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Foliations transverse to the turbulized foliations of punctured torus bundles over a circle

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§ 1. Introduction

Tamura and Sato [7] began the study of codimension-one foliations transverse to given codimension-one foliations. They regarded such foliations as structures of given foliated manifolds. At this viewpoint, the fundamental probelms are to determine whether such foliations exist or not and to classify them when they exist.

CONVENTION. In this paper, foliations are always transversely orientable, of codimension one and of class C^{∞}, unless stated otherwise.

In order to state the known results and ours, we introduce some notations. Let $\Sigma_{g}(h)$ be a compact manifold obtained from the closed surface Σ_{g} of genus g by deleting h small disjoint open 2-disks, where h is a positive integer. Take an orientation preserving C^{∞} diffeomorphism $\phi:\Sigma_{g}(h)\rightarrow\Sigma_{g}(h)$ and consider an equivalence relation $\sim$ on $\boldsymbol{R}\times\Sigma_{g}(h)$ determined by

$$(t, x)\sim(t', x') \quad \text{if} \quad t'=t+1 \text{ and } x'=\phi(x),$$

where $t, t'\in\boldsymbol{R}$ and $x, x'\in\Sigma_{g}(h)$. Then the quotient space $E(\Sigma_{g}(h);\phi)=\boldsymbol{R}\times\Sigma_{g}(h)/\sim$ is a $\Sigma_{g}(h)$ bundle over $S^{1}=\boldsymbol{R}/\boldsymbol{Z}$ with the projection $\pi: E(\Sigma_{g}(h);\phi)\rightarrow S^{1}$ defined by $\pi([t, x])=[t]$ for $(t, x)\in\boldsymbol{R}\times\Sigma_{g}(h)$. We treat $\boldsymbol{R}$ and Σ_{g} as oriented manifolds. Hence $\Sigma_{g}(h)$ and $E(\Sigma_{g}(h);\phi)$ are consequently oriented. Take a continuous map $\sigma:\partial E(\Sigma_{g}(h);\phi)\rightarrow\{1, -1\}$. We have a foliation $\mathscr{F}(\Sigma_{g}(h);\phi)^{\sigma}$ of $E(\Sigma_{g}(h);\phi)$ by turbulizing the bundle foliation $\{\pi^{-1}(x)\}_{x\in S^{1}}$ as in Figure 1.1 (see Nishimori [4] for the precise definition).

We have $\Sigma_{0}(1)=S^{2}(1)=D^{2}$ and $E(D^{2};\mathrm{id})=S^{1}\times D^{2}$. Note that $\mathscr{F}(D^{2};\mathrm{id})^{1}$ (or $\mathscr{F}(D^{2};\mathrm{id})^{-1}$) is a plus (or minus) Reeb component $\mathscr{F}_{R}^{+}$ (or $\mathscr{F}_{R}^{-}$) in Tamura-Sato [7]. For mainfold E with a diffeomorphism $f: E\rightarrow E(\Sigma_{g}(h);\phi)$, we denote by $\mathscr{F}(E)^{\sigma\circ f}$ the induced foliation $f^{*}\mathscr{F}(\Sigma_{g}(h);\phi)^{\sigma}$. Note that $\mathscr{F}(E)^{\sigma\circ f}$ is unique up to C^{0} isomorphism.

The known results are as follows. Tamura and Sato [7] classify the foliations of $S^{1}\times D^{2}$ transverse to the Reeb component $\mathscr{F}_{R}^{+}$ $(=\mathscr{F}(S^{2}(1);\mathrm{id})^{1})$ by introducing the notion of TS diagrams. Furthermore they proved that,

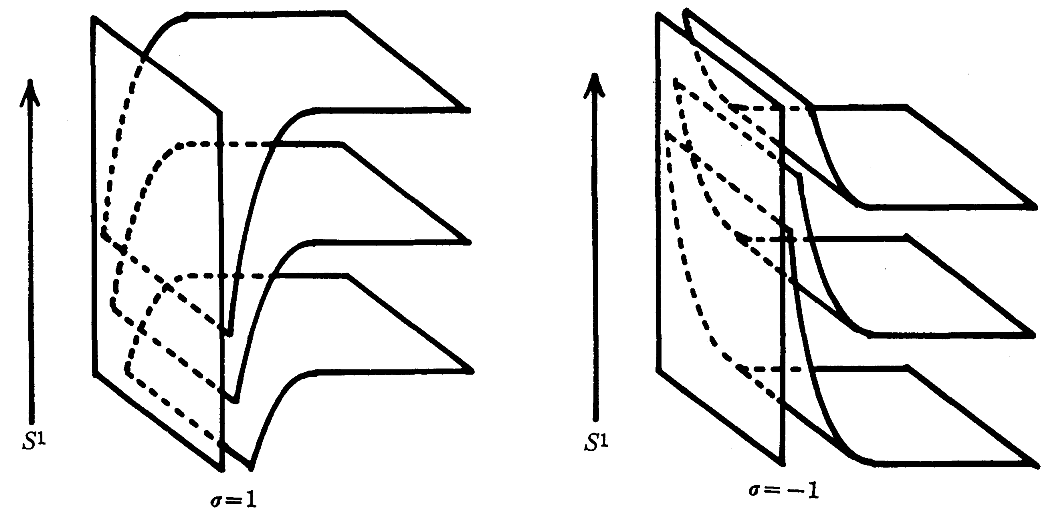

Figure 1. 1.

for any non-trivial fibered knot k of S^3, the patched foliation $\mathscr{F}_R^+ \cup \mathscr{F}_k$ of S^3 has no transverse foliations, where $\mathscr{F}_R^+$ is a Reeb component of a tubular neighborhood $N(k)$ of k, and $\mathscr{F}_k = \mathscr{F}(S^3 - \text{int}\, N(k))^{\pm 1}$, where $S^3 - \text{Int}\, N(k)$ is regarded as the total space of a surface bundle over a circle. Note that all the foliations of S^3 have transverse 2-plane fields (see [7]). In Nishimori [4], the author generalized the classification of transverse foliations to $\mathscr{F}(S^2(h)\,;\, \text{id})^\sigma$ for all h and σ. Furthermore he considered the existence problem of transverse foliations for closed foliated 3-manifolds obtained by patching a finite number of foliated manifolds of the form $(E(S^2(h)\,;\,\text{id}), \mathscr{F}(S^2(h)\,;\,\text{id})^\sigma)$, and gave two criterions.

The starting point of this paper is given as follows. Let k be a fibered knot in S^3 with fiber of genus $g>0$ and $N(k)$ a tubular neighborhood of k. Note that $S^3 - \text{Int}\, N(k)$ is diffeomorphic to $E(\Sigma_g(1)\,;\,\phi)$ for some $\phi \in \text{Diff}_+(\Sigma_g(1))$. The result of Tamura-Sato [7] suggests the following.

Problem A. Does there exist a foliation transverse to the turbulized foliation $\mathscr{F}_k$ of $S^3 - \text{Int}\, N(k)$?

We have the following contrasting results as to Problem A.

Theorem 1. *If k is a trefoil knot, then $\mathscr{F}_k$ has no transverse foliation.*

Theorem 2. *If k is a figure eight knot, then $\mathscr{F}_k$ has a transverse foliation.*

It is natural to generalize Problem A as follows.

Problem B. Does there exist a foliation of $E(\Sigma_g(h)\,;\,\phi)$ transverse to $\mathscr{F}(\Sigma_g(h)\,;\,\phi)^\sigma$?

As to Problem B, we first obtain the following results.

THEOREM 3. *If ϕ: $\Sigma_g(h)\to\Sigma_g(h)$ is isotopic to the identity (that is, $E(\Sigma_g(h)\,;\,\phi)$ is a trivial bundle), then $\mathscr{F}(\Sigma_g(h)\,;\,\phi)^\sigma$ has transverse foliations for all g, h and σ.*

THEOREM 4. *If $\partial E(\Sigma_g(h)\,;\,\phi)$ is connected and $2g-2$ is not divisible by h, then $\mathscr{F}(\Sigma_g(h)\,;\,\phi)^\sigma$ has no transverse foliation.*

When $g=0$ and $h=1$, the fiber $\Sigma_g(h)$ is diffeomorphic to D^2, and hence ϕ is isotopic to the identity. Therefore we can apply Theorem 3 to this case. There are two ways to make progress. The first one is to treat $\mathscr{F}(S^2(h)\,;\,\phi)^\sigma$, and the second one to treat $\mathscr{F}(T^2(1)\,;\,\phi)^\sigma$. At present it is difficult to treat $\mathscr{F}(\Sigma_g(h)\,;\,\phi)^\sigma$ in the most general setting. For the first case, we have the following (see § 2 for the definition of TS diagrams of $(\Sigma_g(h)\,;\,\phi)^\sigma$).

THEOREM 5. *$\mathscr{F}(S^2(h)\,;\,\phi)^\sigma$ has a transverse foliation if and only if there exists a TS diagram of $(S^2(h)\,;\,\phi)^\sigma$.*

Now consider $\mathscr{F}(T^2(1)\,;\,\phi)^\sigma$, whose underlying manifold $E(T^2(1)\,;\,\phi)$ is a fiber bundle over a circle with fiber $T^2(1)=T^2-\mathrm{Int}\,D^2$ (that is, a punctured torus). For a diffeomorphism ϕ: $T^2(1)\to T^2(1)$, we denote by $H_1(\phi)$ the induced isomorphism ϕ^*: $H_1(T^2(1)\,;\,\boldsymbol{Z})\to H_1(T^2(1)\,;\,\boldsymbol{Z})$. It is well known that if $H_1(\phi)=H_1(\phi')$ for diffeomorphisms ϕ, ϕ' : $T^2(1)\to T^2(1)$, then ϕ and ϕ' are isotopic. The following theorem is our main result.

THEOREM 6. *Let ϕ : $T^2(1)\to T^2(1)$ be a diffeomorphism and σ : $\partial E(T^2(1)\,;\,\phi)\to\{1,\,-1\}$ a continuous map (hence σ is constant). Then the turbulized foliation $\mathscr{F}(T^2(1)\,;\,\phi)^\sigma$ admits a transverse foliation if and only if Trace $H_1(\phi)\geqq 2$.*

Now Theorem 1 and Theorem 2 follow from Theorem 6, as follows. Let $k\subset S^3$ be a trefoil knot or a figure eight knot and $N(k)$ a tubular neighborhood of k. Then the fiber of the associated bundle π_k : $S^3-\mathrm{Int}\,N(k)\to S^1$ is diffeomorphic to $T^2(1)$. Let ϕ_k: $T^2(1)\to T^2(1)$ be the twisting map of the bundle π_k : $S^3-\mathrm{Int}\,N(k)\to S^1$. By taking the appropriate basis of $H_1(T^2(1)\,;\,\boldsymbol{Z})$ the isomorphism $H_1(\phi_k)$ corresponds to the matrix $\begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix}$ if k is a trefoil knot and to $\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$ if k is a figure eight knot. Therefore Theorem 6 implies Theorem 1 and Theorem 2.

The proof of the "only if" part of Theorem 6 is divided into two cases, that is,

(a) all the leaves of $\mathscr{G}|F$ are proper,

(b) $\mathscr{G}|F$ has a non-porper leaf,

where $\mathscr{G}$ is a foliation transverse to $\mathscr{F}(T^2(1)\,;\,\phi)^\sigma$ and F is a non-compact leaf of $\mathscr{F}(T^2(1);\phi)^\sigma$. For the case (a), we use the following.

THEOREM 7. *Suppose that* $\mathscr{F}(\Sigma_g(h);\phi)^\sigma$ *has a transverse foliation* $\mathscr{G}$. *If all the leaves of* $\mathscr{G}|F$ *are proper for some non-compact leaf* F *of* $\mathscr{F}(\Sigma_g(h);\phi)^\sigma$, *then a TS diagram of* $(\Sigma_g(h);\phi)^\sigma$ *can be attached to* $\mathscr{G}$.

Applying Theorem 7 to $\mathscr{F}(T^2(1);\phi)^\sigma$ and analyzing the corresponding *TS* diagram, we obtain the following.

THEOREM 8. *Suppose that* $\mathscr{F}(T^2(1);\phi)^\sigma$ *has a transverse foliation* $\mathscr{G}$ *such that all the leaves of* $\mathscr{G}|F$ *are proper for some non-compact leaf* F *of* $\mathscr{F}(T^2(1);\phi)^\sigma$. *Then Trace* $H_1(\phi)=2$.

For the case (b), we have the following.

THEOREM 9. *Suppose that* $\mathscr{F}(T^2(h);\phi)^\sigma$ *has a transverse foliation* $\mathscr{G}$ *such that* $\mathscr{G}|F$ *has a non-proper leaf for some non-compact leaf* F *of* $\mathscr{F}(T^2(h);\phi)^\sigma$. *Then* $H_1(\bar{\phi})=id$ *or Trace* $H_1(\bar{\phi})>2$, *where* $\bar{\phi}: T^2\to T^2$ *is the extension of* ϕ *by the Alexander trick.*

Now the "only if" part of Theorem 6 follows from Theorems 8 and 9.

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§ 2. Preliminaries

In this paper the familiarity with Nishimori [4] is basically supposed. First we describe the turbulized foliation $\mathscr{F}(\Sigma_g(h);\phi)^\sigma$. For simplicity let $\Sigma=\Sigma_g(h)$ and $E=E(\Sigma_g(h);\phi)$. Let $\hat{k}: \partial\Sigma\times I\to\Sigma$ be a collar with $\hat{k}(y,0)=y$ for all $y\in\partial\Sigma$, where I is the closed interval $[0,1]$. Replacing ϕ by an isotopic diffeomorphism if necessary, we may suppose the following.

(1) If $y\in\partial\Sigma$ and $\phi^n(y)$ belong to the same connected component of $\partial\Sigma$ for some $n\in\boldsymbol{Z}$, then $\phi^n(y)=y$.

(2) $\phi(\hat{k}(y,t))=\hat{k}(\phi(y),t)$ for all $y\in\partial\Sigma$ and $t\in I$.

Hereafter we may consider only such ϕ. Define $k: \boldsymbol{R}\times\partial\Sigma\times I\to\boldsymbol{R}\times\Sigma$ by $k(x,y,t)=(x,\hat{k}(y,t))$ for all $x\in\boldsymbol{R}$, $y\in\boldsymbol{R}$, $y\in\partial\Sigma$ and $t\in I$. Now define a foliation $\tilde{\mathscr{F}}$ of $\boldsymbol{R}\times\Sigma$ in such a way that

(1) $\tilde{\mathscr{F}}|(\boldsymbol{R}\times\Sigma-\mathrm{Im}\,k)$ is the restriction of the trivial foliation $\{\{x\}\times\Sigma\}_{x\in\boldsymbol{R}}$, and

(2) the leaves of $\tilde{\mathscr{F}}|\mathrm{Im}\,k$ are the connected components of $R\times\partial\Sigma$ and the subsets

$$\left\{\left(\hat{\sigma}(y)f(t)+x,y,t\right)\middle|y\in\partial\Sigma,\ t\in[0,1]\right\}$$

for $x\in\boldsymbol{R}$, where $\hat{\sigma}$ is naturally defined by σ and f is the function in [4, § 2].

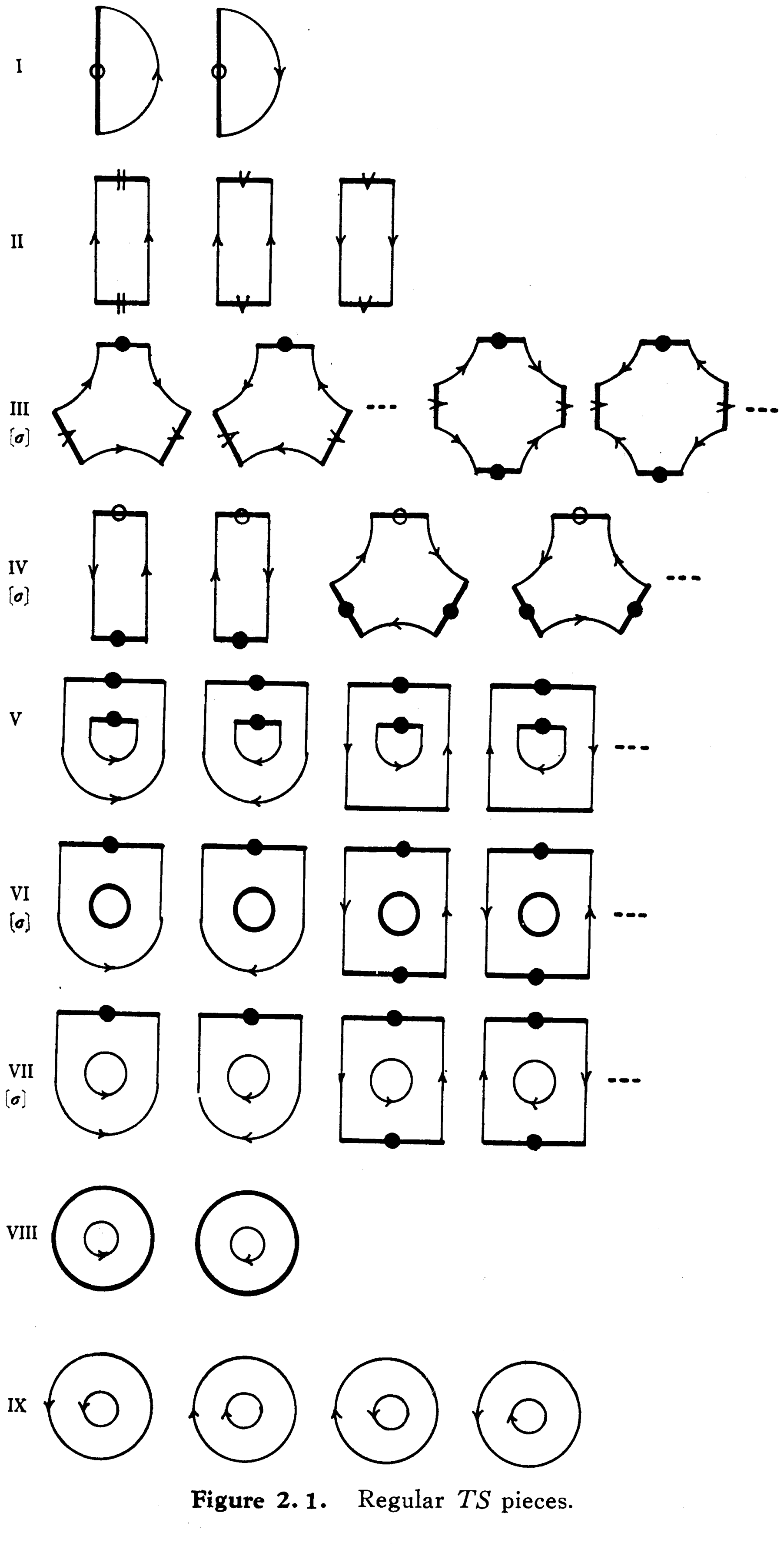

Figure 2. 1. Regular *TS* pieces.

Thus we have $\mathscr{F}(\Sigma_g(h)\,;\,\phi)^\sigma$ as the quotient foliation of $\tilde{\mathscr{F}}$ by the relation $\sim$ in § 1.

Here we give a list of figures which represent the regular TS pieces defined in [4], in Figure 2.1. We omit the explanation of them.

We generalize the notions of *TS* diagram by modifying Definition 13.3 in [4], as follows.

DEFINITION 2.1. A *TS diagram* of $(\Sigma_g(h)\,;\,\phi)^\sigma$ is a triad $\mathscr{T}=(\hat{\mathscr{T}}, \{\phi_t: \Sigma_g(h)\to\Sigma_g(h)\}_{t\in I}, (a, b\,;\,r): \Gamma_g(h)\to(\boldsymbol{N}\times\boldsymbol{Z})^*\times 2\boldsymbol{Z})$ (where $\Gamma_g(h)$ is the set of connected components of $\partial\Sigma_g(h)$ and $(\boldsymbol{N}\times\boldsymbol{Z})^*$ was defined in [4, § 1]) satisfying the following conditions.

$(R1)^*$ $\hat{\mathscr{T}}=(S, \{P_\lambda\}_{\lambda\in\Lambda}, \{\iota_\lambda\}_{\lambda\in\Lambda})$ is a pre *TS* diagram of $\Sigma_g(h)$, and $\{\phi_t\}_{t\in I}$ is a C^∞ isotopy of diffeomorphisms such that ϕ_0 is the identity and ϕ_1 is an isomorphism from $\hat{\mathscr{T}}$ to $\hat{\mathscr{T}}$. Furthermore ϕ is an isomorphism from $\hat{\mathscr{T}}$ to $\hat{\mathscr{T}}$ with $(a, b\,;\,r)\circ\phi=(a, b\,;\,r)$, where we regard ϕ as a map: $\Gamma_g(h)\to\Gamma_g(h)$ in a natural way.

$(R2)^*$ Let $P_\lambda=(\Delta_\nu, \nu_\lambda, s: \mathscr{J}_\lambda\to\mathscr{S}, \omega: K_\lambda\to\{1, -1\})$. Let $C\in\Gamma_g(h)$ and put $p(C)$ (or $q(C)$)$=\#\{J|J\in\mathscr{J}_\lambda$ for some $\lambda\in\Lambda$, $*J\subset C$, $s(J)=\circ$ (or $\bullet$)$\}$.

(i) $(a(C), b(C))=(0, 1)$ or (∞, ∞), then $r(C)=0$ and there are $\lambda\in\Lambda$ and $K\in\mathscr{K}(\Delta_\lambda)-\mathscr{K}_\lambda$ with $*K=C$.

(ii) If $(a(C), b(C))=(a, b)\in(\boldsymbol{N}\times\boldsymbol{Z})^{\text{coprime}}$, then $r(C)=(p(C)-q(C))/a$, there are $\lambda\in\Lambda$ and $J\in\mathscr{J}_\lambda$ with $*J\subset C$, the map $(\phi_1|C)^a$ is the identity, $(\phi\circ\phi_1|C)^a$ is the identity, $(\phi\circ\phi_1|C)^{a'}$ has no fixed point for $0<a'<a$, and the degree of $\eta: [0, a]/\{0, a\}\to C$ equals to b, where η is defined by

$$\eta\big([t]\big)=\phi^{-k}\circ\phi_{t'}\circ(\phi\circ\phi_1)^k(y_0)$$

for $t=k+t'$, $k\in\boldsymbol{Z}$, $0\leqq t'<1$ and a fixed point $y_0\in C$.

(s) Let S_i be a circle in S, and take $C, C'\in\Gamma_g(h)$ as in [4, Definition 13.1 (PR3)]. Then $(a(C'), b(C'))\neq(a(C), -b(C))$.

$(R3)$ (The condition on σ). In the below, J and J' are elements of $\mathscr{J}_\lambda$ with $*J$, $*J'\subset\partial\Sigma_g(h)$.

(iii) If $\nu_\lambda=\mathrm{III}$, then $\sigma(*y')=\sigma(*y)$ (or $-\sigma(*y)$) for $y\in J$ and $y'\in J'$ such that J and J' are contained in the same connected component of $\partial\Delta_\lambda-\cup\{J''|J''\in\mathscr{J}_\lambda, s(J'')=\vee$ or $\wedge\}$.

(iv) If $\nu_\lambda=\mathrm{IV}$, then $\sigma(*y')=-\sigma(*y)$ for $y\in J$ and $y'\in J'$ such that $s(J)=\circ$ and $s(J')=\bullet$.

(vi) If $\nu_\lambda=\mathrm{VI}$ and $(a(C), b(C))=(0, 1)$ for $C\supset *K$, $K\in\mathscr{K}(\Delta_\lambda)-\mathscr{K}_\lambda$, then $\sigma(*y')=-\sigma(*y)$ for $y\in K$ and $y'\in J$.

(viii) If $\nu_\lambda=\mathrm{VIII}$, then $\sigma(*y')=\sigma(*y')$ for $y\in J$ and $y'\in J'$.

We can define an isomorphism between two TS diagrams of $(\Sigma_g(h)\,;\,\phi)^{\sigma}$ as in [4, § 13], but we omit it.

§ 3. The proof of Theorem 3

We may suppose that $\phi \in \mathrm{Diff}_+(\Sigma_g(h))$ is the identity. It is sufficient to construct a TS diagram of $(\Sigma_g(h)\,;\,\mathrm{id})^{\sigma}$. For we can construct a transverse foliation by taking suitable components in the table of [4, Theorem 3] for each TS pieces of the obtained TS diagram.

Let $\Sigma_g(h) = \Sigma_g - (\mathrm{Int}\, D_1 \cup \cdots \cup \mathrm{Int}\, D_h)$, where $D_1, \cdots, D_h$ are disjoint 2-disks in Σ_g. Take disjoint circles $C_1, \cdots, C_g \subset \Sigma_g(h)$ in such a way that the closure T_i of a connected component of $\Sigma_g(h) - C_i$ is homeomorphic to $T^2(1)$ and that $\Sigma_g(h) - T_i$ contains $C_1, \cdots, \hat{C}_i, \cdots, C_g$ and $\partial D_1, \cdots, \partial D_h$. Furthermore take disjoint simple curves $K_1, \cdots, K_g$ and $L_2, \cdots, L_h$ such that one endpoint of K_i belongs to ∂D_1 and the other belongs to C_i, and that one endpoint of L_i belongs to ∂D_1 and the other belongs to ∂D_i. Let M_i (or N_i) be a compact regular neighborhood of $C_i \cup K_i$ (or $D_i \cup L_i$) in $\Sigma_g(h) - \bigcup_{j=1}^{g} (T_j - C_j)$. Then the closure Σ^* of $\Sigma_g(h) - \bigcup_{i=1}^{g} (M_i \cup T_i) - \bigcup_{i=2}^{h} N_i$ in $\Sigma_g(h)$ is a polygon with $2(g+h)$ sides (see Figure 3. 1).

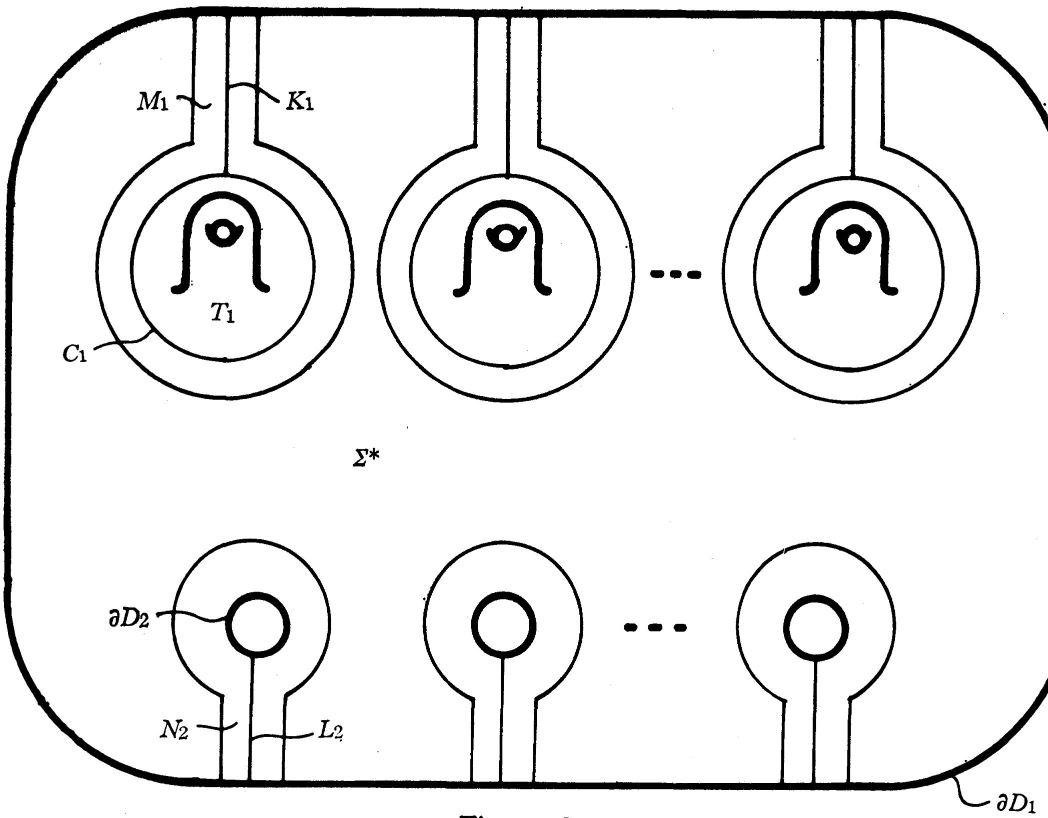

Figure 3. 1.

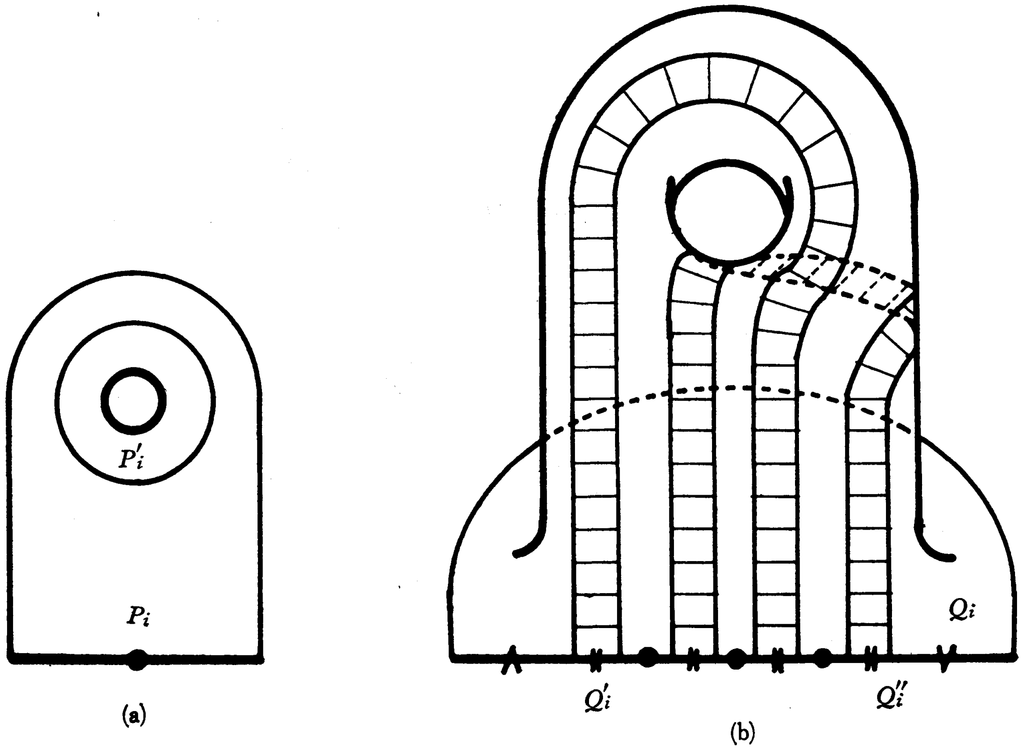

Figure 3. 2.

Choose a TS piece P_0 of type III with $|P_0|=\Sigma^*$. On N_i, we take two TS pieces P_i, P_i' as in Figure 3. 2 (a). On $M_i \cup T_i$, we take three TS pieces Q_i, Q_i', Q_i'' as in Figure 3. 2 (b).

Now we have easily a TS diagram containing P_0, $P_1, \cdots, P_h$, $P_1', \cdots, P_h'$, $Q_2, \cdots, Q_g$, $Q_2', \cdots, Q_g'$, $Q_2'', \cdots, Q_g''$. This completes the proof of Theorem 3.

§ 4. The proof of Theorem 4

For simplicity let $E=E(\Sigma_g(h)\,;\,\phi)$ and $\mathscr{F}=\mathscr{F}(\Sigma_g(h)\,;\,\phi)^\sigma$. Suppose that ∂E is connected and $\mathscr{F}$ has a transverse foliation $\mathscr{G}$. Let p (or q) be the number of positive (or negative) Reeb components of $\mathscr{G}|\partial E$ (see [4] for the definition). Take a basis α, β of $H_1(\partial E\,;\,\boldsymbol{Z})\cong\boldsymbol{Z}\oplus\boldsymbol{Z}$ such that $\pi_*'(\alpha)=\sigma(\partial E)\cdot[S^1]$ and $\pi_*'(\beta)=0$, where $\pi'=\pi|\partial E\colon \partial E\to S^1$. When $\mathscr{G}|\partial E$ has no compact leaf, let $a=\infty$. When $\mathscr{G}|\partial E$ has a compact leaf L, determine a non-negative integer a by $[L]=a\alpha+b\beta$ in $H_1(\partial E\,;\,\boldsymbol{Z})$, where L is oriented in such a way that $a>0$ or $(a, b)=(0, 1)$. Then for a non-compact leaf F of $\mathscr{F}$ the induced foliation $\mathscr{G}|F$ can be illustrated as Figure 4. 1.
Therefore we have

$$ha(p-q)=4-2h-4g$$

as in Tamura-Sato [7] and Nishimori [4]. Since $\mathscr{G}|\partial E$ is transversely ori-

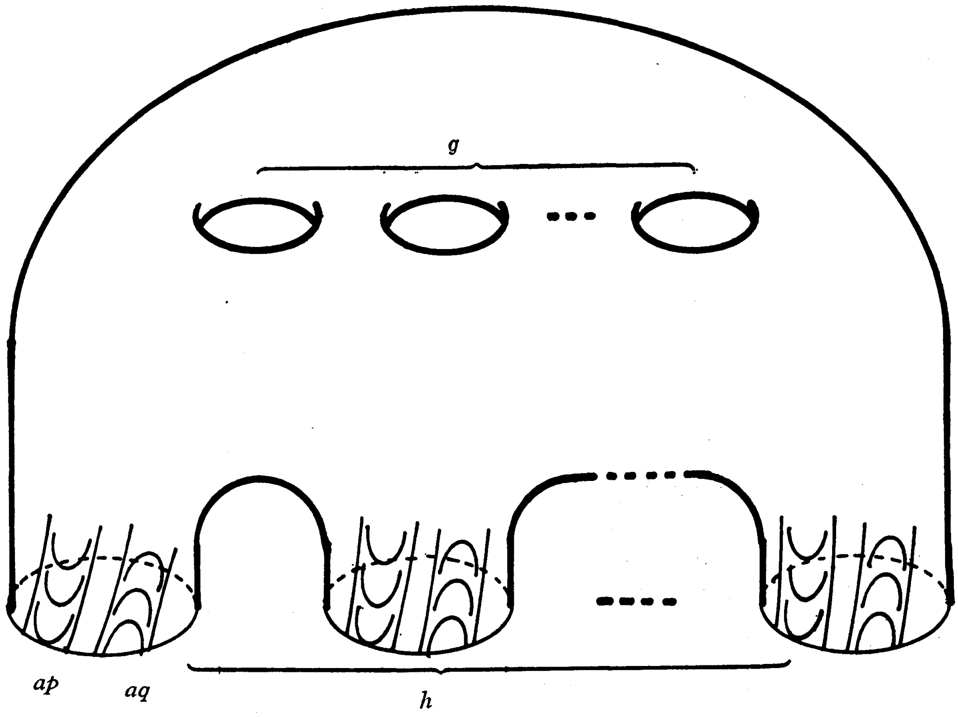

Figure 4. 1.

entable, it follows that $p+q$ is even. This implies that $p-q=2r$ for some $r\in \boldsymbol{Z}$. Hence we have $h(ar+1)=2-2g$, which contradicts the assumption of Theorem 4.

§ 5. Outline of the proof of Theorem 5 and Theorem 7

We only sketch the proof of Theorem 7 and omit the detail. Theorem 5 follows from Theorem 7, since non-compact leaf F of $\mathscr{F}(S^2(h)\,;\,\phi)^{\sigma}$ have genus 0 and the Poincare-Bendixson theorem implies that all the leaves of $\mathscr{G}|F$ are proper for all the foliations $\mathscr{G}$ transverse to $\mathscr{F}(S^2(h)\,;\,\phi)^{\sigma}$.

Suppose that $\mathscr{F}(\Sigma_g(h)\,;\,\phi)^{\sigma}$ has a transverse foliation $\mathscr{G}$ such that $\mathscr{G}|F$ has no non-proper leaf for some non-compact leaf F of $\mathscr{F}(\Sigma_g(h)\,;\,\phi)^{\sigma}$. By the arguments using the *characteristic diffeomorphism* of $\mathscr{G}$ and the *projection* of a leaf of $\mathscr{G}|\partial E(\Sigma_g(h)\,;\,\phi)$ to F (see [4] for the definition of the words in italics), we see that for each *strange negative Reeb cycle* $\mathscr{C}$ there exists a *separating torus* $S(\mathscr{C})$ satisfying the conditions in [4, Proposition 6. 4]. Let $\mathscr{C}_1, \cdots, \mathscr{C}_{\mu}$ be the strange negative Reeb cycles of $\mathscr{G}$ and take a separating torus $S(\mathscr{C}_i)$ for each $\mathscr{C}_i$. Let A be the compact manifold obtained from $E(\Sigma_g(h)\,;\,\phi)$ by deleting a sufficiently small collar of $\partial E(\Sigma_g(h)\,;\,\phi)$. In the same way as in [4, § 9], construct *TS decompositions* $\Omega\cup\Theta$ and $\Omega^x\cup\Theta^x$, and define the *characteristic hexad* $ch(D)=(l(D), m(D), n(D)\,;\,p(D), q(D), s(D))$ of $D\in\Omega$. In the present case, we must take the genus $g(D)$ of $D_1^{[0]}$ also into consideration. Now construct the double W of $D_{\mathrm{I}}^{[0]}$ as in [4, § 11]. Since W is diffeomorphic to a compact manifold obtained from a closed surface of genus $2g(D)+n(D)-1$ by deleting $2(l(D)+m(D))+p(D)+q(D)+s(D)$ open 2-disks, we have the modified equation

$$4g(D)+2\big(l(D)+m(D)+n(D)+p(D)\big)+s(D)=4\,.$$

Since $l(D)+m(D)+n(D)>0$, it follows that $g(D)=0$. Therefore the modification of the equation is not essential and we can prove the similar decomposition theorem as [4, Theorem 3]. Then Theorem 7 can be proved in the similar way as in [4].

§ 6. Pre *TS* diagrams of $T^2(1)$ containing an annular piece

The purpose of this and next sections is to make some preparations for the proof of Theorem 8. Let $\hat{\mathscr{T}}$ be a pre TS diagram of $T^2(1)$ and fix it throughout this section.

Let P be a TS piece contained in $\hat{\mathscr{T}}$. We call P a *disklike piece* if $|P|$ is homeomorphic to D^2 (equivalently, the type of P is I, II, III or IV), and an *annular piece* if $|P|$ is homeomorphic to $S^1\times \mathrm{I}$ (equivalently, the type of P is V, VI, VII, VIII or IX). In this section, we investigate $\hat{\mathscr{T}}$ containing an annular piece.

DEFINITION 6.1. *A subset A of $T^2(1)$ is called a special annulus with respect to $\hat{\mathscr{T}}$ if A satisfies the following conditions (1)-(3).*

(1) $A=|P_1|\cup\cdots\cup|P_k|$ for some TS pieces $P_1,\cdots,P_k$ contained in $\hat{\mathscr{T}}$.

(2) A is homeomorphic to $S^1\times I$.

(3) Each connected component of ∂A intersects $\partial T^2(1)$.

NOTATION. Let A be a subset of $T^2(1)$ satisfying the condition (1) of Definition 6.1. We denote by $\mathrm{Int}\,A$ the interior of A as a manifold, and by $\mathrm{Int}_*\,A$ the interior of A as a subset of $T^2(1)$.

We identify $T^2(1)$ and $T^2-\mathrm{Int}\,D^2$. Then $\partial T^2(1)=\partial D^2$. We have the following lemmas.

LEMMA 6.2. *Let A be a special annulus. Then each connected component of ∂A does not bound a disk in T^2.*

PROOF. Let K_1 and K_2 be the connected components of ∂A. Suppose that K_1 bounds a disk D_* in T^2. Note that T^2-K_1 has two connected components $\mathrm{Int}\,D_*$ and T^2-D_*, and that $D^2\cap\mathrm{Int}\,A=\emptyset$. When $\mathrm{Int}\,A\subset\mathrm{Int}\,D_*$, the fact $D^2\cap K_2\neq\emptyset$ implies that $D^2\subset\mathrm{Int}\,D_*$, which contradicts the fact $D^2\cap K_1\neq\emptyset$. When $\mathrm{Int}\,A\subset T^2-\mathrm{Int}\,D_*$, it follows that $D^2\subset T^2-D_*$, which contradicts the fact $D^2\cap K_1\neq\emptyset$. This completes the proof of Lemma 6.2.

LEMMA 6.3. *Let A be a special annulus. Then each connected component of $T^2(1)-\mathrm{Int}_* A$ is homeomorphic to D^2.*

PROOF. Let $C_1, \cdots, C_n$ be the connected components of $T^2(1)-\mathrm{Int}_* A$. Then the connected components of $C_j \cap A$ are homeomorphic to I. Denote by c_j the number of the connected components of $C_j \cap A$. Since the connected components of A do not bound a disk in T^2 by Lemma 6.2, the complement T^2-A is connected (see Rolfsen [6]). Hence there is an arc ω in $T^2-\mathrm{Int}_* A$ such that $\mathrm{Int}\,\omega \cap A=\emptyset$ and the endpoints of ω are contained in distinct connected components of ∂A. By the general position argument, we may suppose that $\omega \subset T^2(1)-\mathrm{Int}_* A$, and furthermore that $\omega \subset C_1$. This implies that $c_1 \geqq 2$. Let $b_j=\mathrm{rank}\, H_1(C_j)$, where the omitted coefficient is $\boldsymbol{Z}$. From the Mayer-Vietoris sequence

$$
\begin{aligned}
0=\bigoplus_{j=1}^{n} H_1(C_j \cap A) &\to H_1(A) \oplus \Big(\bigoplus_{j=1}^{n} H_1(C_j)\Big) \to H_1\big(T^2(1)\big)\\
\to \bigoplus_{j=1}^{n} H_0(C_j \cap A) &\to H_0(A) \oplus \Big(\bigoplus_{j=1}^{n} H_0(C_j)\Big) \to H_0\big(T^2(1)\big) \to 0\,,
\end{aligned}
$$

we have

$$\Big(1+\sum_{j=1}^{n} b_j\Big)+\sum_{j=1}^{n} c_j+1=2+(1+n)\,.$$

Since $\sum_{j=1}^{n} c_j \geqq n+1$, it follows that

$$b_1=\cdots=b_n=0\,, \qquad c_1=2\,, \qquad c_2=\cdots=c_n=1\,.$$

Therefore each C_j is homeomorphic to D^2.

LEMMA 6.4. *If there are two special annuli A_1 and A_2 with respect to $\hat{\mathscr{T}}$, then* $\mathrm{Int}\, A_1 \cap \mathrm{Int}\, A_2 \neq \emptyset$.

PROOF. Let A_1 and A_2 be special annuli with respect to $\hat{\mathscr{T}}$. Suppose that $\mathrm{Int}\, A_1 \cap \mathrm{Int}\, A_2=\emptyset$. Then A_2 is contained in some connected component of $T^2(1)-\mathrm{Int}_* A_1$. By Lemma 6.3, each connected component K of ∂A_2 is contained in a disk$\subset T^2(1)$. Hence K bounds a disk in T^2 ($\supset T^2(1)$). On the other hand, K does not bound a disk in T^2 by Lemma 6.2, which is a contradiction.

Now suppose that $\hat{\mathscr{T}}$ contains an annular piece. We investigate $\hat{\mathscr{T}}$ case by case in order to find something made invariant by all the automorphisms of $\hat{\mathscr{T}}$.

LEMMA 6.5. *If $\hat{\mathscr{T}}$ contains a TS piece P of type V, then P is the unique TS piece of type V in $\hat{\mathscr{T}}$.*

Proof. Let P' be another TS piece of type V in $\hat{\mathscr{T}}$. Since $|P|$ and $|P'|$ are special annuli, it follows that $\operatorname{Int}|P| \cap \operatorname{Int}|P'| \neq \emptyset$ by Lemma 6.4. Therefore $P=P'$.

Lemma 6.6. *If $\hat{\mathscr{T}}$ contains a TS piece P of type VI, then P is the unique TS piece of type VI in $\hat{\mathscr{T}}$.*

Proof. Let P' be another TS piece of type VI in $\hat{\mathscr{T}}$. Since σ is constant, P and P' are separated by some circles S and S' respectively. Furthermore neither S nor S' bounds a disk in T^2. Suppose that $P \neq P'$. Then $S \cap S' = \emptyset$. Therefore S and S' bound an annulus in $T^2(1)$ (see Rolfsen [6]). This contradicts the condition (PR3) in [4, Definition 13.1].

Lemma 6.7. (1) *$\hat{\mathscr{T}}$ contains no TS piece of type VIII.*

(2) *If $\hat{\mathscr{T}}$ contains a TS piece of type IX, then $\hat{\mathscr{T}}$ contains a TS piece of type VII.*

Proof. (1) Suppose that $\hat{\mathscr{T}}$ contains a TS piece of type VIII. By constructing cannonically non-singular vector fields for all the TS pieces, we find a non-singular vector field on $T^2(1)$ tangent to $\partial T^2(1)$. Since $\chi(T^2(1)) = -1$, this is a contradiction.

(2) follows directly from the configuration of TS pieces and the fact (1).

Lemma 6.8. *If $\hat{\mathscr{T}}$ contains a TS piece of type VII, then $\hat{\mathscr{T}}$ contains exactly two TS pieces P_1 and P_2 of type VII. Furthermore connected components of $\partial|P_1|$ and $\partial|P_2|$ are isotopic in $T^2(1)$, and neither of them bounds a disk in T^2.*

Proof. Let P_1 be a TS piece of type VII in $\hat{\mathscr{T}}$. Considering the configuration of TS pieces, we find a finite sequence $Q_1, \cdots, Q_k$ of TS pieces of type IX and a TS piece P_2 of type VII such that $A=|P_1| \cup |Q_1| \cup \cdots \cup |Q_k| \cup |P_2|$ is a special annulus. Using Lemma 6.4 as in the proof of Lemma 6.5, we see that $\hat{\mathscr{T}}$ contains no TS piece of type VII other than P_1 and P_2. Clearly all the connected components of $\partial|P_1|$, $\partial|Q_1|, \cdots, \partial|Q_k|$, $\partial|P_2|$ are isotopic in $T^2(1)$, and neither of them bounds a disk in T^2 by Lemma 6.2.

§ 7. Pre *TS* diagrams of $T^2(1)$ containing no annular piece

Let $\hat{\mathscr{T}}$ be a pre TS diagram of $T^2(1)$ containing no annular piece and fix it throughout this section.

Definition 7.1. Let P be a TS piece in $\hat{\mathscr{T}}$. A connected component of $|P| \cap \partial T^2(1)$ is called a *boundary side* of P. The closure of a connected component of $\partial|P| \cap \operatorname{Int} T^2(1)$ is called a *gluing side* of P.

The following lemma is clear and we omit the proof.

LEMMA 7.2. *Let $P_1, \cdots, P_m, Q_1, \cdots, Q_n$ be distinct TS pieces in $\hat{\mathscr{T}}$. Then each connected component of*

$$\left(|P_1| \cup \cdots \cup |P_m|\right) \cap \left(|Q_1| \cup \cdots \cup |Q_n|\right)$$

is the common gluing side of some P_i and some Q_j.

DEFINITION 7.3. A *primary piece* in $\hat{\mathscr{T}}$ is a TS piece P in $\hat{\mathscr{T}}$ such that P has more than two gluing sides and each connected component of $T^2(1)-\mathrm{Int}_* |P|$ is homeomorphic to D^2.

The purpose of this section is to show that $\hat{\mathscr{T}}$ has exactly one or two primary pieces. Note that every automorphism of $\hat{\mathscr{T}}$ shifts a primary piece to another one. First we show that $\hat{\mathscr{T}}$ has a primary piece.

LEMMA 7.4. *If $\hat{\mathscr{T}}$ has a special annulus, then $\hat{\mathscr{T}}$ has a primary piece.*

PROOF. Let A be a special annulus. We may suppose that A is *minimal*, that is, for each TS piece P in $\mathscr{T}$ with $|P| \subset A$, each connected component of $A-\mathrm{Int}_* |P|$ is homeomorphic to D^2. Let $C_1, \cdots, C_k$ be the connected components of $T^2(1)-\mathrm{Int}_* A$. By Lemma 6.3, each C_j is homeomorphic to D^2. Furthermore we may suppose that $C_1 \cap A$ has exactly two connected components K_1 and K_2 and that $C_j \cap A$ is connected for all $j>1$, by renumbering C_j's if necessary. By Lemma 7.2, for $i=1, 2$ there is a TS piece P_i in $\hat{\mathscr{T}}$ such that $|P_i| \subset A$ and K_i is a gluing side of P_i. Clearly P_i is not of type I. If P_i has exactly two gluing sides, then $A'=A-\mathrm{Int}_* |P_i|$ is a special annulus, which contradicts the minimality of A. Therefore P_i has more than two gluing sides. Since $C_j \cap (A-\mathrm{Int}_* |P_i|)$ is homeomorphic to I, each connected component of $T^2(1)-\mathrm{Int}_* |P_i|$ is homeomorphic to D^2. Thus we have primary pieces P_1 and P_2.

LEMMA 7.5. *$\hat{\mathscr{T}}$ has a primary piece.*

PROOF. Let $P_1, \cdots, P_n$ be the TS pieces with more than two gluing sides in $\hat{\mathscr{T}}$. Put $A_k=T^2(1)-\mathrm{Int}_*\left(\bigcup_{i=1}^{k} |P_i|\right)$. Note that each connected component of A_k is homeomorphic to D^2, $S^1\times \mathrm{I}$ or $T^2(1)$ and that all of them are homeomorphic to D^2 if $k=n$. If each connected component of A_1 is homeomorphic to D^2, then P_1 is a primary piece and we are done.

Now suppose that there exists k such that some connected component of A_k is not homeomorphic to D^2. Let κ be the maximum of such k's. Then $\kappa<n$. When a connected component A of A_κ is homeomorphic to $S^1\times \mathrm{I}$, it is easy to see that A is a special annulus. In this case $\hat{\mathscr{T}}$ has a primary

piece by Lemma 7.4. When a connected component B of A_κ is homeomorphic to $T^2(1)$, we see that $P_{\kappa+1}$ is a primary piece, as follows.

Clearly $|P_{\kappa+1}|\subset B$. Furthermore each connected component of $B-\mathrm{Int}_*|P_{\kappa+1}|$ is homeomorphic to D^2. Let $C_1, \cdots, C_m$ be the connected component of $T^2(1)-\mathrm{Int}_* B$. Let $b_j=\mathrm{rank}\, H_1(C_j)$. Denote by c_j the number of the connected components of $C_j\cap B$. From the Mayer-Vietoris sequence

$$\begin{aligned} 0=\bigoplus_{j=1}^{m} H_1(C_j\cap B)&\to H_1(B)\oplus\Big(\bigoplus_{j=1}^{m} H_1(C_j)\Big)\to H_1\big(T^2(1)\big)\\ \to\bigoplus_{j=1}^{m} H_0(C_j\cap B)&\to H_0(B)\oplus\Big(\bigoplus_{j=1}^{m} H_0(C_j)\Big)\to H_0\big(T^2(1)\big)\to 0\,, \end{aligned}$$

we have

$$2+\sum_{j=1}^{m} b_j+\sum_{j=1}^{m} c_j+1=2+1+m\,.$$

Since $c_j\geqq 1$ for all j, it follows that $b_j=0$ and $c_j=1$ for all j. Therefore C_j is homoeomorphic to D^2 and $C_j\cap B$ is connected. By Lemma 7.2, we see that if C_j intersects a connected component C of $B-\mathrm{Int}_*|P_{\kappa+1}|$, then $C_j\cap C$ is connected and homeomorphic to I. Therefore each connected component of $T^2(1)-\mathrm{Int}_*|P_{\kappa+1}|$ is homeomorphic to D^2. Hence $P_{\kappa+1}$ is a primary piece. This completes the proof of Lemma 7.5.

Now we have the following.

PROPOSITION 7.6. *$\hat{\mathscr{T}}$ has exactly one or two primary pieces.*

PROOF. Let P be a primary piece in $\hat{\mathscr{T}}$. Let $C_1, \cdots, C_m$ be the connected components of $T^2(1)-\mathrm{Int}_*|P|$. Using the Mayer-Vietoris sequence of $\Big(T^2(1)\,;\ |P|,\ \bigcup_{j=1}^{m} C_j\Big)$, we find that one of the following occurs by renumbering C_j's if necessary.

(A) $C_1\cap|P|$ has exactly three connected components and $C_j\cap|P|$ is connected for all $j>1$.

(B) $C_1\cap|P|$ and $C_2\cap|P|$ have exactly two connected components and $C_j\cap|P|$ is connected for all $j>2$.

In the case (B), we see that there is no primary piece other than P, as follows. Suppose that $\hat{\mathscr{T}}$ has a primary piece $Q\neq P$. Then $|Q|$ is contained in some C_j. If $|Q|\subset C_1$, then the connected component of $T^2(1)-\mathrm{Int}_*|Q|$ containing $|P|\cup C_2$ is homeomorphic to $S^1\times I$, which is a contradiction. If $|Q|\subset C_2$, then we have a contradiction in the same way. If $|Q|\subset C_j$ for some $j>2$, then the connected component of $T^2(1)-\mathrm{Int}_*|Q|$ containing $\mathrm{Int}_*(|P|\cup C_1\cup C_2)$ is homeomorphic to $T^2(1)$, which is a contradiction.

Hereafter consider the case (A). By the same arguments as above, C_j contains no primary piece for any $j>1$. Let J_1, J_2 and J_3 be the connected components of $C_1 \cap |P|$. See Figure 7.1.

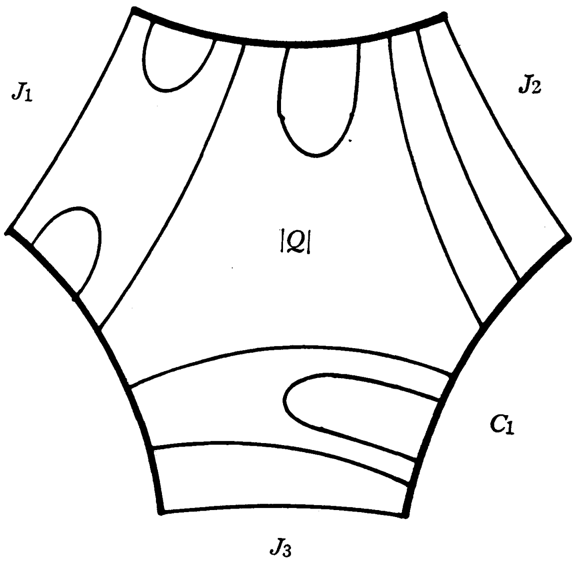

Figure 7.1.

Then we can take a finite sequence $K_1=J_1, K_2, \cdots, K_\nu$ satisfying the following conditions, where ν possibly equals to 1.

(1) Let $1 \leqq i \leqq \nu$. Then K_i and K_{i+1} are gluing sides of a common TS piece P_i with more than one gluing sides such that $|P_i| \subset C_1$.

(2) For $i>1$, the connected component of $C_1 - K_i$ containing J_1 contains neither J_2 nor J_3.

(3) for each gluing side $K \neq K_\nu$ of the TS piece Q such that $|Q| \supset K_\nu$ and $|Q| \not\supset K_{\nu-1}$, the connected component of $C_1 - K$ containing J_1 contains either J_2 or J_3.

We are going to show that Q is a primary piece. It is easy to see that Q has more than two gluing sides. Let B_j be the connected component of $C_1 - \mathrm{Int}_* |Q|$ containing J_j for $j=1, 2, 3$. Clearly $B_1 \neq B_2$ and $B_1 \neq B_3$. Since $|Q| \cup B_1 \cup B_2 \cup B_3$ is homeomorphic to D^2, the intersection $L = |Q| \cap B_2$ is connected. Hence L is a gluing side of Q. Clearly $L \neq K$. By the condition (3), the curve L separates $\mathrm{Int}_* B_2$ and B_3. Therefore $B_2 \neq B_3$. Let B be the connected component of $(|P| \cup C_1) - \mathrm{Int}_* |Q|$ containing $\mathrm{Int}_* |P|$. Then $B \cap |Q|$ has exactly three connected components $|Q| \cap B_1$, $|Q| \cap B_2$ and $|Q| \cap B_3$. The similar arguments using the Mayer-Vietoris sequence as before imply that each connected component of $(|P| \cup C_1) - \mathrm{Int}_* |Q|$ is homeomorphic to D^2. Now we easily see that each connected component of $T^2(1) - \mathrm{Int}_* |Q|$ is homeomorphic to D^2. Therefore Q is a primary piece. Furthermore by the similar arguments as in the case (B), we can show that $\hat{\mathscr{T}}$ has no primary piece other than P and Q. This completes the proof of Proposition 7.6.

§ 8. The proof of Theorem 8

Suppose that $\mathscr{F}(T^2(1)\,;\,\phi)^\sigma$ has a transverse foliation $\mathscr{G}$ such that all the leaves of $\mathscr{G}|F$ are proper for some non-compact leaf F of $\mathscr{F}(T^2(1)\,;\,\phi)^\sigma$. By Theorem 7, we have a TS diagram $\mathscr{T}$ of $(T^2(1)\,;\,\phi)^\sigma$. Then ϕ is an automorphism of the pre TS diagram $\hat{\mathscr{T}}$ subordinated to $\mathscr{T}$.

Part I. First suppose that $\hat{\mathscr{T}}$ has an annular piece.

LEMMA 8.1. *Under the above assumption, there exists a circle K in $T^2(1)$ such that $\phi(K)$ is isotopic to K in $T^2(1)$ and K does not bound a disk in $T^2=T^2(1)\cup D^2$.*

PROOF. When $\hat{\mathscr{T}}$ contains a TS piece P of type V, it follows from Lemma 6.5 that $\phi(|P|)=|P|$. Let K be a connected component of $\partial|P|$. Since ϕ preseerves the orientations of $T^2(1)$ and gluing sides of TS pieces, ϕ maps K to K. By Lemma 6.2, K does not bound a disk in T^2. When $\hat{\mathscr{T}}$ contains a TS piece of type VI, VII or IX, we can find K in the similar way as above using Lemma 6.6, Lemma 6.7 and Lemma 6.8.

Let K be as in Lemma 8.1. Taking an element $\beta\in H_1(T^2(1))$, we can obtain a basis $\alpha=[K]$, β of $H_1(T^2(1))=\boldsymbol{Z}\oplus\boldsymbol{Z}$, where the omitted coefficient is $\boldsymbol{Z}$. Since $H_1(\phi)(\alpha)=\alpha$, the matrix Φ respesenting $H_1(\phi)$ with respect to the basis α, β has the form $\begin{pmatrix}1 & 0\\ r & s\end{pmatrix}$ for some r, $s\in\boldsymbol{Z}$. Since $\det\Phi=1$, we have $s=1$. Therefore Trace $H_1(\phi)=2$.

Part II. Hereafter suppose that $\hat{\mathscr{T}}$ contains no annular piece. By Proposition 7.6, $\hat{\mathscr{T}}$ has exactly one or two primary pieces. Clearly ϕ maps a primary piece to another one. Since $\partial T^2(1)$ is connected, $\mathscr{T}$ has no TS piece of type IV. Therefore a primary piece is of type III.

Suppose that $\hat{\mathscr{T}}$ has exactly one primary piece, say P. Then we have $\phi(|P|)=|P|$. Note that if ϕ fixes a TS piece of type III, then ϕ fixes all of its sides (because ϕ preserves the symbols of boundary sides) and furthermore all the TS piece in $\hat{\mathscr{T}}$ (because $\hat{\mathscr{T}}$ contains no annular piece). Let $C_1, \cdots, C_m$ be the connected components of $T^2(1)-\mathrm{Int}_*|P|$ satisfying the condition (B) in the proof of Proposition 7.6. Denote by L_i and L_i' the connected components of $|P|\cap C_i$ for $i=1, 2$. See Figure 8.1.

Take circles K, L as in Figure 8.1. Then the homology classes $[K]$ and $[L]$ generate $H_1(T^2(1))$. Furthermore we have

$$H_1(\phi)\big([K]\big)=[K]\,,\qquad H_1(\phi)\big([L]\big)=[L]\,.$$

Therefore $H_1(\phi)=\mathrm{id}$ and Trace $H_1(\phi)=2$.

Now suppose that $\hat{\mathscr{T}}$ has exactly two primary TS pieces, say P and Q. When $\phi(|P|)=|P|$ and $\phi(|Q|)=|Q|$, it is easy to see that $H_1(\phi)=\mathrm{id}$ as above and we are done. Hence suppose that $\phi(|P|)=|Q|$ and $\phi(|Q|)=|P|$. Take gluing sides J_1, J_2, J_3 of P as in the case (A) of Proposition 7.6. Let K_j be the gluing side of Q belonging to the connected component of $T^2(1)-(\mathrm{Int}_*|P|$

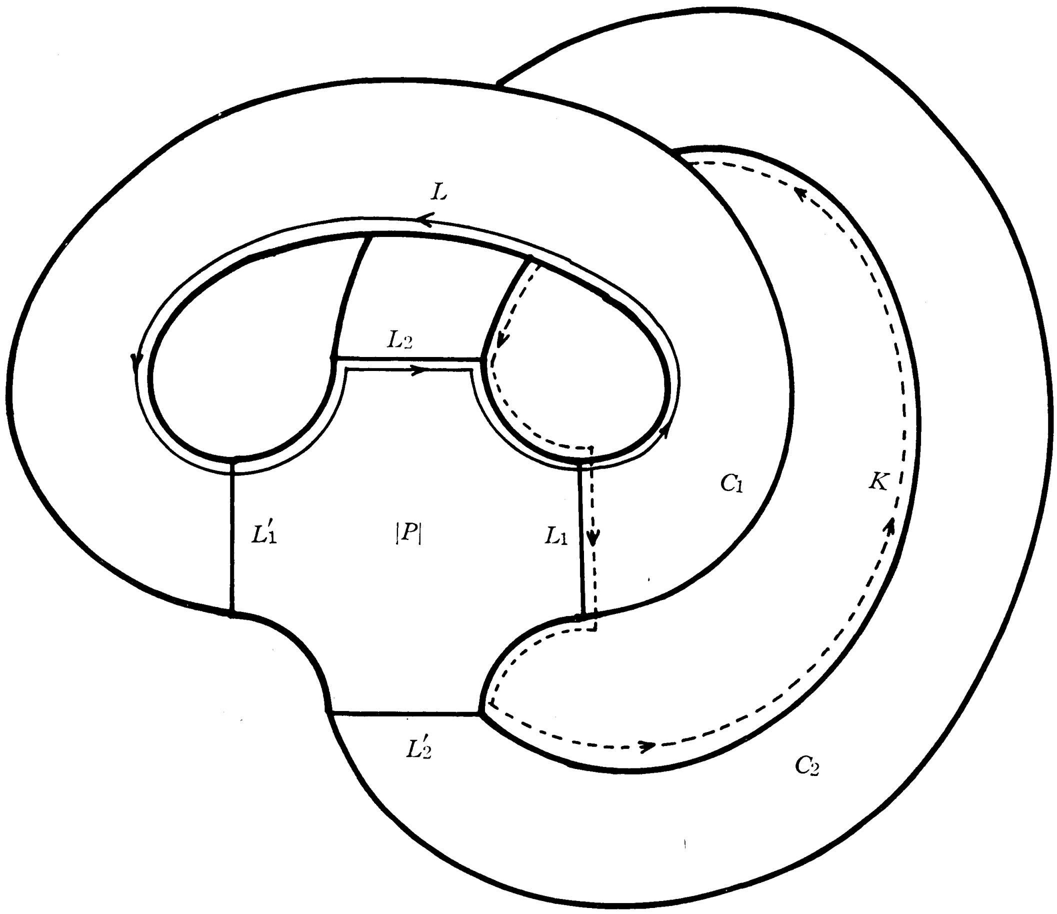

Figure 8. 1.

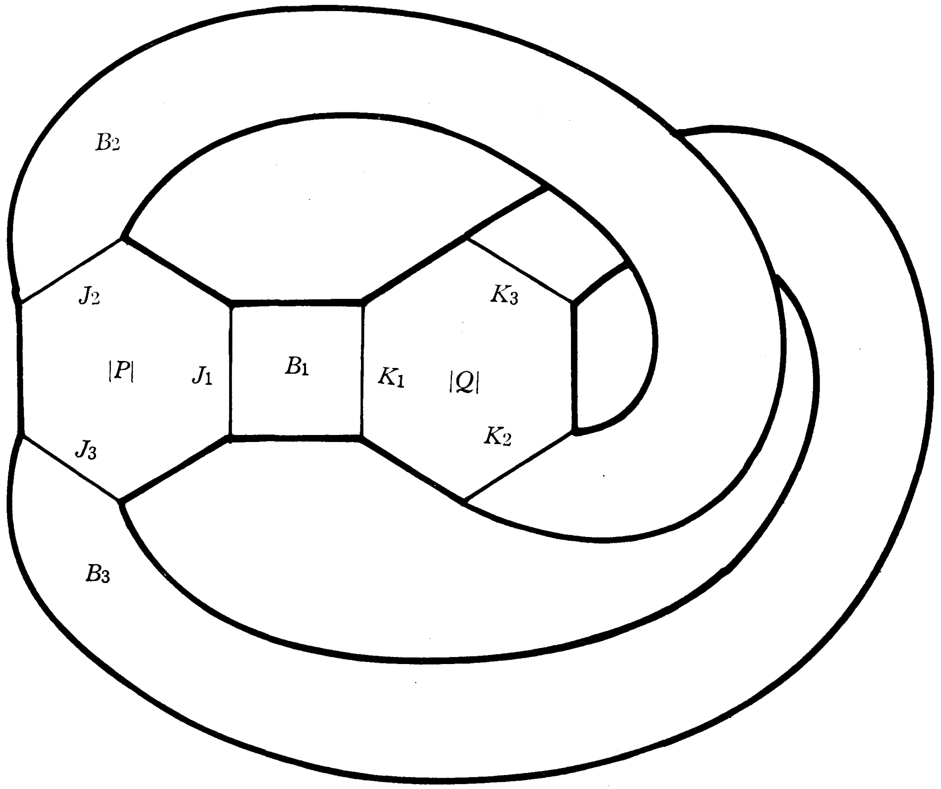

Figure 8. 2.

$\cup \mathrm{Int}_* |Q|)$ containing J_j. We may suppose that J_1, J_2, J_3 are arranged in the order compatible to the orientation of $\partial |P|$. Then K_1, K_2, K_3 are arranged in the order compatible to the orientation of $\partial |Q|$. Denote by B_i the connected component of $T^2(1)-\mathrm{Int}_* |P| \cup \mathrm{Int}_* |Q|)$ containing J_i for $i=1, 2, 3$. See Figure 8.2.

We have possibly three cases, that is,

(i) $\phi(J_1)=K_1$,

(ii) $\phi(J_1)=K_2$,

(iii) $\phi(J_1)=K_3$.

Let us check these cases one by one.

Case (i). Suppose that $\phi(J_1)=K_1$. Then $B_1 \supset K_1$. Note that $J_1 \neq K_1$, for otherwise ϕ could not preserve the orientations of $T^2(1)$ and J_1. Therefore B_1 is homeomorphic to D^2. Let $\mathscr{L}$ be the set of gluing sides of TS pieces in $\hat{\mathscr{T}}$ connecting the different connected components of $\partial B_1-(J_1 \cup K_1)$. We give $L \in \mathscr{L}$ an orientation following to [4, Remark 12.7]. Let $\mathscr{X}$ be the set of the closures X of connected components of $B_1-\bigcup_{L \in \mathscr{L}} L$ satisfying the following condition (*).

(*) For $L_1, L_2 \in \mathscr{L}$ with $L_1 \cup L_2 \subset \partial X$ and $L_1 \neq L_2$, the orientation of L_1 coincides with that of ∂X if and only if that of L_2 coincides with that of ∂X.

Since $\phi(J_1)=K_1$, the orientation of J_1 coincides with that of ∂B_1 if and only if that of K_1 coincides with that of ∂B_1. Therefore the number $\sharp(\mathscr{X})$ is odd. Since $\phi(X) \in \mathscr{X}$ for all $X \in \mathscr{X}$, there exists $X_0 \in \mathscr{X}$ with $\phi(X_0)=X_0$. It is easy to see that X_0 contains exactly one TS pieces P_0 of type III with two gluing sides in $\mathscr{L}$. Therefore $\phi(|P_0|)=|P_0|$. Then ϕ fixes all the TS pieces in $\hat{\mathscr{T}}$, which is a contradiction. We conclude that the case (i) does not occur.

Case (ii). Suppose that $\phi(J_1)=K_2$. Then $\phi(B_1)=B_2$, $\phi(B_2)=B_3$ and $\phi(B_3)=B_1$. Since $\phi^2(|P|)=|P|$, the automorphism ϕ^2 fixes all the TS pieces in $\hat{\mathscr{T}}$. Then we have $B_1=\phi^2(B_1)=\phi(B_2)=B_3$, which is a contradiction. Therefore the case (ii) does not occur.

Case (iii). Suppose that $\phi(J_1)=K_3$. Then we have a contradiction as above and the case (iii) does not occur.

Now we come to a conclusion that if $\hat{\mathscr{T}}$ contains no annular piece, then ϕ fixes all the TS pieces in $\hat{\mathscr{T}}$ and $H_1(\phi)$ is the identity. This completes the proof of Theorem 8.

§ 9. The proof of Theorem 9

Suppose that $\mathscr{F}=\mathscr{F}(T^2(h)\,;\,\phi)^\sigma$ has a transverse foliation $\mathscr{G}$ such that $\mathscr{G}|F$ has a non-proper leaf G_0 for some non-compact leaf F of $\mathscr{F}$.

Take a non-singular vector field X on $E(T^2(h)\,;\,\phi)$ tangent to $\mathscr{G}$ and transverse to $\mathscr{F}$. For each $x\in F$, let $\psi(x)$ be the first intersecting point of the orbit of X strating from x with F. Then $\psi: F\to F$ is a diffeomorphism. Clearly $\psi^*(\mathscr{G}|F)=\mathscr{G}|F$. Let $F^*=F\cup\mathscr{E}(F)$, where $\mathscr{E}(F)$ is the space of ends of F, and extend ψ to $\bar{\psi}$: $F^*\to F^*$. Then F^* is homeomorphic to T^2 and the induced isomorphism $H_1(\bar{\psi}): H_1(T^2;\,\boldsymbol{Z})\to H_1(T^2;\,\boldsymbol{Z})$ coincides with $H_1(\bar{\phi})$ when F^* and T^2 are adequately identified.

Hereafter we fix an identification $F=T^2-h\cdot D^2$. Let L be an oriented circle in F transverse to $\mathscr{G}|F$ and intersecting G_0. Since G_0 is not proper, it follows that $\#(G_0\cap L)=\infty$. Therefore L does not bound a disk in T^2. We can take an oriented circle K in F such that $K\cap L$ is a single point and the homology classes $[K]$ and $[L]$ generate $H_1(T^2;\,\boldsymbol{Z})$. We identify T^2 with $(\boldsymbol{R}/\boldsymbol{Z})\times(\boldsymbol{R}/\boldsymbol{Z})$ in such a way that $K=(\boldsymbol{R}/\boldsymbol{Z})\times\{[0]\}$ and $L=\{[0]\}\times(\boldsymbol{R}/\boldsymbol{Z})$, where $[0]$ means 0 mod 1. Let $\pi: \boldsymbol{R}^2\to(\boldsymbol{R}/\boldsymbol{Z})^2$ be the projection. On the open subset $\pi^{-1}(F)$ of $\boldsymbol{R}^2$, we have the induced foliation $\pi^*(\mathscr{G}|F)$. For each $n\in\boldsymbol{Z}$, the line $\tilde{L}_n=\{n\}\times\boldsymbol{R}$ is transverse to $\pi^*(\mathscr{G}|F)$ because $\pi(\tilde{L}_n)=L$.

In our situation, we can still define the rotation number as follows. Let Ω be the set of $y\in\boldsymbol{R}$ such that the leaf $\tilde{G}_y$ of $\pi^*(\mathscr{G}|F)$ passing through $(0, y)$ intersects $\tilde{L}_n$ for all $n\in\boldsymbol{Z}$. For all $y\in\Omega$, the leaf $\tilde{G}_y$ intersects $\tilde{L}_n$ at a single point $(n, z(y, n))$.

LEMMA 9.1. $\Omega\neq\emptyset$.

PROOF. Let $\mathscr{A}$ be the set of leaves of $\mathscr{G}|F$ contained in the closure of the non-proper leaf G_0 as a subset of F. It follows that $\mathscr{A}$ is uncountable. On the other hand, $\mathscr{A}$ contains at most two compact leaves, because n compact leaves of $\mathscr{G}|F$ separates F into n connected components and G_0 must be contained in one of them. Denote by $\mathscr{B}$ the set of non-compact leaves $G\in\mathscr{A}$ such that $\tilde{G}\cap\tilde{L}_n=\emptyset$ for some lift $\tilde{G}$ of G and some $n\in\boldsymbol{Z}$. Let $G\in\mathscr{B}$. Then an end ε of G is cofinal with an end $\bar{\varepsilon}$ of F. Since G is contained in the closure of G_0, the limit set of ε in $E(T^2(h)\,;\,\phi)$ consists of compact leaves of $\mathscr{G}|\partial E(T^2(h)\,;\,\phi)$ contained in $\partial|\mathscr{N}|$ for some negative Reeb component $\mathscr{N}$ in $\mathscr{G}|\partial E(T^2(h)\,;\,\phi)$. Since only a finite number of leaves of $\mathscr{G}|F$ can correspond in this way to each compact leaf of $\mathscr{G}|\partial E(T^2(h)\,;\,\phi)$ and $\mathscr{G}|\partial E(T^2(h)\,;\,\phi)$ has at most a finite number of negative Reeb components, we have $\#(\mathscr{B})<\infty$. Take a non-compact leaf $G\in\mathscr{A}-\mathscr{B}$. Then a lift $\tilde{G}$ of G intersects $\tilde{L}_n$ for all $n\in\boldsymbol{Z}$. Therefore $\Omega\neq\emptyset$.

We can prove the following lemma by the similar arguments as in Nitecki [5], and we omit the proof.

LEMMA 9.2. *Let* $y\in\Omega$. *Then the sequence* $z(y,1)/1$, $z(y,-1)/-1$, $z(y,2)/2$, $z(y,-2)/-2,\cdots$ *converges and the limit* $\rho(y)=\lim_{|n|\to\infty} z(y,n)/n$ *does not depend on* y.

We call $\rho=\rho(y)$ the *rotation number* of $\mathscr{G}|F$ with respect to $[K]$ and $[L]$. As to the rationality of ρ, we have the following.

LEMMA 9.3. *If* ρ *is rational, then all the leaves of* $\mathscr{G}|F$ *are proper.*

PROOF. Suppose that ρ is rational, say k/m for some k, $m\in\boldsymbol{Z}$. Replacing T^2 by a covering space if necessary, we may suppose that $\rho=k$. Define a homeomorphism $g:\Omega\to\Omega$ by $g(y)=z(y,1)$ for all $y\in\Omega$. Then g has the following property.

(1) If $y_1<y_2$ for y_1, $y_2\in\Omega$, then $g(y_1)<g(y_2)$.

(2) $g(y+n)=g(y)+n$ for all $y\in\Omega$ and $n\in\boldsymbol{Z}$.

Let G be a leaf of $\mathscr{G}|F$. If G is non-proper, then there is a non-proper leaf G' in the closure of G such that $G'=\pi(\tilde{G}_y)$ for some $y\in\Omega$. Hence we may suppose that $G=\pi(\tilde{G}_0)$ and $0\in\Omega$.

We see that $k-1<g(0)<k+1$, as follows. If $g(0)\geqq k+1$, then $g^n(0)\geqq n(k+1)$ and $\rho\geqq k+1$, which is a contradiction. If $g(0)\leqq k-1$, then $g^n(0)\leqq n(k-1)$ and $\rho\leqq k-1$, which is a contradiction.

Case I. Suppose that $g(0)=k$. Then G is a compact leaf and we are done.

Case II. Suppose that $k<g(0)<k+1$. Then $-k-1<g^{-1}(0)<-k$. We have $nk<g^n(0)<nk+1$ for all $n\in N$ and hence $-nk-1<g^{-n}(0)<-nk$ for all $n\in N$. For, if $g^n(0)\geqq nk+1$, then $g^{mn}(0)\geqq m(nk+1)$ for all $m\in N$ and $\rho\geqq k+(1/n)$, which is a contradiction.

CLAIM 9.4. $G\cap\pi(\{0\}\times]0,\ g(0)-k[)=\emptyset$.

PROOF. Suppose the contrary. Then there are n, $N\in\boldsymbol{Z}$ with $N<g^n(0)<N+g(0)-k$. Clearly $n\neq 0$.

(i) Suppose that $n>0$. Then $N=nk$ and $g^n(0)<(n-1)k+g(0)$. On the other hand, that $g(0)>k$ implies that $g^{m+1}(0)>g^m(k)=g^m(0)+k$ for all $m\in\boldsymbol{Z}$. Then we have

$$g^n(0)>g^{n-1}(0)+k>\cdots>g(0)+(n-1)k,$$

which is a contradiction.

(ii) Suppose that $n<0$. Let $n=-n'$. Then $N=-n'k-1$ and $g^{-n'}(0)<-(n'+1)k-1+g(0)$. Hence $0<g^{n'}(-(n'+1)k-1+g(0))=-(n'+1)k-1+$

$g^{n'+1}(0)$. Therefore $g^{n'+1}(0)>(n'+1)k+1$ for some $n'\in N$, which is a contradiction.

From Claim 9.4 and the following claim, we conclude that G is proper in Case II.

CLAIM 9.5. $G\cap\pi(\{0\}\times]g^{-1}(0)+k, 0[)=\emptyset$.

PROOF. Suppose the contrary. Then there are m, $M\in\boldsymbol{Z}$ with $M+g^{-1}(0)+k<g^{m}(0)<M$. It follows that $M+k<g^{m+1}(0)<g(0)+M$. Put $n=m+1$ and $N=M+k$. Then we have the same equation as in the proof of Claim 9.4 and a contradiction.

Case III. Suppose that $k-1<g(0)<k$. By the similar arguments as in Case II, we see that G is proper. This completes the proof of Lemma 9.3.

Here we give another description of the rotation number ρ. Let G be a non-proper leaf of $\mathscr{G}|F$. Then $\#(G\cap L)=\infty$. Let x_0 be an accumulation point of $G\cap L$. Take an infinite sequence $z_1, z_2, \cdots$ of points in $G\cap L$ converging to x_0. Denote by P_n the closed path in $T^2=T^2(h)\cup h\cdot D^2$ consisting of the segment of G between z_n and z_{2n} and the shorter segment of L between z_n and z_{2n}. We give P_n the orientation compatible with that of G. Then the homology class $[P_n]\in H_1(T^2;\boldsymbol{Z})$ is represented as $\alpha_n[K]+\beta_n[L]$ for some α_n, $\beta_n\in\boldsymbol{Z}$. We obtain the following lemma and omit the proof.

LEMMA 9.6. $\rho=\lim_{n\to\infty}\beta_n/\alpha_n$.

Now return to the proof of Theorem 9. By Lemma 9.3, it follows that ρ is irrational. Denote by $\begin{pmatrix} p & q \\ r & s \end{pmatrix}$ the matrix corresponding to $H_1(\bar{\phi})$ with respect to the basis $[K]$, $[L]$. Let G be the above non-proper leaf of $\mathscr{G}|F$. Since $\phi(G)$ is also a non-proper leaf of $\mathscr{G}|F$ and $\phi(P_n)$ can play the role of P_n, it follows that

$$\rho=\lim_{n\to\infty}\frac{r\alpha_n+s\beta_n}{p\alpha_n+q\beta_n}=\frac{r+s\rho}{p+q\rho}\,.$$

Hence we have an equation

$$(*)\qquad q\rho^2+(p-s)\rho-r=0\,.$$

When $q=0$, the irrationality of ρ implies that $p=s$ and $r=0$. Since $ps-qr=1$, it follows that $p=s=\pm1$, that is, $H_1(\bar{\phi})=\pm\mathrm{id}$.

When $q\neq0$, the equation (*) is quadratic with respect to ρ. Since ρ is irrational, the descriminant of (*) must be positive. Hence we have

$$0<(p-s)^2+4qr=(p-s)^2+4(ps-1)=(p+s)^2-4\,.$$

Therefore $|\mathrm{Trace}\, H_1(\bar{\phi})|=|p+s|>2$.

Now suppose that Trace $H_1(\bar{\phi}) \leqq 0$, from which we will bring out a contradiction. First we give a useful algebraic lemma. The simple proof given below is due to T. Tanisaki.

LEMMA 9.7. *Consider the canonical action of $SL(2, \boldsymbol{Z})$ on $\boldsymbol{Z} \oplus \boldsymbol{Z}$. Let $A \in SL(2, \boldsymbol{Z})$ satisfy* Trace $A \leqq 0$. *Let S be a subsemigroup of $\boldsymbol{Z} \oplus \boldsymbol{Z}$ satisfying the following conditions.*

(1) *If $s \in S$, then $As \in S$.*

(2) $S \neq \emptyset$.

Then $0 \in S$.

PROOF. Take an element $s \in S$. Since

$$A^2 - (\mathrm{Trace}\ A) \cdot A + E = 0\,,$$

it follows that $0 = (A^2 - (\mathrm{Trace}\ A) \cdot A + E)\, s \in S$, where E is the identity matrix.

Now take a non-proper leaf G_0 of $\mathcal{G}|F$ and a point x_0 in G_0. We fix a transverse orientation of $\mathcal{G}|F$. Let S be the set of elements s in $\pi_1(T^2, x_0)$ having, as representatives, closed paths transverse to $\mathcal{G}|F$ oriented in the same direction as the transverse orientation of $\mathcal{G}|F$. In other words, $S = \iota_*(\Pi S(x_0, \mathcal{G}|F))$, where $\Pi S(x_0, \mathcal{G}|F)\ (\subset \pi_1(F, x_0))$ is the homotopy secant of $\mathcal{G}|F$ at x_0 (see Lamoureux [3], Inaba [2]) and ι is the inclusion map of F into T^2. Then S is a subsemigroup of $\pi_1(T^2, x_0) \cong \boldsymbol{Z} \oplus \boldsymbol{Z}$. Clearly $S \neq \emptyset$. Let $A \in SL(2, \boldsymbol{Z})$ be the matrix corresponding to $H_1(\bar{\phi})$. Then we have $AS = S$. Since Trace $A =$ Trace $H_1(\bar{\phi}) \leqq 0$ by the assumption, it follows that $0 \in S$ by Lemma 9.7. Therefore there is a closed path $C \ni x_0$ transverse to $\mathcal{G}|F$ such that $[C] = 0$ in $\pi_1(T^2, x_0) \cong H_1(T^2; \boldsymbol{Z})$. By the general position arguments, we may suppose that C is of class C^∞ and has only a finite number of self intersections, which are all double. Let $\tilde{C}$ be a lift of C. Since C is null homotopic, the curve $\tilde{C}$ is closed. We may suppose that $G_0 = \pi(\tilde{G}_y)$ for some $y \in \Omega$ and $\tilde{C} \cap \tilde{G}_y \neq \emptyset$. Now we have the following lemma.

LEMMA 9.8. *In the above situation, there is a simple closed curve P in $\pi^{-1}(F) \subset \boldsymbol{R}^2$ such that*

(1) *P is transverse to $\pi^*(\mathcal{G}|F)$,*

(2) $P \cap \tilde{G}_y \neq \emptyset$.

PROOF. When $\tilde{C}$ has no self intersection, we can take $\tilde{C}$ as P and we are done. Suppose that $\tilde{C}$ has a self intersection point p. Then $\tilde{C}$ is divided by p into two closed curves P_1 and P_1'. We may suppose that $P_1 \cap \tilde{G}_y \neq \emptyset$. Clearly P_1 has less self intersection points than $\tilde{C}$. In this way, we have a finite sequence $P_1, \cdots, P_n$ of closed curves in $\pi^{-1}(F)$ such that

(1) P_i is transverse to $\pi^*(\mathcal{G}|F)$ for all i,

(2) $P_i \cap \tilde{G}_y \neq \emptyset$ for all i,

(3) P_n is simple.

Then $P = P_n$ is the desired curve.

Let P be the simple closed curve in $\pi^{-1}(F) \subset \boldsymbol{R}^2$ obtained in Lemma 9. 8. Since P bounds a disk in $\boldsymbol{R}^2$ and P is transverset o $\pi^*(\mathscr{G}|F)$, each leaf $\tilde{G}$ of $\pi^*(\mathscr{G}|F)$ can intersect P at most once and if $\tilde{G} \cap P \neq \emptyset$, then an end of $\tilde{G}$ is captured in the domain of $\boldsymbol{R}^2$ surrounded by P. On the other hand, $\tilde{G}_y \cap \tilde{L}_n \neq \emptyset$ for all $n \in \boldsymbol{Z}$, a contradiction. Therefore Trace $H_1(\bar{\phi}) > 0$. We conclude that $H_1(\bar{\phi}) = \mathrm{id}$ or Trace $H_1(\bar{\phi}) > 2$. This completes the proof of Theorem 9.

§ 10. The construction of transverse foliations (the completion of the proof of Theorem 6)

Suppose that Trace $H_1(\phi) \geqq 2$. The purpose of this section is to construct a foliation $\mathscr{G}$ transverse to $\mathscr{F} = \mathscr{F}(T^2(1)\,;\,\phi)^\sigma$. The construction comes from a construction in Franks-Williams [1] and an observation of K. Yano.

Define a linear map $\tilde{\Phi}: \boldsymbol{R}^2 \to \boldsymbol{R}^2$ by $\tilde{\Phi}(x, y) = (px + qy,\ rx + sy)$, x, $y \in \boldsymbol{R}$, where $\begin{pmatrix} p & q \\ r & s \end{pmatrix}$ is the matrix corresponding to $\mathrm{H}_1(\phi)$ with respect to the basis $[S^1 \times \{*\}]$, $[\{*\} \times S^1]$ of $H_1(T^2(1)\,;\,\boldsymbol{Z})$. Then we have a diffeomorphism Φ : $T^2 \to T^2$ making the diagram

$$\begin{array}{ccc} R^2 & \xrightarrow{\tilde{\Phi}} & R^2 \\ {\scriptstyle\pi}\downarrow & & \downarrow{\scriptstyle\pi} \\ T^2 & \xrightarrow[\Phi]{} & T^2 \end{array}$$

commute, where $\pi: \boldsymbol{R}^2 \to T^2 = \boldsymbol{R}^2/\boldsymbol{Z}^2$ is the projection.

Case I. Suppose that Trace $H_1(\phi) > 2$.

Let μ, ν be the eigenvalues of $\tilde{\Phi}$. Since $\mu + \nu =$ Trace $H_1(\phi) > 2$ and $\mu\nu = 1$, we may suppose that $0 < \mu < 1 < \nu$. Let V be the unit eigenvector corresponding to μ. Denote by $\tilde{\mathscr{G}}$ the foliation of $\boldsymbol{R}^2$ whose leaves are lines parrallel to V. Then $\tilde{\Phi}$ preserves $\tilde{\mathscr{G}}$, that is $\tilde{\Phi}^* \tilde{\mathscr{G}} = \tilde{\mathscr{G}}$. We can construct a diffeomorphism $\theta: \boldsymbol{R}^2 \to \boldsymbol{R}^2$ satisfying the following conditions (1)-(4).

(1) $\theta^* \tilde{\mathscr{G}} = \tilde{\mathscr{G}}$.

(2) Support $\theta = Cl(\{x \in \boldsymbol{R}^2 | \theta(x) \neq x\})$ is contained in $U_{3\nu\varepsilon}(0)$ for sufficiently small $\varepsilon > 0$, where we denote by $U_r(x)$ the open disk in $\boldsymbol{R}^2$ of radius r and with center at x.

(3) θ is connected to id by an isotopy fixing all the points in $\boldsymbol{R}^2 -$ Support θ.

(4) $\theta\cdot\tilde{\Phi}(x)=x$ for all $x\in U_{2\varepsilon}(0)$.

In order to construct $E(T^2(1)\,;\,\phi)$, consider $K=J\times J\times I$, where $J=[-1/2,\ 1/2]$ and $I=[0,1]$. Let $Z=\{(x,y,z)\in K|x^2+y^2\leqq\varepsilon^2\cdot\exp 2\alpha z\}$, where $\alpha=\log\nu>0$. Identifying $(-1/2,y,z)$ with $(1/2,y,z)$ and $(x,\ -1/2,z)$ with $(x,1/2,z)$ for all $x,\ y\in J$ and $z\in I$, we have $T^2\times I$. Furthermore we identify $(x,y,0)\in K$ with $(\nu x,\nu y,1)$ if $x^2+y^2\leqq 4\varepsilon^2$, and otherwise with $(x',y',1)$ where (x',y') is determined by $\pi\circ\theta\circ\tilde{\Phi}(x,y)=\pi(x',y')$. Denote by E the quotient space obtained as above. Let Π: $K\to E$ be the projection. Then $\Pi(Z)$ is diffeomorphic to $D^2\times S^1$. The foliation $(\tilde{\mathscr{G}}\times I)|K$ induces a foliation $\mathscr{G}$ of E with $\Pi^*\mathscr{G}=(\tilde{\mathscr{G}}\times I)|K$. Clearly $\mathscr{G}$ is transverse to $\partial\Pi(Z)$. See Figure 10.1.

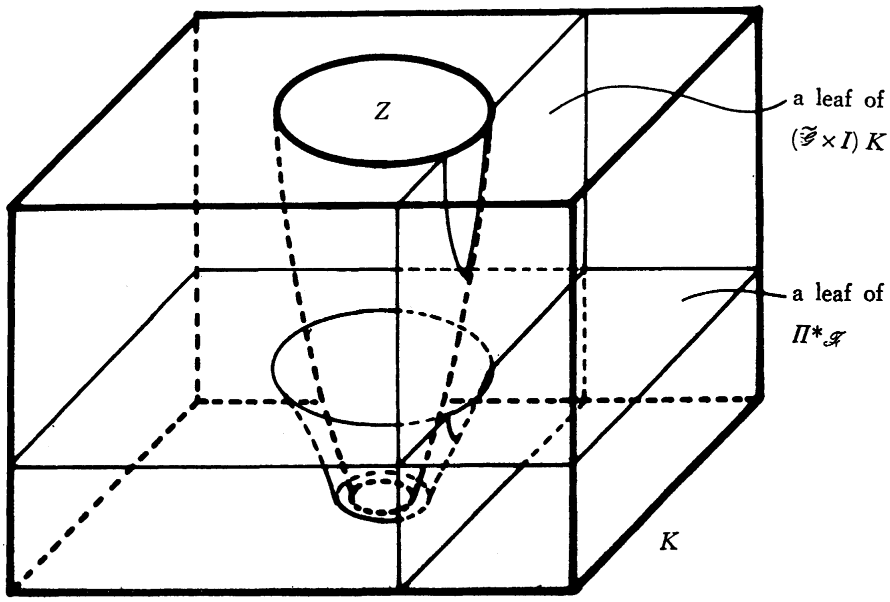

Figure 10. 1.

On the other hand, we have a foliation $\mathscr{F}$ of $E_0=E-\mathrm{Int}\,\Pi(Z)$ by turbulizing the trivial foliation

$$\left\{\Pi\left(J\times J\times\{z\}\right)\cap E_0\right\}_{z\in[0,1]},$$

as indicated in Figure 10.1. Then we see that $\mathscr{G}_0=\mathscr{G}|E_0$ is transverse to $\mathscr{F}$. Since there exists a diffeomorphism f: $E(T^2(1)\,;\,\phi)\to E_0$ with $f^*\mathscr{F}=\mathscr{F}(T^2(1)\,;\,\phi)^{\sigma}$, the induced foliation $f^*\mathscr{G}_0$ is the desired one transverse to $\mathscr{F}(T^2(1)\,;\,\phi)^{\sigma}$.

Case II. Suppose that Trace $H_1(\phi)=2$.

With respect to an appropriate basis of $H_1(T^2(1)\,;\,\boldsymbol{Z})$, the automorphism $H_1(\phi)$ corresponds to a matrix $\begin{pmatrix}1 & q\\ 0 & 1\end{pmatrix}$ for some $q\in\boldsymbol{Z}$. Then $\tilde{\Phi}$ constructed by $\begin{pmatrix}1 & q\\ 0 & 1\end{pmatrix}$ preserves the foliation $\tilde{\mathscr{G}}=\{\boldsymbol{R}\times\{y\}\}_{y\in\boldsymbol{R}}$ of $\boldsymbol{R}^2$. We can construct a diffeomorphism θ: $\boldsymbol{R}^2 \to \boldsymbol{R}^2$ satisfying the following conditions (1)-(4). Let $\nu = \mathrm{Max}\,\{2, |q|\}$.

(1) $\theta^* \tilde{\mathscr{G}} = \tilde{\mathscr{G}}$.

(2) There is a diffeomorphism η: $\boldsymbol{R} \to \boldsymbol{R}$ with Support $\eta \subset V_{4\nu\varepsilon}(0)$ for sufficiently small $\varepsilon > 0$ such that $\theta \cdot \tilde{\Phi}(x, y) = \tilde{\Phi}(x, \eta(y))$ for all $(x, y) \in \boldsymbol{R}^2 - U_{3\varepsilon}(0)$, where we denote by $V_r(y)$ the open interval in $\boldsymbol{R}$ of radius r and with center at y.

(3) θ is connected to *id* by an isotopy fixing all the points in $\boldsymbol{R}^2-$ Support θ.

(4) $\theta \circ \tilde{\Phi}(x) = \nu x$ for all $x \in U_{2\varepsilon}(0)$.

By the similar construction as in Case I, we have a transverse foliation of $\mathscr{F}(T^2(1); \phi)^{\sigma}$. This completes the proof of Theorem 6.

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