# A remark on 2-transitive groups of odd degree

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Let $G$ be a 2-transitive group on $\Omega=\{1, 2, \cdots, n\}$, $n$ odd. Let $G_{a,b}$ be the stabilizer of the points $a$, $b$ and $g_1^*(2)$ the number of involutions in $G_1$ which fix only the point 1. Let $F(H)$ denote the set of all points fixed by a subset H of G and $\alpha(H)$ the number of points in $F(H)$. In this note we shall prove the following.

THEOREM *If* $|G_{1,2}|$ *is even and* $\alpha(G_{1,2})$ *is odd, then* $g_1^*(2)=1$ *and* $G$ *has a regular normal subgroup or every involution of* $G$ *is conjugate to an involution of* $G_{1,2}$.

PROOF. Let $I$ be an involution of $G$ with the cycle structure $(1, 2)\cdots$. Then $I$ normalizes $G_{1,2}$. Let $d$ be the number of elements of $G_{1,2}$ inverted by $I$. Then $d$ is the number of involutions with cycle structures $(1, 2)\cdots$. Let $g(2)$ and $g_1(2)$ denote the number of involutions in $G$ and $G_1$, respectively. Since $G$ is 2-transitive, $G=G_1+G_1IG_1$ and hence $g(2)=g_1(2)+d(n-1)$. $d-g_1^*(2)=\{(g(2)-g_1^*(2)n)-(g_1(2)-g_1^*(2))\}/(n-1)$ is the number of involutions with the cycle structures $(1, 2)\cdots$ which are conjugate to an involution of $G_{1,2}$. Thus $g_1^*(2)$ is the number of involutions with cycle structures $(1, 2)\cdots$ which are not conjugate to any involution of $G_{1,2}$. Since $F(G_{1,2})^I=F(G_{1,2})$ and $\alpha(G_{1,2})$ is odd, $\alpha(\langle G_{1,2}, I\rangle)=1$. Let $a$ be the point in $F(\langle G_{1,2}, I\rangle)$. Every involution in $IG_{1,2}$ fixes $a$. Assume $g_1^*(2)\neq 0$. Let $L$ be the subgroup of $G_a$ generated by $g_1^*(2)$ involutions with the cycle structures $(1, 2)\cdots$ which fix only $a$. Then $L$ is characteristic in $G_a$ and hence $L$ is 1/2-transitive on $\Omega-\{a\}$. Since $\{1, 2\}^L=\{1, 2\}$, $L$ is 2-group and every $L$-orbits in $\Omega-\{a\}$ is of length 2. If $g^*(2)\geq 2$, then there exists a $L$-orbit of length $>2$. Thus $g^*(2)=1$, and by $Z^*$-theorem $0(G)\neq 1$ and $G$ has a regular normal subgroup. This proves Theorem.

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## References

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