On surfaces in 3-sphere: Prime decompositions

By Shin'ichi SUZUKI

0. Introduction

Throughout this paper, we work in the piecewise linear category, consisting of simplicial complexes and piecewise linear maps. The theorems concern "knot types" of a connected, closed (=compact, without boundary), oriented surface (=2-dimensional manifold) F in the 3-dimensional sphere S^3 with a fixed orientation.

In the previous paper [25], we showed a unique prime decomposition theorem for special linear graphs in S^3 as generalization of knots [23] and links [12], see [20] and also [2], [10], [26], [27]. In the paper, we shall formulate a prime decomposition theorem for pairs $(F \subset S^3)$'s as the same way as that of [25] and [27] except for obvious modifications, and discuss the uniqueness of the prime decompositions.

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1. Prime Decompositions for $(F \subset S^3)$

In the paper, homeomorphism and isomorphism are denoted by the same symbol $\cong$, while $\approx$, $\simeq$ and $\sim$ refer, respectively, to isotopy, homotopy and homology. ∂X, $cl(X)$ and $°X$ denote, respectively, the boundary, the closure and the interior of a manifold X, and when applied to oriented objects these respect orientations. By $\boldsymbol{Z}$ we shall denote the infinite cyclic group.

We shall say that a submanifold X of a manifold Y is *properly embedded* (or simply *proper*) if $X \cap \partial Y = \partial X$.

By D^n and S^{n-1} we shall denote the standard n-cell and the standard $(n-1)$-sphere ∂D^n, respectively. We always assume that S^3 has the right-handed orientation.

For a connected surface F, $g(F)$ stands for the genus of F.

We shall now formulate the prime decomposition for pairs $(F \subset S^3)$ of closed, connected and oriented surfaces in S^3.

1. 1. Definition. Two pairs $(F_1 \subset S^3)$ and $(F_2 \subset S^3)$ are said to be *congruent*, denoted by $(F_1 \subset S^3) \cong (F_2 \subset S^3)$, if there is an orientation-preserving

homeomorphism $\psi: S^3 \to S^3$ such that $\psi(F_1)=F_2$ and $\psi|_{F_1}$ is also orientation-preserving.

Then it is trivial that the relation of congruence is an equivalence relation. By Fisher [6], this definition is the same as that of Tsukui [27], cf. Gugenheim [11]. We call the congruence class of a pair $(F\subset S^3)$ the *knot type* of $(F\subset S^3)$. For a pair $(F\subset S^3)$, we denote the pair having the opposite orientation to F by $(-F\subset S^3)$. Of course, $(F\subset S^3)$ and $(-F\subset S^3)$ are not always congruent.

1.2. *Composition*: Let $(F_1\subset S^3)$ and $(F_2\subset S^3)$ be pairs, and let $D_1^3\subset S^3$ and $D_2^3\subset S^3$ be 3-cells with $D_1^3\cap F_1\cong D^2$ and $D_2^3\cap F_2\cong D^2$. Then, the *composition* $(F_1\subset S^3)\#(F_2\subset S^3)$ of two pairs $(F_1\subset S^3)$ and $(F_2\subset S^3)$ is a new pair $(F\subset S^3)$ obtained by matching the boundaries $\partial(S^3-{}^\circ D_1^3)$ and $\partial(S^3-{}^\circ D_2^3)$ using an orientation-reversing homeomorphism ζ such that $\zeta(\partial(F_1-{}^\circ D_1^3))=\partial(F_2-{}^\circ D_2^3)$ and $\zeta|_{(F_1-{}^\circ D_1^3)}$ is also orientation-reversing.

By the Alexander's theorem [1] and the homogeneity theorem of Newman-Gugenheim [11], up to congruence, the operation $\#$ of composition is well-defined, associative and commutative.

Conversely, we shall say that $(F_1\subset S^3)\#(F_2\subset S^3)$ is a *decomposition* for $(F\subset S^3)$, and that such the 2-sphere $\partial D_1^3=\partial D_2^3$ gives the decomposition.

For any pair $(F\subset S^3)$, the existence of a 3-cell $D_0^3\subset S^3$ with $D_0^3\cap F\cong D^2$ is obvious. Let $D_1^3\cup\cdots\cup D_n^3$ be mutually disjoint 3-cells in S^3 with $D_i^3\cap F\cong D^2$. Then, it will be convenient to call the proper pair $(F\cap(S^3-\cup{}^\circ D_i^3)\subset(S^3-\cup{}^\circ D_i^3))$ is *equivalent* to $(F\subset S^3)$.

From 1.2, we obtain at once the

1.3. Proposition. *If* $(F\subset S^3)\cong(F_1\subset S^3)\#(F_2\subset S^3)$, *then* $g(F)=g(F_1)+g(F_2)$.

1.4. Definition. We call a pair $(F\subset S^3)$ *non-trivial* if $g(F)\neq 0$, that is, $(F\subset S^3)\not\cong(S^2\subset S^3)$. A non-trivial pair $(F\subset S^3)$ is said to be *prime* if there is no decomposition $(F\subset S^3)\cong(F_1\subset S^3)\#(F_2\subset S^3)$ with both $(F_1\subset S^3)$ and $(F_2\subset S^3)$ non-trivial.

1.5. Proposition. *Every* $(F\subset S^3)$ *with* $g(F)=1$ *is prime.*

By Propositions 1.3. and 1.5 and the finiteness of genus, we have the following:

1.6. Theorem. (Existence of Prime Decomposition) *Every non-trivial pair* $(F\subset S^3)$ *has a prime decomposition*

$$(F\subset S^3)\cong(F_1\subset S^3)\#\cdots\#(F_u\subset S^3)$$

of prime pairs $(F_i\subset S^3)$.

The following question immediately come to mind.

1.7. Question. *Is the prime decomposition for* $(F\subset S^3)$ *unique?* That is, *are the summands* $(F_i\subset S^3)$ *in* 1.6 *uniquely determined up to order and congruence?*

This has been shown to be true for some kind of pairs in [27] and [30].

1.8. Proposition. (Tsukui [27, Th. 2]) *For any pair* $(F\subset S^3)$ *with* $g(F)=2$, *the prime decomposition in* 1.6 *is unique.*

In order to state our version of Waldhausen's result [30], we need some preparation.

1.9. Let $(F\subset S^3)$ be a pair of a connected, closed, oriented suaface F in S^3. Then, S^3-F consists of two oriented open 3-manifolds. We denote the closures of these manifolds in S^3 by V_F and W_F, and in particular, we always assume that the orientation of ∂V_F is consistent with that of F. It will be noticed that $V_F\cup W_F=S^3$, $V_F\cap W_F=F$ and $V_F=S^3-{}^\circ W_F=cl(S^3-W_F)$, $W_F=S^3-{}^\circ V_F=cl(S^3-V_F)$, see Edward [3].

1.10. Definition. A non-trivial pair $(F\subset S^3)$ is said to be *unknotted* if both V_F and W_F are solid-tori of genus $g(F)$. Here, a solid-torus of genus p is a 3-manifold homeomorphic to a regular neighborhood in S^3 of a connected compact 1-dimensional complex of Euler characteristic $1-p$. (Refer to 2.12, 2.17 and 2.18 below.)

1.11. Proposition. *For any unknotted pairs* $(F\subset S^3)$ *and* $(F'\subset S^3)$ *with* $g(F)=g(F')=1$, $(F\subset S^3)\cong(F'\subset S^3)$.

The proof of 1.11 is by the Dehn's lemma [14], [22], or the loop theorem [21], see [27], [30], etc..

This Proposition enables us to denote an unknotted pair of genus 1 by $(T\subset S^3)$, and we also denote $(n-1)\,(T\subset S^3)\#(T\subset S^3)$ simply by $n(T\subset S^3)$.

If a pair $(F\subset S^3)$ is unknotted, it forms a Heegaard-splitting of S^3, and so we have:

1.12. Proposition. (Waldhausen [30, (3.1)]) *If* $(F\subset S^3)$ *is unknotted, then* $(F\subset S^3)$ *has the unique prime decomposition*

$$(F\subset S^3)\cong g(F)\,(T\subset S^3)\,.$$

We will study unknotted pairs in the forthcoming papers.

In the remainder of this paper, we shall give in §2 and 3 some elementary properties of V_F and W_F, and in §4 an affirmative answer to Question 1.7 in a special case, and in §5 some examples of prime pairs.

2. Preliminary Remarks

In this section, let us explain several definitions and well-known facts to be used freely in the sequel.

2. 1. 3-*manifolds* are to be compact, connected and oriented.

We shall call a homeomorphic image of S^1 (resp. of D^1) a *simple loop* (resp. a *simple arc*).

For a subcomplex X of a complex Y, by $N(X; Y)$ we denote a regular neigoborhood of X in Y, that is, we construct its second derived and take the closed star of X. It will be noted that if Y is a manifold, $N(X; Y)\cap \partial Y=N(X\cap\partial Y; \partial Y)$.

An *isotopy* (i) of a homeomorphism $\phi: Y\to Y'$ is a homeomorphism $H: Y\times[0, 1]\to Y'\times[0, 1]$ such that $H(y, t)=(\eta_t(y), t)$, where $\eta_t: Y\to Y'$ is a homeomorphism, and $\eta_0=\phi$;

(ii) of subcomplexes X_1 and X_2 in Y is an isotopy of the identity map on Y such that $\eta_1(X_1)=X_2$.

2. 2. *Convention*: In the paper, we often consider two 2-manifolds X_1 and X_2, which may not be connected, properly embedded in a 3-manifold M. The well-known general position argument asserts that there is an isotopy of the identity map on M so that $\eta_1(X_1)$ and X_2 intersect transversally. From now on, unless otherwise specified, we assume that $X_1\cap X_2$ *consists of a finite number of mutually disjoint simple loops and simple arcs proper in both* X_1 *and* X_2.

We make full use of socalled innermost curves. A simple loop Γ in $X_1\cap X_2$ is said to be an *innermost loop* on X_1 if Γ bounds a 2-cell C^2 on X_1 so that $°C^2\cap X_2=\emptyset$, and a simple arc γ in $X_1\cap X_2$ is said to be an *innermost arc* on X_1 if γ cuts off a 2-cell C^2 on X_1 so that $°C^2\cap X_2=\emptyset$. It will be noticed that if $X_1\cong S^2$ or $X_1\cong D^2$, there is at least one innermost curves on X_1 provided $X_1\cap X_2\neq\emptyset$, and moreover there is at least one innermost loop on X_1 provided that $X_1\cap X_2$ contains simple loops.

2. 3. Definition. A 3-manifold M is said to be *irreducible* if every 2-sphere in M bounds a 3-cell in M, and to be ∂-*irreducible* if for any proper 2-cell C^2 in M, ∂C^2 bounds a 2-cell on ∂M.

There are several properties of irreducible and ∂-irreducible 3-manifolds with boundary, see [22], [26], [29], etc.. Some of them will be recorded below.

2. 4. Lemma. (Papakyriakopoulos [21], Stallings [24], etc.) *A 3-manifold M is* ∂-*irreducible if and only if the homomorphism* $\iota_*: \pi_1(\partial M)\to\pi_1(M)$, *induced by the natural inclusion, is a monomorphism.*

2. 5. Proposition. (Fox [7], Homma [13]) *For every non-trivial pair* $(F\subset S^3)$, *at least one of* V_F *and* W_F *is not* ∂-*irreducible.* (Refer to Kinoshita [17]).

2.6. Proposition. *Let M be an irreducible 3-manifold, and let C_1^2 and C_2^2 be proper 2-cells in M with $\partial C_1^2 = \partial C_2^2$. Then, there exists an isotopy of C_1^2 and C_2^2 in M keeping ∂M fixed.*

This follows from the irreducibility of M. The proof, which is omitted here, is by an induction on the number of components in $C_1^2 \cap C_2^2$.

2.7. Definition. (1) Let J and K be systems of mutually disjoint simple loops on a 2-manifold F. We shall say that *J and K are in reduced position*, if $J \cap K$ consists of a finite number of points crossing one another, and there is no 2-cell on F whose boundary consists of an arc in J and arc in K.

(2) Let A and B be systems of mutually disjoint proper 2-cells in a 3-manifold M. We shall say that *A and B are in reduced position*, if ∂A and ∂B are in reduced position on ∂M, and $A \cap B$ consists no simple loops.

2.8. Proposition. (Epstein [4]) *Let J and K be systems of mutually disjoint simple loops on a closed 2-manifold F. Then, there is an isotopy of the identity map on F such that $\eta_1(J)$ and K are in reduced position.*

2.9. Proposition. *Let M be an irreducible 3-manifold, and let A and B be systems of mutually disjoint proper 2-cells in M such that ∂A and ∂B are in reduced position on ∂M. Then, there is an isotopy of the identity map on M so that $\eta_1(A)$ and B are in reduced position.*

2.10. Definition. Let M and M' be 3-manifolds with ∂M and $\partial M'$ connected. The *disk-sum* $M \natural M'$ of M and M' is a 3-manifold obtained by matching a 2-cell on ∂M with a 2-cell on $\partial M'$, using an orientation-reversing homeomorphism. The operation $\natural$ of disk-sum is well-defined up to homeomorphism, and associative and commutative. The reader is refered to Dohi [2], Gross [10], Swarup [26]. A 3-manifold M with connected boundary is said to be *∂-prime*, if $M \not\cong D^3$ and there is no decomposition $M \cong M_1 \natural M_2$ with both $M_1 \not\cong D^3$ and $M_2 \not\cong D^3$.

2.11. Proposition. (Dohi [2], Gross [10], Swarup [26]) *Let M be a 3-manifold with connected boundary. If $M \not\cong D^3$, then M is homeomorphic to a disk-sum $P_1 \natural \cdots \natural P_u$ of ∂-prime 3-manifolds, and the summands P_i are uniquely determined up to order and homeomorphism.*

2.12. Definition. Let SPC denote the class of 3-manifolds M with connected boundary such that M can be embedded in S^3. A 3-manifold U in the class SPC is called a *solid-torus of genus p* if $U \cong p(D^2 \times S^1) = (p-1)(D^2 \times S^1) \natural (D^2 \times S^1)$; a disk-sum of p copies of $D^2 \times S^1$.

2.13. Proposition. (Fox [7]) *For a 3-manifold M in the class* SPC, *there exists a pair $(F \subset S^3)$ with $V_F \cong M$ and $W_F \cong g(F)(D^2 \times S^1)$.*

2.14. Proposition. (Papakyriakopoulos [22]) *3-manifolds in the class* SPC *are irreducible.* (Refer to [26, Prop. 2.7].)

2.15. Proposition. *Let M and M' be 3-manifolds in the class* SPC. *Then, we have the followings:*

(1) *The disk-sum $M \natural M'$ is also in the class* SPC.
(2) *If $g(\partial M)=1$, then M is ∂-prime.*
(3) *If $g(\partial M)\geqq 2$, then M is ∂-prime if and only if M is ∂-irreducible.*
(4) *$M\cong D^2\times S^1$ is an only 3-manifold in the class* SPC *that is ∂-prime but not ∂-irreducible.*
(5) (Jaco [15]) *M is ∂-prime if and only if $\pi_1(M)$ is indecomposable with respect to free products.*

2.16. *Meridian and Meridian-Disk*: Let M be a 3-manifold with connected boundary ∂M. A simple loop J on ∂M will be called a *meridian* of M if $J\simeq 1$ in M and $\partial M-J$ is connected. A system of mutually disjoint n meridians $J_1\cup\cdots\cup J_n$ of M is called a *system of meridians* of M if $\partial M-(J_1\cup\cdots\cup J_n)$ is connected, whence it is a 2-manifold of genus $g(\partial M)-n$ with $2n$ holes. A proper 2-cell A in M and a system of mutually disjoint n proper 2-cells $A_1\cup\cdots\cup A_n$ in M will be called a *meridian-disk* and a *system of meridian-disks*, respectively, if ∂A and $\partial A_1\cup\cdots\cup\partial A_n$ are a meridian and a system of meridians. By Dehn's lemma and the well-known cut-and-exchange method, for any system of meridians $J_1\cup\cdots\cup J_n$ of M there is a system of meridian-disks $A_1\cup\cdots\cup A_n$ of M with $\partial A_1\cup\cdots\cup\partial A_n=J_1\cup\cdots\cup J_n$, and if M is irreducible this system of meridian-disks is unique up to isotopy by 2.6.

We have the following well-known characterization of the solid-torus.

2.17. Proposition. *Let U be a 3-manifold in the class* SPC *with $g(\partial M)=p$. Then the followings are equivalent.*

(1) *$U\cong p(D^2\times S^1)$; a solid-torus of genus p.*
(2) *There is a system of meridians $J_1\cup\cdots\cup J_p$ of U.*
(3) *$\pi_1(U)$ is a free group of rank p.*

2.18. Proposition. (Feustel [5], Griffiths [9], etc.) *Let U be a solid-torus of genus p with $p>0$, and let γ be a simple loop on ∂U. Then, the followings are eqivalent.*

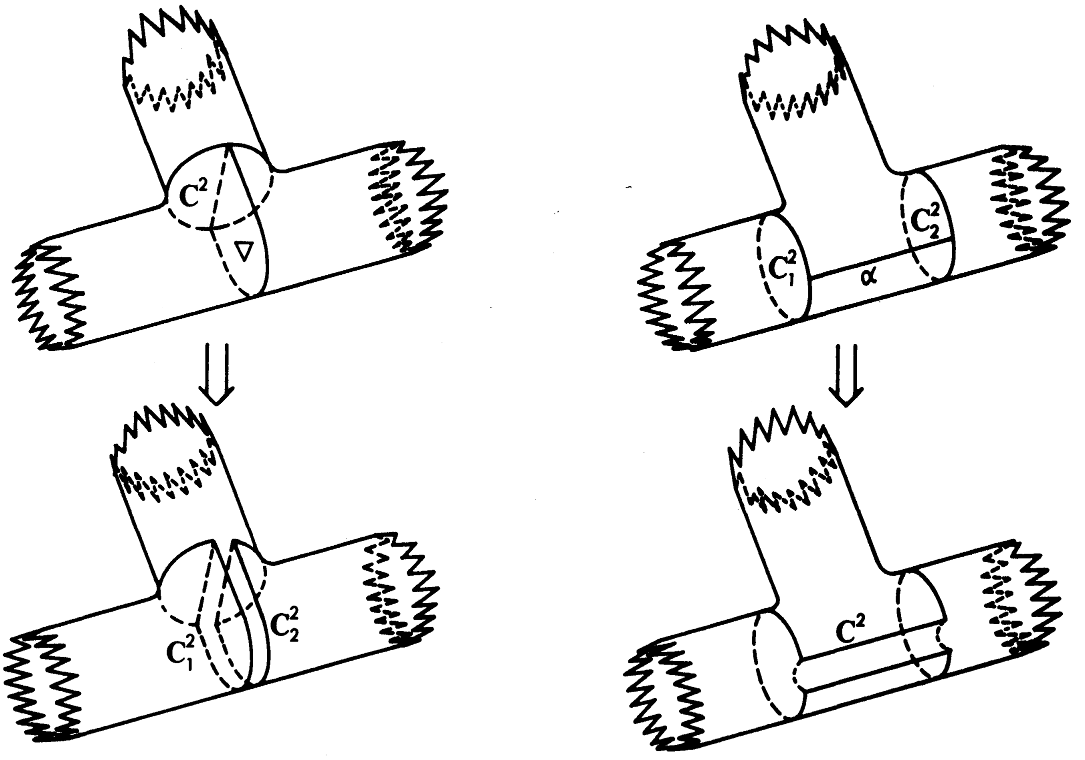
(1) *The 3-manifold obtained by attaching a 3-cell to U along γ is a solid-torus of genus $p-1$.*
(2) *There exists a system of meridians $J_1\cup\cdots\cup J_p$ of U such that $\gamma\cap(J_1\cup\cdots\cup J_p)=\gamma\cap J_1$ consists of one crossing point.*

(3) *The quotient group* $\pi_1(U)/\{\gamma\}^\nu$ *is a free group of rank* $p-1$, *where* $\{\gamma\}^\nu$ *is the smallest normal subgroup of* $\pi_1(U)$ *containing the homotopy class* $[\gamma]$.

3. Modifications of Proper 2-cells

3.1. Definition. Let C^2 be a proper 2-cell in a 3-manifold M. Suppose that there is a 2-cell ∇ in M such that $\nabla \cap C^2 = \partial\nabla \cap C^2$ consists of a simple arc and $\nabla \cap \partial M = \partial\nabla \cap \partial M = cl(\partial\nabla - C^2)$, see Fig. 1 (a). Then, using a regular neighborhood of $C^2 \cup \nabla$ we have disjoint proper 2-cells, say $C_1^2 \cup C_2^2$, in M as illustrated in Fig. 1 (a). More precisely, $cl(\partial N(C^2 \cup \nabla\,;\, M) \cap {}^\circ M)$ consists of three proper 2-cells in M, and one of which is parallel to C^2; let $C_1^2 \cup C_2^2$ be the others. We say that C_1^2 *and* C_2^2 *are obtained from* C^2 *by a modification* (*of type*) ∇ (*along the* 2-*cell* ∇). It should be noticed that $(C_1^2 \cup C_2^2) \cap (C^2 \cup \nabla) = \emptyset$.

Conversely, let C_1^2 and C_2^2 be disjoint proper 2-cells in a 3-manifold M, and let α be a simple arc on ∂M such that $\alpha \cap (\partial C_1^2 \cup \partial C_2^2) = \partial\alpha$ and $\partial\alpha \cap \partial C_1^2 \neq \emptyset \neq \partial\alpha \cap \partial C_2^2$. Then, $cl(\partial N(C_1^2 \cup \alpha \cup C_2^2\,;\, M) \cap {}^\circ M)$ consists of three proper 2-cells in M, and one of which is parallel to C_1^2 and another to C_2^2; let C^2 be the third, see Fig. 1 (b). We say that C^2 *is obtained from* C_1^2 *and* C_2^2



(a) Modification ∇ (b) Modification $\bar{\nabla}$

Fig. 1.

by a modification (*of type*) $\bar{\nabla}$ (*along the simple arc* α). It should be noticed that $C^2 \cap (C_1^2 \cup \alpha \cup C_2^2) = \emptyset$.

3.2. Lemma. (F. Hosokawa) *Let* U *be a solid-torus of genus* p *with a system of meridian-disks* $A_1 \cup \cdots \cup A_p$, *and let* C^2 *be a proper 2-cell in* U. *Then,* C^2 *can be obtained* (*up to isotopy*) *by a finite sequence of modifications* $\bar{\nabla}$*'s from mutually disjoint proper 2-cells* $E_1, \cdots, E_\nu$, $(1 \leqq \nu < \infty)$, *in* U, *where each* E_i *is isotopic to one of* $A_1, \cdots, A_p$ *in* U.

Proof. If $\partial C^2 \simeq 1$ on ∂U, then it is easily checked that C^2 is obtained by a modification $\bar{\nabla}$ from two proper 2-cells E_1 and E_2, where $E_1 \approx A_1 \approx E_2$ in U, and so we may assume that $\partial C^2 \not\simeq 1$ on ∂U. By Propositions 2.14, 2.8 and 2.9, we may assume that C^2 and $A_1 \cup \cdots \cup A_p$ are in reduced position.

If $C^2 \cap (A_1 \cup \cdots \cup A_p) = \emptyset$, then C^2 is contained properly in the 3-cell $D_0^3 = cl(U - N(A_1 \cup \cdots \cup A_p\,;\, U))$. Because ∂C^2 bounds a 2-cell on ∂D_0^3, it is easy to see that C^2 is obtained (up to isotopy) from some of 2-cells $D_0^3 \cap N(A_1 \cup \cdots \cup A_p\,;\, U) = \partial D_0^3 \cap \partial N(A_1 \cup \cdots \cup A_p\,;\, U) = A_1' \cup A_1'' \cup \cdots \cup A_p' \cup A_p''$ by a finite sequence of modifications $\bar{\nabla}$'s. Here, $A_i' \cup A_i'' = \partial D_0^3 \cap \partial N(A_i\,;\, U)$, and of course, $A_i' \approx A_i \approx A_i''$ in U for $i = 1, \cdots, p$.

If $C^2 \cap (A_1 \cup \cdots \cup A_p) \neq \emptyset$, then we choose an innermost arc, say γ_1, on one of $A_1, \cdots, A_p$, say A_1. Let $\nabla_1 \subset A_1$ be the 2-cell cut off by γ_1 so that $°\nabla_1 \cap C^2 = \emptyset$. Then, we have disjoint proper 2-cells $C_1^2 \cup C_2^2$ in U from C^2 by a modification ∇ along ∇_1, so that

$$(C_1^2 \cup C_2^2) \cap (A_1 \cup \cdots \cup A_p) = C^2 \cap (A_1 \cup \cdots \cup A_p) - \gamma_1\,.$$

Repeating of this procedure, we have a finite number of mutually disjoint proper 2-cells, say $C_1^2 \cup \cdots \cup C_\lambda^2$, in U with $(C_1^2 \cup \cdots \cup C_\lambda^2) \cap (A_1 \cup \cdots \cup A_p) = \emptyset$. According to the first case, we have now a required collection of proper 2-cells $E_1 \cup \cdots \cup E_\nu$ from $C_1^2 \cup \cdots \cup C_\lambda^2$ by a finite sequence of modifications ∇'s, and from the definitions of the $\bar{\nabla}$ and ∇ we complete the proof.

3.3. In order to generalize 3.2, we consider the following special decomposition. Suppose $M \cong P_1 \natural \cdots \natural P_u$, where $P_1, \cdots, P_u$ are ∂-prime 3-manifolds with connected boundary. Let D_0^3 be a 3-cell, and let $D_1 \cup \cdots \cup D_u$ be mutually disjoint 2-cells on ∂D_0^3. The 3-manifold M^* is obtained by pasting a 2-cell on ∂P_i to D_i, for $i = 1, \cdots, u$. Since $M^* \cong P_1 \natural \cdots \natural P_u \cong M$, there is a system of mutually disjoint proper 2-cells, say $D_1 \cup \cdots \cup D_u$, in M so that

(*) Each D_i divides M into two 3-manifolds $M_{i1} \cong P_i$ and $M_{i2} \cong P_1 \natural \cdots \natural P_{i-1} \natural P_{i+1} \natural \cdots \natural P_u$.

3.4. Theorem. *Let* M *be a 3-manifold in the class* SPC *having a* ∂*-prime decomposition*

$$M \cong P_1 \natural \cdots \natural P_r \natural P_{r+1} \natural \cdots \natural P_u$$

with $P_i \not\cong D^2 \times S^1$ *for* $i=1, \cdots, r$, *and* $P_j \cong D^2 \times S^1$ *for* $j=r+1, \cdots, u$. *Let* $D_1 \cup \cdots \cup D_u$ *be a system of mutually disjoint proper 2-cells in M satisfying* (*) *in* 3.3, *and let* $A_{r+1}, \cdots, A_u$ *be meridian-disks of* $M_{r+1,1} \cong P_{r+1}, \cdots, M_{u1} \cong P_u$, *respectively, such that* $A_j \cap D_j = \emptyset$ *for* $j=r+1, \cdots, u$. *Let* C^2 *be a proper 2-cell in M. Then,* C^2 *can be obtained (up to isotopy) by a finite sequence of modifications* $\bar{V}$'s *from mutually disjoint proper 2-cells* $E_1 \cup \cdots \cup E_\nu$, $(1 \leqq \nu < \infty)$, *where each* E_i *is isotopic to one of* $D_1, \cdots, D_r$, $A_{r+1}, \cdots, A_u$ *in M.*

The proof of Theorem 3.4, which is omitted here, is the same as that of Lemma 3.2 except for obvious modifications. We remark that $cl(M - N(D_1 \cup \cdots \cup D_r \cup A_{r+1} \cup \cdots \cup A_u\,;\, M))$ consists of $r+1$ 3-manifolds homeomorphic to $D^3, P_1, \cdots, P_r$, and that since $P_1, \cdots, P_r$ are ∂-irreducible by 2.15, every proper 2-cell C^2 in $M_{i1} \cong P_i$ is isotopic to D_i, $i=1, \cdots, r$, provided that $\partial C^2 \not\simeq 1$ on ∂M. In Theorem 3.4, the condition (*) is not always essential, and one of $D_1, \cdots, D_r$ can be omitted.

Remark. It is interesting to remark that the uniqueness of the ∂-prime decomposition for a 3-manifold M in the class SPC is easily proved by 3.4. Note that in 3.3 and 3.4 we did not use the uniqueness.

3.5. Corollary to 3.4. *Let M be a 3-manifold in the class* SPC, *and suppose that* $\pi_1(M) \cong G_1 * G_2$ *and both* G_1 *and* G_2 *are indecomposable with respect to free products and not free. Let* C_1^2 *and* C_2^2 *be proper 2-cells in M with* $\partial C_i^2 \not\simeq 1$ *on* ∂M *for* $i=1, 2$. *Then,* $C_1^2 \approx C_2^2$ *in M.*

Proof. It may be remarked that the existence of such the C_i^2 follows from 2.16, and that $\partial C_i^2 \sim 0$ on ∂M. Thus, C_1^2 divides M into two ∂-prime, ∂-irreducible 3-manifolds P_1 and P_2 with $\pi_1(P_1) \cong G_1$ and $\pi_1(P_2) \cong G_2$. By Theorem 3.4 and the note following 3.4, C_2^2 can be obtained by a finite sequence of modifications $\bar{V}$'s from proper 2-cells $E_1 \cup \cdots \cup E_\nu$ in M with $E_i \approx C_1^2$ in M for $i=1, \cdots, \nu$.

If $\nu=1$, then $C_1^2 \approx E_1 \approx C_2^2$ in M, and we are finished.

On the other hand, when we performed a modification $\bar{V}$ for two isotopic 2-cells E_1 and E_2, for the result C^2 it is easily checked that $\partial C^2 \simeq 1$ on ∂M. So, we can omit these E_1 and E_2, and so on. Thus, we can conclude that $\nu=1$.

3.6. Corollary to 3.4. *Let M be a 3-manifold in the class* SPC, *and suppose that* $\pi_1(M) \cong \boldsymbol{Z} * G$ *and G is indecomposable with respect to free products and not free. Let* C_1^2 *and* C_2^2 *be proper 2-cells in M with* $\partial C_i^2 \not\sim 0$ *on* ∂M *for* $i=1, 2$. *Then,* $C_1^2 \approx C_2^2$ *in M.*

Proof. Note that the existence of C_i^2 follows from 2.16, and that C_i^2 is a meridian-disk of M. Using the C_1^2 we can choose a proper 2-cell D_1 in M so that D_1 divides M into $P_1 \cong D^2 \times S^1$ and a ∂-prime, ∂-irreducible 3-manifold P_2 with $\pi_1(P_2) \cong G$, and C_1^2 is a meridian-disk of P_1. By 3.4, C_2^2 is obtained by a finite sequence of modifications $\bar{\nabla}$'s from proper 2-cells $E_1 \cup \cdots \cup E_\mu \cup E_1' \cup \cdots \cup E_\nu'$ with $E_i \approx C_1^2$ in M for $i=1, \cdots, \mu$, and $E_j' \approx D_1$ in M for $j=1, \cdots, \nu$.

When we performed a modification $\bar{\nabla}$ for E_1' and E_2', for the result E, $\partial E \simeq 1$ on ∂M. When we performed a modification $\bar{\nabla}$ for E_1 and E_1', for the result E it is easily checked that $E \approx E_1 \approx C_1^2$ in M. We therefore assume that $\nu = 0$.

On the other hand, let E be a proper 2-cell obtained from E_1 and E_2 by a modification $\bar{\nabla}$. Then, it is easy to see that either $\partial E \simeq 1$ on ∂M or E divides M into $P_1' \cong D^2 \times S^1$ and $P_2' \cong P_2$. In the first case we can omit these E_1 and E_2, and in the second case we can again omit E_1 and E_2 by replacing D_1 by E because we can assume that C_1^2 is a meridian-disk of P_1' and P_1' contains the rest $E_3 \cup \cdots \cup E_\mu$ as proper 2-cells.

Since $\partial C_?^2 \not\simeq 0$ on ∂M, we conclude that $\mu = 1$, and completing the proof.

4. Pairs $(F \subset S^3)$ of a Special Kind

In this section, using Theorem 3.4 we shall give an affirmative answer to Question 1.7 for a special kind of pairs.

4.1. Theorem. *Let $(F \subset S^3)$ be a non-trivial pair having a prime decomposition $(F \subset S^3) \cong (F_1 \subset S^3) \# \cdots \# (F_u \subset S^3)$ such that*

(**) *V_{F_i} (or W_{F_i}) is ∂-irreducible for all $i=1, \cdots, u$.*

Then, the prime decomposition for $(F \subset S^3)$ is unique.

By virtue of Proposition 2.15, the condition (**) may be equivalent to (**)' $\pi_1(V_{F_i}) \not\cong \boldsymbol{Z}$ is indecomposable with respect to free products. About the condition (**) we refer the reader to §5 below.

We begin with a useful lemma which follows from the definition of the modification $\bar{\nabla}$, and the proof is omitted.

4.2. Lemma. *Let $(F \subset S^3)$ be a pair, and let Σ_1 and Σ_2 be disjoint 2-spheres in S^3 such that $\Sigma_1 \cup \Sigma_2$ gives a decomposition*

$$(F \subset S^3) \cong (F_1 \subset S^3) \# (F_2 \subset S^3) \# (F_3 \subset S^3).$$

We suppose that Σ_1 resp. Σ_2 gives a decomposition

$$(F \subset S^3) \cong (F_1 \subset S^3) \# \big((F_2 \subset S^3) \# (F_3 \subset S^3)\big)$$

resp. $$(F\subset S^3)\cong(F_2\subset S^3)\#\big((F_1\subset S^3)\#(F_3\subset S^3)\big).$$

Let α be a simple arc on F with

$$\alpha\cap\Sigma_i=\partial\alpha\cap\Sigma_i\neq\emptyset \quad for \quad i=1,2,$$

and let Σ be a 2-sphere in S^3 such that $\Sigma\cap V_F$ and $\Sigma\cap W_F$ are obtained by modifications $\bar{V}$'s along α from $(\Sigma_1\cup\Sigma_2)\cap V_F$ and $(\Sigma_1\cup\Sigma_2)\cap W_F$, respectively.

Then, Σ gives the decomposition

$$(F\subset S^3)\cong\big((F_1\subset S^3)\#(F_2\subset S^3)\big)\#(F_3\subset S^3).$$

As in [20], [26], etc., Theorem 4.1 will clearly follow from the following lemma.

4.3. Lemma. *With $(F\subset S^3)$ as in* Theorem 4.1, *suppose $(F\subset S^3)$ has a decomposition $(G_1\subset S^3)\#(G_2\subset S^3)$. Then, we can rearrange the $(F_i\subset S^3)$ so that*

$$(G_1\subset S^3)\cong(F_1\subset S^3)\#\cdots\#(F_t\subset S^3)$$

and $$(G_2\subset S^3)\cong(F_{t+1}\subset S^3)\#\cdots\#(F_u\subset S^3)$$

for some t with $0\leqq t\leqq u$.

Proof. From the hypothesis, we can choose mutually disjoint 3-cells $D_1^3\cup\cdots\cup D_u^3$ in S^3 so that $(F\cap D_i^3\subset D_i^3)$ is equivalent to $(F_i\subset S^3)$ for $i=1,\cdots,u$. Let D_i be the 2-cell $\partial D_i^3\cap V_F$ for $i=1,\cdots,u$. Then, from the hypothesis, the system of mutually disjoint proper 2-cells $D_1\cup\cdots\cup D_u$ in V_F satisfies the condition (*) in 3.3 for a ∂-prime decomposition

$$V_F\cong V_{F_1}\natural\cdots\natural V_{F_u}.$$

On the other hand, from the hypothesis, there exists a 2-sphere Σ in S^3 which gives the decomposition $(G_1\subset S^3)\#(G_2\subset S^3)$. By Theorem 3.4, the proper 2-cell $\sigma=\Sigma\cap V_F$ in V_F can be obtained (up to isotopy) by a finite sequence of modifications $\bar{V}$'s from mutually disjoint proper 2-cells $\sigma_1\cup\cdots\cup\sigma_\nu$ in V_F, where each σ_j is isotopic to one of $D_1,\cdots,D_u$ in V_F. Since each ∂D_i bounds the 2-cell $\partial D_i^3\cap W_F$ in W_F, there is a system of mutually disjoint proper 2-cells $\tau_1\cup\cdots\cup\tau_\nu$ in W_F with $\partial\tau_j=\partial\sigma_j$ for $j=1,\cdots,\nu$. Thus, we have a system of mutually disjoint 2-spheres $\mathscr{S}=\{\sigma_1\cup\tau_1,\cdots,\sigma_\nu\cup\tau_\nu\}$ in S^3, and we may assume that $\mathscr{S}\cap(\partial D_1^3\cup\cdots\cup\partial D_u^3)=\emptyset$. Now, it is easy to see that $\mathscr{S}$ gives a decomposition

$$(F\subset S^3)\cong(F_1'\subset S^3)\#\cdots\#(F_{\nu+1}'\subset S^3)$$

such that each $(F_k'\subset S^3)$ belongs to, up to congruence, the following set

$$\mathscr{T}=\left\{\text{compositions of some of } (S^2\subset S^3),\ (F_1\subset S^3), \cdots, (F_u\subset S^3)\right\}$$

and each $(F_i\subset S^3)$ is contained in exactly one of $(F_1'\subset S^3), \cdots, (F_{\nu+1}'\subset S^3)$ as a prime component, for $k=1, \cdots, \nu+1$ and $i=1, \cdots, u$.

Now we perform the first modification $\bar{\Gamma}$ for two of $\sigma_1\cup\cdots\cup\sigma_\nu$, say $\sigma_{\nu-1}$ and σ_ν, along a simple arc α_1 on $F=\partial V_F$, and let us denote the result by $\sigma_{\nu-1}'$. Moreover, we perform a modification $\bar{\nabla}$ at once for the corresponding 2-cells $\tau_{\nu-1}$ and τ_ν along the same arc α_1, and let us denote the result by $\tau_{\nu-1}'$. We have now a new system of mutually disjoint 2-spheres $\mathscr{S}'=\{\sigma_1\cup\tau_1, \cdots, \sigma_{\nu-2}\cup\tau_{\nu-2}, \sigma_{\nu-1}'\cup\tau_{\nu-1}'\}$ in S^3, which gives a decomposition

$$(F\subset S^3)\cong(F_1''\subset S^3)\#\cdots\#(F_\nu''\subset S^3).$$

By Lemma 4.2, we know that exactly one of $(F_1''\subset S^3), \cdots, (F_\nu''\subset S^3)$, say $(F_\nu''\subset S^3)$, is a composition of two of $(F_1'\subset S^3), \cdots, (F_{\nu+1}'\subset S^3)$, say $(F_\nu'\subset S^3)$ and $(F_{\nu+1}'\subset S^3)$, and for every other $k=1, \cdots, \nu-1$, $(F_k''\subset S^3)\cong(F_k'\subset S^3)$. That is, for $k=1, \cdots, \nu$, each $(F_k''\subset S^3)$ belongs to $\mathscr{T}$ up to congruence, and for $i=1, \cdots, u$, each $(F_i\subset S^3)$ is contained in exactly one of $(F_1''\subset S^3), \cdots, (F_\nu''\subset S^3)$ as a prime component.

Repeating of the same procedure as above, we have a 2-sphere $\mathscr{S}^{(\nu-1)}=\sigma_1^{(\nu-1)}\cup\tau_1^{(\nu-1)}$ in S^3, and a decomposition

$$(F\subset S^3)\cong(F_1^{(\nu)}\subset S^3)\#(F_2^{(\nu)}\subset S^3)$$

giving by $\mathscr{S}^{(\nu-1)}$ such that both $(F_1^{(\nu)}\subset S^3)$ and $(F_2^{(\nu)}\subset S^3)$ belong to $\mathscr{T}$ up to congruence, and each $(F_i\subset S^3)$ is contained in exactly one of $(F_1^{(\nu)}\subset S^3)$ and $(F_2^{(\nu)}\subset S^3)$ as a prime component. Since $\sigma_1^{(\nu-1)}\approx\sigma$ in V_F and $\tau_1^{(\nu-1)}\approx\Sigma\cap W_F$ in W_F by 2.6, we conclude that $(F_1^{(\nu)}\subset S^3)\cong(G_1\subset S^3)$ and $(F_2^{(\nu)}\subset S^3)\cong(G_2\subset S^3)$, and completing the proof.

5. Existence of Prime Pairs $(F\subset S^3)$

By 1.2 and 2.10, we have the following.

5.1. Proposition. *For a pair $(F\subset S^3)$, if one of V_F and W_F is ∂-prime, then $(F\subset S^3)$ is prime.*

Using this 5.1, we will prove the following.

5.2. Theorem. *For any positive integer p, there exists a prime pair $(F\subset S^3)$ with $g(F)=p$.*

Proof. The case $p=1$ is Proposition 1.5, and the case $p=2$ has been given in Suzuki [25, §5] and Tsukui [27, §7], and see Jaco [16], etc.. Now, we note again Example 5.6 of [25] below, which will be used to construct another examples. It should be noted that 5.6 [25] and 4.11 [25] implies

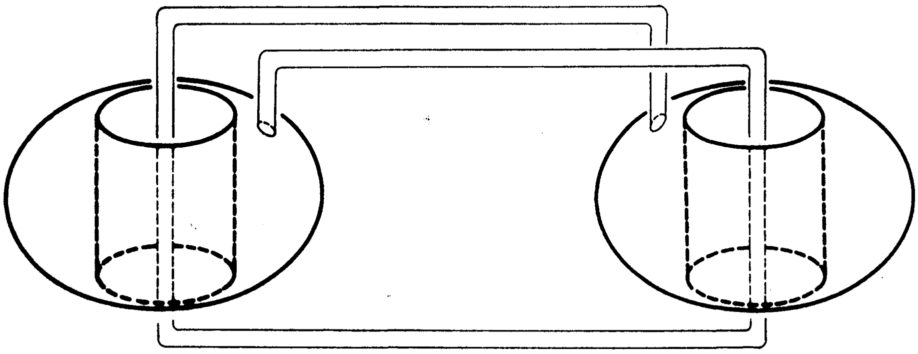


Fig. 2. $(F^* \subset S^3)$

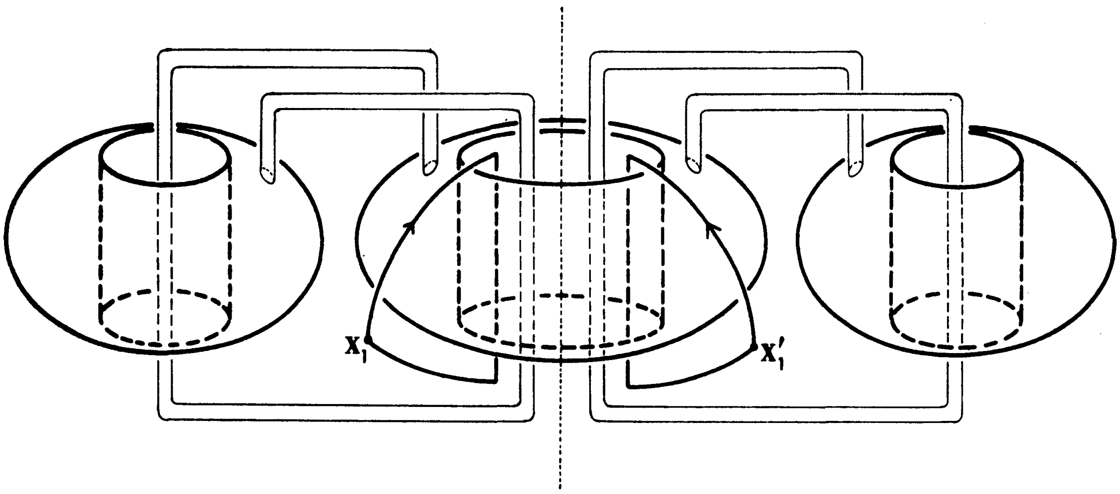


Fig. 3. $(F_1 \subset S^3)$

that $\pi_1(V_{F^*})$ is indecomposable; that is, V_{F^*} is ∂-prime by 2.15. (We refer to Kinoshita [17] for the Alexander polynomial of graphs.)

Now, we give the following pair $(F_1 \subset S^3)$ in Fig. 3. From the construction, we can easily check that $\pi_1(V_{F_1})$ is the free product of two copies of $\pi_1(V_{F^*})$ with the subgroup $\boldsymbol{Z}(x_1)$ and $\boldsymbol{Z}(x_1')$ amalgamated under the map $x_1 \to x_1'$, where $\boldsymbol{Z}(x_1)$ and $\boldsymbol{Z}(x_1')$ are infinite cyclic groups generated by x_1 and x_1', respectively. By a corollary of the Kurosh Subgroup Theorem, we conclude that $\pi_1(V_{F_1})$ is indecomposable with respect to free products; see Magnus et al [19, p. 243 and p. 246]. That is, $(F_1 \subset S^3)$ is prime with $g(F_1)=3$ by 2.15 and 5.1.

By the same way as above, from prime pairs $(F^* \subset S^3)$ and $(F_1 \subset S^3)$ with $g(F^*)=2$ and $g(F_1)=3$, we can construct a prime pair $(F_2 \subset S^3)$ with $g(F_2)=4$ such that V_{F_2} is ∂-prime. In general, we can construct inductively a prime pair $(F_i \subset S^3)$ with $g(F_i)=i+2$ such that V_{F_i} is ∂-prime, from prime pairs $(F^* \subset S^3)$ and $(F_{i-1} \subset S^3)$ with $g(F_{i-1})=i+1$, and completing the proof.

On the other hand, for pairs $(F \subset S^3)$ with $g(F)=2$, we have the following:

5. 3. Proposition. (Tsukui [28]) *A pair* $(F\subset S^3)$ *with* $g(F)=2$ *is prime if and only if either* V_F *or* W_F *is* ∂*-prime.*

Contrary to 5.3, we record the following, which follows from Examples 5.5 and 5.6 below.

5. 4. Theorem. *For every integer* p *with* $p\geqq 3$, *there is a prime pair* $(F\subset S^3)$ *such that both* V_F *and* W_F *are not* ∂*-prime.*

5. 5. Example. (Fig. 4) *For every integer* p *with* $p\geqq 3$, *there is a prime pair* $(F\subset S^3)$ *such that* $V_F\cong P\natural(D^2\times S^1)$, $W_F\cong p(D^2\times S^1)$ *and* P *is* ∂*-irreducible.*

Proof. $(F\subset S^3)$ in Fig. 4 shows the case $p=3$, and in the other cases constructions are analogously by using the pairs $(F_i\subset S^3)$ given in the proof of 5.2. We only remark that for the pair $(F_i\subset S^3)$ in 5.2, W_{F_i} is a solid-torus.

To show that the pair $(F\subset S^3)$ has required properties, we use the following pair $(G\subset S^3)$ in Fig. 5. From the construction, we have $V_F\cong V_G\cong V_{F^*}\natural(D^2\times S^1)$, here V_{F^*} is in Fig. 2 and ∂-irreducible by 2.15. Using 2.17, we can easily see that $W_F\cong 3(D^2\times S^1)$. Now we consider the simple loop α on F given in Fig. 4. It will be noticed that α is meridian of $V_F\cong V_{F^*}\natural(D^2\times S^1)$ which is unique up to isotopy by 3.6. Let A be a meridian-disk with $\partial A=\alpha$. It is clear that $W_F\cup N(A;V_F)$ is a disk-sum

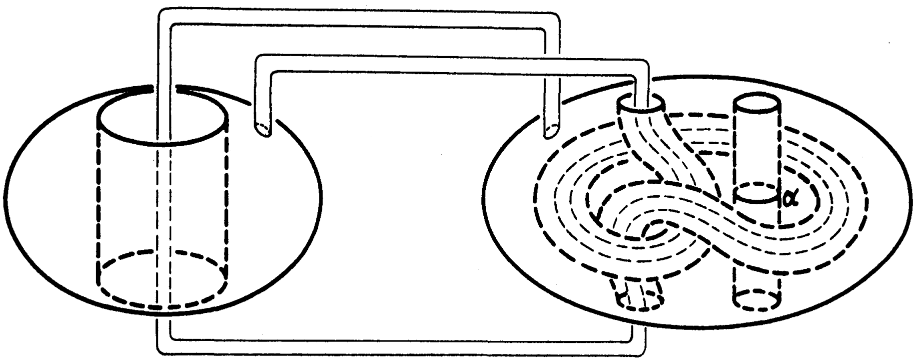


Fig. 4. $(F\subset S^3)$

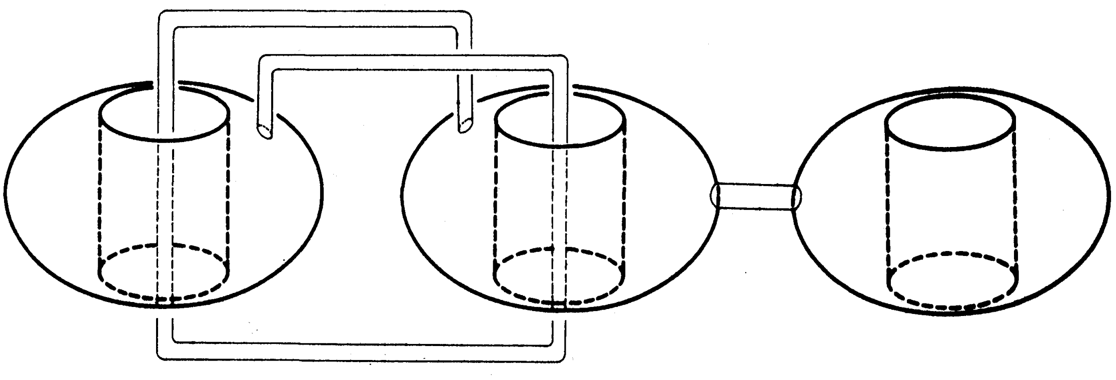


Fig. 5. $(G\subset S^3)$

of $D^2\times S^1$ and the closed complement V_K of the clover-leaf knot, that is, $W_F\cup N(A; V_F)\not\cong 2(D^2\times S^1)$. From 2.18, we can conclude that $(F\subset S^3)$ is prime.

5.6. Example. (Fig. 6) *For every integer p with $p\geqq 3$, there is a prime pair $(F\subset S^3)$ such that $V_F\cong P_1\natural P_2$, $W_F\cong p(D^2\times S^1)$, and both P_1 and P_2 are ∂-irreducible.*

Proof. $(F\subset S^3)$ in Fig. 6 shows the case $p=3$, and in the other cases we can construct required pairs analogously.

To show that the $(F\subset S^3)$ has required properties, we refer to the pair $(H\subset S^3)$ in Fig. 7. From the construction, we have $V_F\cong V_H\cong V_{F'}\natural V_K$, here $V_{F'}$ is in Fig. 2 and V_K is the closed complement of the clover-leaf knot. Note that both V_{F*} and V_K are ∂-irreducible. By virtue of 2.17, we can check that $W_F\cong 3(D^2\times S^1)$.

We consider the simple loop β on F given in Fig. 6. It is easy to see that $\beta\simeq 1$ in V_F and $\beta\sim 0$ on F. By 3.5, such the loop is unique up to isotopy. To show that $(F\subset S^3)$ is prime, it is enough to show that $\beta\not\simeq 1$ in W_F. Assume the contrary, then β bounds a proper 2-cell in W_F which must divide $W_F\cong 3(D^2\times S^1)$ into $Q_1\cong D^2\times S^1$ and $Q_2\cong 2(D^2\times S^1)$. So, we

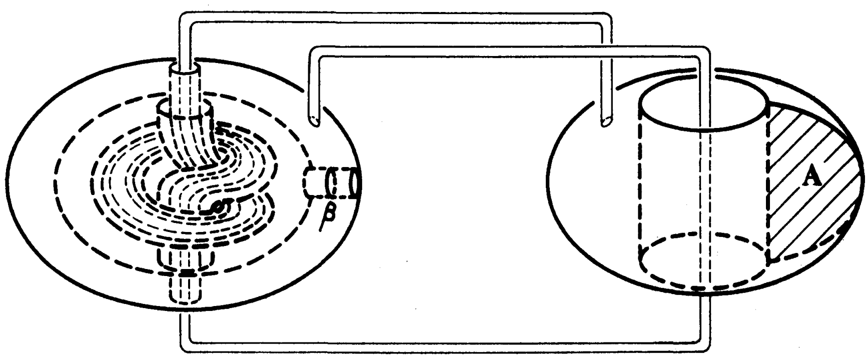


Fig. 6. $(F\subset S^3)$

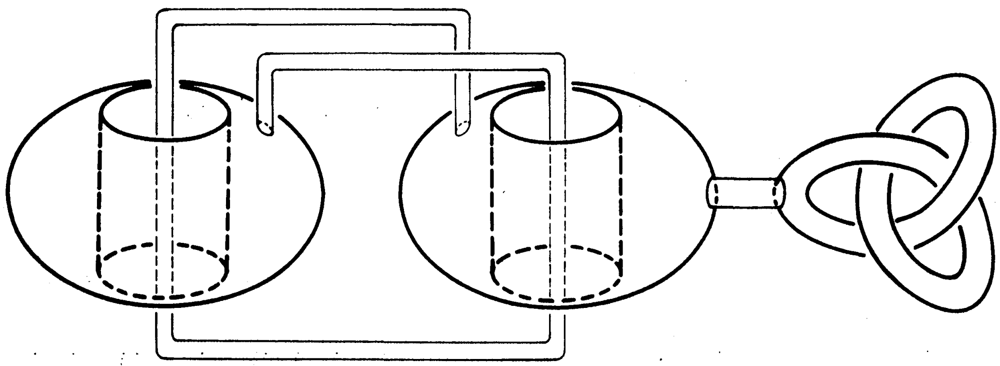


Fig. 7. $(H\subset S^3)$

may choose a system of meridians $J_1 \cup J_2 \cup J_3$ of W_F with $\beta \cap (J_1 \cup J_2 \cup J_3) = \emptyset$. Of course, A in Fig. 6 is a meridian-disk of W_F with $\beta \cap A = \emptyset$. From simple observations of the surface $(\partial(cl(W_F - N(A; W_F)) \subset S^3)$, it is easy to see that there does not exist such the system of meridians.

From the pairs $(F \subset S^3)$ in Fig. 4 and $(G \subset S^3)$ in Fig. 5, and the pairs $(F \subset S^3)$ in Fig. 6 and $(H \subset S^3)$ in Fig. 7, we have:

5.7. Proposition. *The knotting problem of a closed oriented surface in S^3 is not reducible. That is, even if $V_F \cong V_{F'}$ and $W_F \cong W_{F'}$* (i.e. $S^3 - F \cong S^3 - F'$), *$(F \subset S^3)$ and $(F' \subset S^3)$ are not always congruent.* (Refer to Fox [8, Prob. 7].)

Department of Mathematics
Kobe University

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