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# On the relation between Radonifying mappings and kernels of probability measures on Banach spaces

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## § 1. Introduction

The present paper is devoted to the study of Radonifying mappings on Banach spaces. In particular we investigate the relation between Radonifying mappings and kernels of probability measures.

Let $E$ be a Banach space, $E^{*}$ be its topological dual space and $\mu$ be a Borel probability measure on $E$. Denote by $\tau_{\mu}$ the topology on $E^{*}$ induced by the convergence in probability on $L^{0}(E, \mu)$. then the kernel of $\mu$ is defined as the topological dual of $(E^{*}, \tau_{\mu})$ and denoted by $K_{\mu}$. The notion of kernel has been introduced by Borell [1]. Let $F$ be another Banach space and $T$ be a continuous linear mapping from $F$ into $E$. After Schwartz [10], $T: F\rightarrow E$ is Radonifying provided that if $\nu$ is a cylindrical measure on $F$ of type 0, then $T(\nu)$ is a Radon measure on $E$ of order 0.

The purpose of this paper is to prove the followings.

THEOREM 1.1. *Let $E$ and $F$ be Banach spaces, and $T$ be a continuous linear mapping from $F$ into $E$. Suppose that $E$ is isomorphic to a subspace of $L^{p}$, $1<p<\infty$. Then the following statements are equivalent.*

(1) *There exists a Borel probability measure $\mu$ on $E$ such that $K_{\mu}\supset T(F)$.*

(2) *The adjoint map $T^{*}: E^{*}\rightarrow F^{*}$ is $q$-summing for some $q$, where $q<p$ if $p<2$, and $q=2$ if $p\geqq 2$.*

(3) *$T^{*}: E^{*}\rightarrow F^{*}$ is Radonifying.*

THEOREM 1.2. *Let $E$ and $F$ be Banach spaces, and $T$ be a continuous linear mapping from $F$ into $E$. Suppose that $E$ is separable and of type 2. Then the following statements are equivalent.*

(1) *There exists a Borel probability measure on $E$ which is quasi-invariant with respect to $T(F)$.*

(2) *There exists a Borel probability measure $\mu$ on $E$ such that $K_{\mu}\supset T(F)$.*

(3) *$T^{*}: E^{*}\rightarrow F^{*}$ is 2-summing.*

(4) *$T^{*}: E^{*}\rightarrow F^{*}$ is Radonifying.*

These two theorems generalize the results of Xia [15] and the author

[13]. Furthermore, as an application to Theorem 1. 1, we shall study Radonfying mappings on function spaces and give conditions to be Radonifying for the multiplication by a function from $L^q$ into $L^p$ in terms of $r$-summing mappings and kernels of probability measures.

Throughout this paper, we assume that all linear spaces are with real coefficients.

## § 2. Preliminaries

Let $E$ and $F$ be Banach spaces, and $T$ be a continuous linear mapping from $F$ into $E$.

DEFINITION 2. 1. For $0<p<\infty$, $T:F\to E$ is called $p$-summing if for each weakly $p$-summable sequence $\{x_n\}$ of $F$, $\{T(x_n)\}$ is an absolutely $p$-summable sequence of $E$. $T$ is called completely summing if it is $p$-summing for all $p>0$. (For the details of $p$-summing mappings; see Pietsch [6] and [7].)

DEFINITION 2. 2. For $0\leqq p<\infty$, $T:F\to E$ is called $p$-Radonifying if for each cylindrical measure $\nu$ on $F$ of type $p$, the image $T(\nu)$ is a Radon measure on $E$ of order $p$. We shall call Radonifying instead of 0-Radonifying. (For the details of $p$-Radonifying mappings; see Schwartz [8], [9], [10], [11] and [12].)

We shall now introduce a fundamental lemma.

LEMMA 2. 1. *Let $E$ and $F$ be Banach spaces, $T$ be a continuous linear mapping from $F$ into $E$, and let $\mathfrak{F}$ be the smallest $\sigma$-algebra of $E$ making every $x^*\in E^*$ measurable. Suppose that there exists a probability measure $\mu$ on $(E, \mathfrak{F})$ such that $K_\mu\supset T(F)$. Then the adjoint map $T^*:E^*\to F^*$ is completely summing.*

PROOF. Let $0<p<\infty$. For a weakly $p$-summable sequence $\{x_n^*\}$ of $E^*$, we define a $\mu$-measurable function $f$ on $E$ by $f(x)=\sum|<x_n^*, x>|^p$, and also define a probability measure $\nu$ on $(E, \mathfrak{F})$ by

$$\nu(A)=C\int_A \exp(-f(x))d\mu(x) \text{ for every } A\in\mathfrak{F},$$

where $C$ is a normalized constant. It is clear that the measures $\mu$ and $\nu$ are mutually equivalent, so that $K_\nu\supset T(F)$ holds. It follows from Theorem 4. 1 of [14] that there exists a positive constant $C_1$ such that for every $x^*\in E^*$, the inequality

$$\|T^*(x^*)\|^p\leqq C_1\int_E|<x^*, x>|^p d\nu(x)$$

holds. Hence in virtue of the choice of $\nu$, we have $\sum\|T^*(x_n^*)\|^p<\infty$. This completes the proof.

REMARK 2. 1. As is shown in this proof, Lemma 2. 1 also holds even in the case where $\mu$ is a probability measure on $E^{**}$ (bidual of $E$), that is, if there exists a weak* Radon probability measure $\mu$ on $E^{**}$ such that $K_\mu \supset T(F)$, then $T^*: E^* \to F^*$ is completely summing.

COROLLARY 2.2. *Let $E$ and $F$ be Banach spaces, and $T$ be a continuous linear mapping from $F$ into $E$. Suppose that there exists a Borel probability measure $\mu$ on $E$ such that $K_\mu \supset T(F)$. Then $T^*: E^* \to F^*$ is $p$-Radonifying for all $p>1$. Furthermore, if we assume that $E$ has the metric approximation property, then $T^*: E^* \to (F^*, \sigma(F^*, F))$ is Radonifying.*

PROOF. The assertion follows from Lemma 2.1 and Schwartz [9, Proposition III. 3.1] and [10, Théorème IX. 3.26].

COROLLARY 2.3. *Let $E$ and $F$ be Banach spaces, and $T$ be a continuous linear mapping from $F$ into $E$. Then the following statements hold.*

(1) *Let $1 \leqq p < q \leqq 2$. Suppose that $F^*$ is isomorphic to a subspace of $L^q$. Then $T: F \to E$ is $p$-summing implies that $T^*: E^* \to F^*$ is completely summing.*

(2) *Suppose that $F^*$ is isomorphic to a subspace of $L^0$ and has the metric approximation property. Then $T: F \to E$ is completely summing implies that $T^*: E^* \to F^*$ is completely summing.*

PROOF. First, we shall prove (1). If $T: F \to E$ is $p$-summing, then by Schwartz [9, Proposition III. 3.1], it is $r$-Radonifying, $p<r<q$. Since there exists a cylindrical measure $\nu$ on $F$ of type $r$ such that $K_\nu \supset F$, if we put $\mu = T(\nu)$, then $\mu$ is a Radon probability measure on $E$ such that $K_\mu \supset T(F)$. Thus the assertion follows from Lemma 2.1. Next, we shall prove (2). If $T: F \to E$ is completely summing, then by Schwartz [10, Théorème IX. 3.26], $T: F \to (E^{**}, \sigma(E^{**}, E^*))$ is Radonifying since $F^*$ has the metric approximation property. Since there exists a cylindrical measure $\nu$ on $F$ of type 0 such that $K_\nu \supset F$, if we put $\mu = T(\nu)$, then $\mu$ is a weak* Radon probability measure on $E^{**}$ such that $K_\mu \supset T(F)$. Thus the assertion follows from Remark 2.1. This completes the proof.

Finally we shall introduce the notion of type, cotype and $p$-Pietsch. Let $\{\varepsilon_n\}$ be a sequence of random signs, i.e. independent identically distributed random variables taking the values $+1$, $-1$ with probability $1/2$.

DEFINITION 2.3. Let $E$ be a Banach space. For $0<p\leqq 2$, $E$ is called of type $p$ if for each sequence $\{x_n\}$ of $E$, $\sum \|x_n\|^p < \infty$ implies $\sum \varepsilon_n x_n$ converges a.s.. For $2 \leqq q < \infty$, $E$ is called of cotype $q$ if for each sequence $\{x_n\}$ of $E$, $\sum \varepsilon_n x_n$ converges a.s. implies $\sum \|x_n\|^q < \infty$.

DEFINITION 2.4. Let $E$ be a Banach space, and let $0<p<\infty$. $E$ is called $p$-Pietsch if for every Banach space $F$, every $p$-summing mapping from $E$ into $F$ is completely summing.

Example 2.1. Consider infinite dimensional $L^r$ spaces.

(1) If $1 \leqq r \leqq 2$, $L^r$ is of type $r$, and no better. $(L^r)^*$ is $p$-Pietsch for $0 < p < r$, and no better.

(2) If $2 \leqq r < \infty$, $L^r$ is of type 2.

(3) If a Banach space $E$ is isomorphic to a subspace of $L^0$, $E$ is of cotype 2 and 2-Pietsch.

It is known that every Banach space is $(1-\varepsilon)$-Pietsch, $\varepsilon > 0$, and every infinite dimensional Banach space is not $(2+\varepsilon)$-Pietsch, $\varepsilon > 0$. (For the details of type, cotype and $p$-Pietsch; see Schwartz [12].)

## § 3. Main results

We shall first consider the following problem.

Problem $(A)$. Let $E$ and $F$ be Banach spaces, and $T$ be a continuous linear mapping from $F$ into $E$. Then are the following four statements equivalent when $E$ and $F$ belong to some suitable class of Banach spaces?

(1) There exists a Borel probability measure on $E$ which is quasi-invariant with respect to $T(F)$.

(2) There exists a Borel probability measure $\mu$ on $E$ such that $K_\mu \supset T(F)$.

(3) The adjoint map $T^*: E^* \to F^*$ is $p$-summing, $0 < p < \infty$.

(4) $T^*: E^* \to F^*$ is Radonifying.

In [15], Xia raised the above problem and proved that if both $E$ and $F$ are separable Hilbert spaces, then the above statements (1), (3) and (4) are equivalent. We shall extend his result to more general Banach spaces.

Remark 3.1. It is easy to see that the followings hold.

(1) Let $E$ be a reflexive Banach space, and let $0 < p < \infty$. If the statements (2) and (3) in Problem $(A)$ are equivalent for every Banach space $F$, then $E^*$ is $p$-Pietsch.

(2) Let $E$ be an infinite dimensional reflexive Banach space, and let $2 < p < \infty$. Then the statements (2) and (3) in Problem $(A)$ are not equivalent for some Banach space $F$ and some continuous linear map $T: F \to E$.

(3) If $E$ is isomorphic to a subspace of $L^p$, and $F$ is a Hilbert space, then the four statements in Problem $(A)$ are equivalent for every $p$, $1 \leqq p < \infty$.

We shall now show the equivalence of (2), (3) and (4) in Problem $(A)$ for every Banach space $F$ when $E$ belongs to some suitable class of Banach spaces. By Remark 3.1 we must assume $0 < p \leqq 2$.

Theorem 3.1. *Let $E$ and $F$ be Banach spaces, $T$ be a continuous linear mapping from $F$ into $E$, and let $1 < p < \infty$. Suppose that $E$ is isomorphic to a subspace of $L^p$. Then the following statements are equivalent.*

(1) *There exists a Borel probability measure $\mu$ on $E$ such that $K_\mu \supset T(F)$.*

(2) *$T^*: E^* \to F^*$ is $q$-summing for some $q$, where $q<p$ if $p<2$, and $q=2$ if $p \geqq 2$.*

(3) *$T^*: E^* \to F^*$ is Radonifying.*

PROOF. (1)$\Longrightarrow$(2) follows from Lemma 2.1. On the other hand, (3)$\Longrightarrow$(2) follows from Schwartz [10, Théorème IX. 3.26]. Hence we may show that the implications (2)$\Longrightarrow$(1) and (2)$\Longrightarrow$(3) hold. We shall first consider the case of $p<2$. Suppose that $T^*: E^* \to F^*$ is $q$-summing, $q<p$. It follows from Pietsch factorization theorem [6] that $T^*: E^* \to F^*$ can be factored as follows;

$$E^* \xrightarrow[J]{} G \xrightarrow[K]{} F^*$$

where $G$ is a Banach space which is isomorphic to a subspace of $L^q$, $J$ is $q$-summing and $K$ is a continuous linear mapping. Here we may assume $q>1$. Since $E$ is isomorphic to a subspace of $L^p$, by Corollary 2.3 $J^*: G^* \to E$ is $r$-summing, $1<r<q$, so that it is $r$-Radonifying (cf. Schwartz [9, Proposition III. 3.1]). Since there exists a cylindrical measure $\nu$ on $G^*$ of type $r$ such that $K_\nu \supset G^*$, if we put $\mu = J^*(\nu)$, then $\mu$ is a Radon probability measure on $E$ such that $K_\mu \supset T(F)$ since $J^*(G^*) \supset T(F)$. Thus (1) certainly holds. For (3), since $T^*: E^* \to F^*$ is $q$-Radonifying, $1<q<p$, the assertion follows from Maurey [4, Théorème 3]. We shall next consider the case of $p \geqq 2$. Suppose that $T^*: E^* \to F^*$ is 2-summing. It follows from Pietsch factorization theorem [6] that $T^*: E^* \to F^*$ can be factored as follows;

$$E^* \xrightarrow[J]{} H \xrightarrow[K]{} F^*$$

where $H$ is a Hilbert space, $J$ is 2-summing and $K$ is a continuous linear mapping. Since $E$ is isomorphic to a subspace of $L^p$, $J^*: H^* \to E$ is $p$-summing, so that it is $p$-Radonifying (cf. Schwartz [9, Proposition III. 3.1]). Let $\gamma$ be the canonical Gaussian cylindrical measure on $H^*$. Since $\gamma$ is of type $p$ and $K_\gamma \supset H^*$, if we put $\mu = J^*(\gamma)$, then $\mu$ is a Gaussian Radon probability measure on $E$ such that $K_\mu \supset T(F)$ since $J^*(H^*) \supset T(F)$. Thus (1) certainly holds. On the other hand, (3) follows from Okazaki's theorem [5]. This completes the proof.

REMARK 3.2. In Theorem 3.1, the statement (2) can not be replaced by the following.

(2′) *$T^*: E^* \to F^*$ is $p$-summing.*

We shall next show the equivalence of four statements in Problem ($A$) for every Banach space $F$ in the case where $E$ is separable and of type 2.

THEOREM 3.2. *Let $E$ and $F$ be Banach spaces, and $T$ be a continuous* *linear mapping from $F$ into $E$. Suppose that $E$ is separable and of type 2. Then the following statements are equivalent.*

(1) *There exists a Borel probability measure on $E$ which is quasi-invariant with respect to $T(F)$.*

(2) *There exists a Borel probability measure $\mu$ on $E$ such that $K_\mu \supset T(F)$.*

(3) $T^*: E^* \to F^*$ *is 2-summing.*

(4) $T^*: E^* \to F^*$ *is Radonifying.*

PROOF. (1)$\Longrightarrow$(2) is clear, and (4)$\Longrightarrow$(3) follows from Schwartz [10, Théorème IX. 3. 26]. On the other hand, (2)$\Longrightarrow$(3) follows from Lemma 2. 1. Hence it suffices to show that (3)$\Longrightarrow$(1) and (3)$\Longrightarrow$(4) hold. Suppose that $T^*: E^* \to F^*$ is 2-summing. It follows from Pietsch factorization thorem[6] that $T^*: E^* \to F^*$ can be factored as follows;

$$E^* \xrightarrow[J]{} G \xrightarrow[K]{} F^*$$

where $G$ is a Hilbert space with the norm $\|\cdot\|_G$, $J$ is 2-summing and $K$ is a continuous linear mapping. We now put $U = \{x^* \in E^*; \|J(x^*)\|_G \leqq 1\}$ and denote by $U^\circ$ the polar of $U$ in $E$. Let $H = \cup\, nU^\circ$, and let $L: H \to E$ be the identity map. Then $H$ is a Hilbert subspace of $E$ and the adjoint map $L^*: E^* \to H^*$ is 2-summing. Since $E$ is separable and of type 2, if we denote by $\gamma$ the canonical Gaussian cylindrical measure on $H$, then the image $L(\gamma)$ is a Gaussian Radon probability measure on $E$ which is quasi-invariant with respect to $H$ (cf. Chobanjan and Tarieladze [2, Corollary 3. 1]). Here the continuity of $K: G \to F^*$ implies $H \supset T(F)$, and so (1) certainly holds. On the other hand, by the closed graph theorem, $T: F \to H$ is continuous, so that (4) follows from Okazaki's theorem [5]. This completes the proof.

REMARK 3. 3. If $E$ is of type 2, then $E^*$ is of cotype 2, and so it is 2-Pietsch. Note that if $E$ is a separable Banach space and $E^*$ is of cotype 2, then in general, Theorem 3. 2 does not hold even in the case where $F$ is a Hilbert space. (For example, $E = c^0$.)

PROPOSITION 3. 3. *Let $E$ be a Banach space, $H$ be a Hilbert space and $T$ be a continuous linear mapping from $H$ into $E$. Suppose that $E$ has an unconditional basis and is of cotype $q$ for some $q$, $2 \leqq q < \infty$. Then the following statements are equivalent.*

(1) *There exists a Borel probability measure on $E$ which is quasi-invariant with respect to $T(H)$.*

(2) *There exists a Borel probability measure $\mu$ on $E$ such that $K_\mu \supset T(H)$.*

(3) $T^*: E^* \to H^*$ *is 1-summing.*

(4) $T^*: E^* \to H^*$ *is Radonifying.*

PROOF. (1)$\Longrightarrow$(2) is clear, (2)$\Longrightarrow$(3) follows from Lemma 2.1, and (4)$\Longrightarrow$(3) follows from Schwartz [10, Théorème IX. 3.26]. Hence it suffices to show that (3)$\Longrightarrow$(1) and (3)$\Longrightarrow$(4) hold. Suppose that $T^*: E^* \rightarrow H^*$ is 1-summing. It follows from Chobanjan and Tarieladze [2, Corollary 2.1] that if we denote by $\gamma$ the canonical Gaussian cylindrical measure on $H$, then the image $T(\gamma)$ is a Gaussian Radon probability measure on $E$ which is quasi-invariant with respect to $T(H)$. Thus (1) certainly holds. On the other hand, (4) follows from Okazaki's theorem [5]. This completes the proof.

PROPOSITION 3.4. *Let $E$ and $F$ be Banach spaces, and $T$ be a continuous linear mapping from $F$ into $E$. Suppose that $E$ is isomorphic to a subspace of $L^p$, $0 \leqq p < \infty$, and $F^*$ is isomorphic to a subspace of $L^0$. Also suppose that both $E$ and $F^*$ have the metric approximation property. Then the following statements are equivalent.*

(1) *There exists a weak* Radon probability measure $\mu$ on $E^{**}$ such that $K_\mu \supset T(F)$.*

(2) *$T^*: E^* \rightarrow F^*$ is completely summing.*

(3) *$T^*: E^* \rightarrow (F^*, \sigma(F^*, F))$ is Radonifying.*

PROOF. (1)$\Longrightarrow$(2) follows from Remark 2.1. On the other hand, by Schwartz [10, Théorème IX. 3.26], (2) and (3) are equivalent. Hence it suffices to show that (3)$\Longrightarrow$(1) holds. Suppose that $T^*: E^* \rightarrow (F^*, \sigma(F^*, F))$ is Radonifying. If $1 < p < \infty$, then the assertion follows from Theorem 3.1. If $0 \leqq p \leqq 1$, then by Corollary 2.3, $T: F \rightarrow E$ is completely summing, so that $T: F \rightarrow (E^{**}, \sigma(E^{**}, E^*))$ is Radonifying since $F^*$ has the metric approximation property. Since $F^*$ is isomorphic to a subspace of $L^0$, there exists a cylindrical measure $\nu$ on $F$ of type 0 such that $K_\nu \supset F$. If we put $\mu = T(\nu)$, then $\mu$ is a weak* Radon probability measure on $E^{**}$ such that $K_\mu \supset T(F)$. Thus (1) holds. This completes the proof.

## § 4. Application

In [8], Schwartz studied Radonifying mappings on sequence spaces and investigated conditions to be Radonifying for a diagonal mapping from $l^q$ into $l^p$. In this section, as an application to our results in Section 3, we shall study Radonifying mappings on function spaces and investigate conditions to be Radonifying for the multiplication by a function from $L^q$ into $L^p$.

Let $(\Omega, \Sigma, \mu)$ and $(\Omega, \Sigma, \nu)$ be $\sigma$-finite measure spaces. Suppose that $\mu$ and $\nu$ are mutually equivalent. For $1 \leqq p < \infty$ and $1 \leqq q < \infty$, we denote by $L^p(\mu)$ and $L^q(\nu)$ usual Banach spaces. Let $g(\omega)$ be a real valued measurable function defined on $\Omega$. Then $T_g$ denotes a mapping from $L^0(\nu)$ into $L^0(\mu)$

defined by $T_g: f(\omega)\to f(\omega)g(\omega)$ ($T_g$ will be called the multiplication by $g$.) We now suppose that $T_g$ operates from $L^q(\nu)$ into $L^p(\mu)$, that is, $T_g(L^q(\nu))\subset L^p(\mu)$. In this case, by the closed graph theorem, $T_g: L^q(\nu)\to L^p(\mu)$ is continuous.

Under the above situation, we have the following.

THEOREM 4.1. *Let* $1\leqq p<\infty$, $1\leqq q<\infty$, *and let* $T_g: L^q(\nu)\to L^p(\mu)$ *be the multiplication by a function* $g$. *Then the following statements are equivalent.*

(1) *There exists a Borel probability measure* $\gamma$ *on* $L^p(\mu)$ *such that* $K_\gamma\supset T_g(L^q(\nu))$.

(2) $(T_g)^*: (L^p(\mu))^*\to(L^q(\nu))^*$ *is* $r$*-summing for some* $r<p$.

(3) $(T_g)^*: (L^p(\mu))^*\to(L^q(\nu))^*$ *is Radonifying.*

PROOF. (1)$\Longrightarrow$(2) follows from Lemma 2.1, and (3)$\Longrightarrow$(2) follows from Schwartz [10, Théorème IX. 3.26]. Hence it suffices to show that (2)$\Longrightarrow$(1) and (2)$\Longrightarrow$(3) hold. Suppose that $(T_g)^*: (L^p(\mu))^*\to(L^q(\nu))^*$ is $r$-summing, $r<p$. Let $\Omega_g=\{\omega\in\Omega\,;\,|g(\omega)|>0\}$. We denote by $\tilde{\mu}$ and $\tilde{\nu}$ the restriction of $\mu$ and $\nu$ on $\Omega_g$, respectively. Without loss of generality, we may assume $\tilde{\mu}(\Omega_g)>0$ (equivalently, $\tilde{\nu}(\Omega_g)>0$.) If we denote by $\tilde{T}_g$ the restriction of $T_g$ on $L^q(\tilde{\nu})$, then $\tilde{T}_g$ is a continuous linear mapping from $L^q(\tilde{\nu})$ into $L^p(\tilde{\mu})$, and $(T_g)^*: (L^p(\tilde{\mu}))^*\to(L^q(\tilde{\nu}))^*$ is $r$-summing. Let $\bar{\mu}$ be a $\sigma$-finite measure on $\Omega_g$ defined by $\bar{\mu}(A)=\int_A|g(\omega)|^p d\tilde{\mu}(\omega)$, and let $S_g$ be a linear isometry from $L^p(\tilde{\mu})$ onto $L^p(\bar{\mu})$ defined by $S_g: f(\omega)\to f(\omega)/g(\omega)$. Since $\bar{\mu}$ is equivalent to $\tilde{\mu}$, it is also equivalent to $\tilde{\nu}$. If we put $T=S_g\cdot\tilde{T}_g$, then $T$ is the natural injection from $L^q(\tilde{\nu})$ into $L^p(\bar{\mu})$, and $T^*: (L^p(\bar{\mu}))^*\to(L^q(\tilde{\nu}))^*$ is $r$-summing. Now we shall prove that there exists a Borel probability measure $\tilde{\gamma}$ on $L^p(\bar{\mu})$ such that $K_{\tilde{\gamma}}\supset T(L^q(\tilde{\nu}))$. For the case of $1<p<\infty$ and $2\leqq q<\infty$, by Theorem 3.1 such a measure certainly exists since every $r$-summing mapping from a Banach space into a Banach space of cotype 2 is 2-summing (cf. Maurey [3, Corollary 2]). For the case of $1\leqq p<\infty$ and $1\leqq q\leqq 2$, the assertion follows from Theorem 3.1 of [13]. Hence it suffices to show only the case of $p=1$ and $2<q<\infty$. It follows from Corollary 2.3 that $T: L^q(\tilde{\nu})\to L^p(\bar{\mu})$ is completely summing, and so it is $s$-Radonifying for all $s>1$ (cf. Schwartz [9, Proposition III. 3.1]). Let $1<s<q^*$, where $1/q+1/q^*=1$. Since there exists a cylindrical measure $\sigma$ on $L^q(\tilde{\nu})$ of type $s$ such that $K_\sigma\supset L^q(\tilde{\nu})$, if we put $\tilde{\gamma}=T(\sigma)$, then $\tilde{\gamma}$ *is a Radon probability measure on* $L^p(\bar{\mu})$ such that $K_{\tilde{\gamma}}\supset T(L^q(\tilde{\nu}))$. Thus in any case, such a measure certainly exists. Since $L^q(\tilde{\nu})$ is separable (cf. [13, Theorem 3.1]), by Corollary 2.2, $T^*: (L^p(\bar{\mu}))^*\to(L^q(\tilde{\nu}))^*$ is Radonifying,

so that $(T_g)^*:(L^p(\mu))^*\to(L^q(\nu))^*$ is also Radonifying. Thus (3) holds. Finally, we shall show that (1) holds. Let $U$ be the canonical linear isometry from $L^p(\tilde{\mu})$ into $L^p(\mu)$, and put $W=U\cdot S_g^{-1}$. Then $W$ is a linear isometry from $L^p(\bar{\mu})$ into $L^p(\mu)$. If we denote by $\gamma$ the image of $\tilde{\gamma}$ *under* $W$, $\gamma$ is a Borel probability measure on $L^p(\mu)$ such that $K_\gamma\supset T_g(L^q(\nu))$. This completes the proof.

REMARK 4.1. A $\mu$-measurable set $A$ of positive measure is called a $\mu$-atom if for any $\mu$-measurable subset $B$ of $A$, we have either $\mu(B)=0$ or $\mu(A\cap B^c)=0$. In the above theorem, it is shown that if $(T_g)^*:(L^p(\mu))^*\to(L^q(\nu))^*$ is $p$-summing, then the set $\Omega_g=\{\omega\in\Omega\,;\,|g(\omega)|>0\}$ is a union of an at most countable number of $\mu$-atoms (cf. [13, Theorem 3.1]). Under the same assumptions as in Theorem 4.1, if we also assume that $\mu$ has no atoms, then $(T_g)^*:(L^p(\mu))^*\to(L^q(\nu))^*$ is Radonifying if and only if $g(\omega)=0$ a. s..

Let $\alpha=(\alpha_n)$ be a sequence of real numbers. Then the multiplication $(\xi_n)\to(\alpha_n\xi_n)$ operates from $l^q$ into $l^p$ under suitable conditions on the sequence $\alpha$. Such a linear mapping will be called diagonal and denoted by $T_\alpha$.

COROLLARY 4.2. *Let* $1\leqq p<\infty$, $1\leqq q<\infty$, *and* $T_\alpha:l^q\to l^p$ *be the diagonal. Then the following statements are equivalent.*

(1) *There exists a Borel probability measure* $\mu$ *on* $l^p$ *such that* $K_\mu\supset T_\alpha(l^q)$.

(2) $(T_\alpha)^*:(l^p)^*\to(l^q)^*$ *is r-summing for some* $r<p$.

(3) $(T_\alpha)^*:(l^p)^*\to(l^q)^*$ *is Radonifying.*

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