathcal{F}}} = A(\overline{\mathfrak{F}})$ from $A(G)|_{\mathcal{F}} = A(\mathfrak{F})$. By the minimal property of G, $G/C^{(i)}$ is a cyclic group of which the order is a prime or a product of distinct primes. Thus if $j \neq i$ then $|P_j| = p_j$, and $P_i \cong C_{p_i} \times C_{p_i}$ or $C_{p_i^2}$. By Proposition 3.7 we get $A(G)|_{\mathcal{F}} \neq A(\mathfrak{F})$, which is a contradiction. □

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