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Singularities of the scattering kernel for several convex obstacles

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§ 1. Introduction.

Let $\mathscr{O}$ be a compact set with a smooth boundary in $\boldsymbol{R}^{n}(n\geqq 2)$ such that $\Omega=\boldsymbol{R}^{n}-\mathscr{O}$ is connected. Let us consider the scattering problem of the following:

$$(1.1)\quad \begin{cases}\square\, u(t,x)=(\partial_t^2-\Delta_x)u(t,x)=0 & \text{in } \boldsymbol{R}^1\times\Omega,\\ \partial_\nu u(t,x)=0 & \text{on } \boldsymbol{R}^1\times\partial\Omega,\\ u(0,x)=f_1(x) & \text{on } \Omega,\\ \partial_t u(0,x)=f_2(x) & \text{on } \Omega,\end{cases}$$

where ν is the unit inner normal to the boundary $\partial\Omega$. Denote by $k_-(s,\omega)$ $(k_+(s,\omega))\in L^2\boldsymbol{R}^1\times S^{n-1})$ the incoming (outgoing) translation representation of the initial data (f_1,f_2). Then the scattering operator S: $k_-(t,\omega)\rightarrow k_+(s,\theta)$ has a distribution kernel $S(s-t,\theta,\omega)$, where $S(s,\theta,\omega)$ is called the scattering kernel (cf. LAX and PHILLIPS [3], [4], SHENK II [9]).

The relation between $\mathscr{O}$ and $S(s,\theta,\omega)$ has been studied by several authors. SOGA [10] and YAMAMOTO [12] have characterized the convexity of $\mathscr{O}$ in terms of the singularities of $S(s,-\omega,\omega)$ as follows:

$\mathscr{O}$ is convex if and only if sing supp $S(\cdot,-\omega,\omega)$ *consists of only one point for any* $\omega\in S^{n-1}$.

NAKAMURA and SOGA [6] and NAKAMURA [7] have examined sing supp $S(\cdot,-\omega,\omega)$ precisely when $\mathscr{O}$ consists of two disjoint convex obstacles. Under suitable assumptions, they have established the relation between the distribution of the singular points of $S(\cdot,-\omega,\omega)$ and the distance of the obstacles.

In this paper our purpose is to study the sing supp $S(\cdot,-\omega,\omega)$ in case $\mathscr{O}$ consists of several disjoint convex obstacles $\{\mathscr{O}_j\}_{j=1,\cdots,J}$. Owing to the Neumann boundary condition, we can easily analyze the singularities of $S(s,-\omega,\omega)$. Set $r_i(\omega)=\min_{y\in\partial\mathscr{O}_i}y\cdot\omega\,(1\leqq i\leqq J)$, and take point $x_0\in P=\{x:x\cdot\omega=\min_{1\leqq i\leqq J}r_i(\omega)-1\}$. We consider the following broken ray:

(*) the ray starts at x_0 in the direction ω and reflects m times at the boundary $\partial\mathscr{O}$according to the law of the geometrical optics, or more specifically, say at the points $\{x_1, \cdots, x_m\}\in\partial\mathscr{O}_{j_1}\times\cdots\times\partial\mathscr{O}_{jm}$ with $j_l\in\{1,\cdots,J\}$ such that $j_l\neq j_{l+1}$ and returns to the point x_{m+1} on P in the direction $-\omega$. Set

(1.2) $s^{\vec{J}}=\sum_{j=0}^{m}|x_j-x_{j+1}|-2$

where $\vec{J}=\{j_1,\cdots,j_m\}$ $(\in\{1,\cdots,J\}^m)$. We assume the followings on the position $\mathscr{O}_j$:

(1.3) For all $\{j_1, j_2, j_3\}\in\{1,\cdots,J\}^3$ with different indices to each other, it holds that
every line going through both $\mathscr{O}_{j_1}$ and $\mathscr{O}_{j_2}$ does not intersect $\mathscr{O}_{j_3}$ and that

(1.4) $s^{n-1}-\bigcup_{j_1\neq j_2}\{\pm\frac{x-y}{|x-y|}\in S^{n-1};\in\partial\mathscr{O}_{j_1},\ y\in\partial\mathscr{O}_{j_2}\}.$

The first main result is the following theorem;

THEOREM 1. *Let* $\{\mathscr{O}_j\}_{j=1,\cdots,J}$ *satisfy the hypotheses* (1.3) *and* (1.4). *Then for*

$$\omega\in S^{n-1}-\bigcup_{j_1\neq j_2}\{\pm\frac{x-y}{|x-y|}\in S^{n-1};x\in\partial\mathscr{O}_{j_1},\ y\in\partial\mathscr{O}_{j_2}\},$$

there exist the broken rays satisfying the propert. (*) *for any positive integer* m *and* $\{j_1,\cdots,j_m\}\in\{1,\cdots,J\}^m$ *with* $j_l\neq j_{l+1}$, *and the following equality holds*

(1.5) sing supp $S(\cdot, -\omega, \omega)=\{s_{\vec{J}};\vec{J}=\{1,\cdots,J\}^m$ *with* $j_l\neq j_{l+1},\ m=1,2,$ $\cdots\}$, *where* $s_{\vec{J}}=-2\min_{1\leq i\leq J}r_i(\omega)-s^{\vec{J}}$.

When each $\mathscr{O}_j$ is strictly convex and the position $\mathscr{O}_j$ satisfies some hypothesis, then IKAWA [2] has shown that there exists a unique periodic ray with the reflection points $\{y_1,\cdots,y_k\}\in\partial\mathscr{O}_{j_1}\times\cdots\times\partial\mathscr{O}_{j_k}$ for any $\{j_1,\cdots,j_k\}\in\{1,\cdots,J\}^k$ such that $j_l\neq j_{l+1}$ and $j_k\neq j_1$. A periodic ray means the reflection points satisfy the relation:

$$y_i=y_{i+Nk}\text{ for } i=1,2,\cdots,k \text{ and } N=1,2,\cdots.$$

Our hypotheses on the position $\mathscr{O}_j$ imply the assumption in [2]: For all $\{j_1, j_2, j_3\}\in\{1,2,\cdots,J\}^3$ such that $j_l\neq j_{l'}$, if $l\neq l'$, the convex hull of $\bar{\mathscr{O}}_{j_1}$ and $\bar{\mathscr{O}}_{j_2}$ has no intersection with $\bar{\mathscr{O}}_{j_3}$ ($\bar{U}$ denotes the closure of U).

Now we set

(1.6) $\tilde{s}_{k,j_1,\cdots,j_k}=\sum_{j=1}^{k}|y_j-y_{j+1}|\ \ (y_{k+1}=y_1)$.

We want to know the distribution of all the points of $\{s^{\mathcal{J}}\}$. In case only two convex obstacles are concerned, the distance of those has been characterized by the distribution of singular points of the scattering kernel. However, in the case of several convex obstacles we may not expect such a simple result as in the case of two convex obstacles. In this paper we show that the broken ray behaving periodically, it can be characterized by its periodicity. Let us explain this fact. We say that $\mathcal{J}=\{j_1,\cdots,j_m\}$ has a periodic part, when there exists a natural number h with $1\leqq h\leqq m$ such that the set of a sequence $\{j_i\}_{h\leqq i\leqq m}$ has a period $k\in\boldsymbol{N}$;

(1.7) $j_{h+l}=j_{h+l+Nk}\quad \begin{matrix}\text{for } l=0,1,2,\cdots,k-1,\\ N=1,2,\cdots\end{matrix}.$

We note that an integer $m-h-k+1$ must be exactly divisible by k. Especially, for this broken ray, we set

$$s_{m,j_1,\cdots,j_{h+k-1}}=s^{\mathcal{J}}.$$

Under the above notations we have the second main result:

THEOREM 2. *Let $\{\mathcal{O}_j\}_{j=1,\cdots,J}$ satisfy the hypotheses in Theorem* 1. *If each $\mathcal{O}_j$ is strictly convex then*

(1.8) $\lim_{m\to+\infty}\{(s_{m+k,j_1,\cdots,j_{h+k-1}}-s_{m,j_1,\cdots,j_{h+k-1}})-\tilde{s}_{k,j_h,\cdots,j_{h+k-1}}\}=0.$

Note that the hypotheses (1.3) *and* (1.4) *on $\mathcal{O}_j$ are satisfied when*

$$\min_{i\neq j}\text{dist}\,(\mathcal{O}_i,\mathcal{O}_j)>\{(\sin\frac{\pi}{J(J-1)})^{-1}-1\}\ \max_{1\leqq j\leqq J}\text{diam}\ \mathcal{O}_j.$$

The main tasks in the proof of Theorem 1 and Theorem 2 are to show that there exist actually the broken rays with the property (*) and to obtain the equality (1.8). Our proofs are essentially based on the ideas used in [6] and [7]. Thus in the present paper we shall show that the methods used in [6] and [7] are applicable to the case of several convex obstacles.

PETKOV and STOJANOV [8] (without proofs) have extended the results in [6] to the case of several disjoint strictly convex obstacles in $\boldsymbol{R}^3$ under some assumptions. But to the author it seems that not all our results can be obtained by their methods.

§ 2. Properties of the broken rays.

First, we define the broken rays according to the law of the geometrical optics. For $x\in\partial\Omega$, denote by $\nu(x)$ the unit inner vector normal to the boundary. We suppose that

$$\{x=x_0+l\xi_0\,;\,l>0\}\cap\partial\Omega\neq\phi$$

for $x_0\in\Omega$ and $\xi_0\in S^{n-1}$, and define l_{j-1}, x_j and ξ_j successively by

$$\begin{aligned}&l_{j-1}=\inf\{l>0\,;\,x_{j-1}+l\xi_{j-1}\in\partial\Omega\},\\&x_j=x_{j-1}+l_{j-1}\xi_{j-1},\\&\xi_j=\xi_{j-1}-2(\xi_{j-1}\cdot\nu(x_j))\nu(x_j),\end{aligned}$$

where $l_{j-1}=\infty$ when $x_{j-1}+l\xi_{j-1}\notin\partial\Omega$ for any $l>0$. We call the set

$$L(x_0,\,\xi_0)=\bigcup_j\{x=x_j+l\xi_j\,;\,0\leqq l<l_j\}$$

the broken rays starting at x_0 in the direction ξ_0 with the reflection points $\{x_j\}$. When there exists an integer $m\geqq 1$ such that $\{x=x_m+l\xi_m\,;\,l>0\}\cap\partial\Omega=\phi$. We set

$$\begin{aligned}&{}^{\#}\mathrm{ref}\ L(x_0,\,\xi_0)=m,\\&\mathrm{dir}_\infty L(x_0,\,\xi_0)=\xi_m.\end{aligned}$$

Let us prove the following existence of the broken ray stated in Theorem 1.

THEOREM 2.1. *Let $\{\mathcal{O}_j\}_{j=1,\cdots,J}$ satisfy the hypotheses* (1.3) *and* (1.4). *We assume that*

$$\omega\in S^{n-1}-\bigcup_{j_1\neq j_2}\{\pm\frac{x-y}{|x-y|}\in S^{n-1}\,;\,x\in\partial\mathcal{O}_{j_1},\ y\in\partial\mathcal{O}_{j_2},\ j_1\neq j_2\}.$$

Then for any m and $\{j_1,\cdots,j_m\}\in\{1,\cdots,J\}^m$ $(j_k\neq j_{k+1},\ k=1,2,\cdots,m-1)$, there exists x_0 on P (stated in Introduction) and the broken ray $L(x_0,\,\omega)$ such that

(i) ${}^{\#}\mathrm{ref}\ L(x_0,\,\omega)=m$, $\mathrm{dir}_\infty L(x_0,\,\omega)=-\omega$,

(ii) *the reflection points $\{x_j\}$ satisfy* $\{x_1,\cdots,x_m\}\in\partial\mathcal{O}_{j_1}\times\cdots\times\partial\mathcal{O}_{j_m}$.

PROOF OF THEOREM 2.1. Our proof is based on the following thoughts. Let ω be the direction given in Theorem 1. Suppose a plane wave propagates in the direction ω and hits a convex obstacle $\mathcal{O}_{j_1}$. Then the wave front reflected by $\mathcal{O}_{j_1}$ will be no focal points and that there will be at least one broken ray reflected in the direction $-\omega$. Suppose the wave front reflected by $\mathcal{O}_{j_l}$ hits $\mathcal{O}_{j_{l'}}$ subsequently and is reflected by $\mathcal{O}_{j_{l'}}$. Then it also has no focal points and there exists at least one reflected ray with the direction $-\omega$. Using Lemma 2.1 in [7], we can check that this process is successively true. Without loss of generality, we may assume that $\omega=(0,0,\cdots,0,1)$. Let us consider the broken ray starting at the point on P in the direction ω. Set

$$S_{\pm}^{n-1}=\{\theta\in S^{n-1}:\pm\theta\cdot\frac{a_{j_{l'}}-a_{j_l}-[(a_{j_{l'}}-a_{j_l})\cdot\omega]\omega}{|a_{j_{l'}}-a_{j_l}-[(a_{j_{l'}}-a_{j_l})\cdot\omega]\omega|}>0,$$

$\mathrm{dist}(\mathcal{O}_{j_{l'}},\mathcal{O}_{j_l})=|a_{j_{l'}}-a_{j_l}|\}$, $U_{1,j_k}=\{x\in\partial\mathcal{O}_{j_k}:\nu(x)\cdot\omega<0\}$ for $k=l,\ l'$, $\xi_{1,j_k}=\omega-2(\omega\cdot\nu(x))\nu(x)$ for $x\in\bar{U}_{1,j_k}$; $k=l,\ l'$.

Here, $\bar{U}$ denotes the closure of the set U. By using Lemma 2.2 in [7] inductively, for any $m(m=2,3,\cdots)$ we have a connected open set $U_{m,j_k}(\subset\partial\mathcal{O}_{j_k},\ k=l,\ l')$ such that

(2.1)
$$\begin{cases}(\mathrm{i})\quad \overline{S_{+}^{n-1}}\subset\{\xi_{m,j_l}(x):x\in U_{m,j_l}\},\\ \qquad \text{and}\\ \qquad \overline{S_{-}^{n-1}}\subset\{\xi_{m,j_{l'}}(\mathrm{x}):x\in U_{m,j_{l'}}\},\\ (\mathrm{ii})\quad \nu(x)\cdot\xi_{m,j_k}(x)>0 \text{ for } x\in U_{m,j_A}(k=l,\ l'),\\ (\mathrm{iii})\quad \text{for } k=l,\ l', \text{ the wave front associated with } \xi_{m,j_k}(x) \text{ is}\\ \text{convex surface,}\end{cases}$$

where $\xi_{m,j_k}(x)$ denotes the m^{th} reflected direction by $\mathcal{O}_{j_k}$ ($k=l,\ l'$) (for detailed definitions and notations see § 2 in [7]). We put

$$S_{l,l'}=\bigcap_m\bigcap_{k=l,l'}\{\xi_{m,j_k}(x):x\in U_{m,j_k}\},$$
$$M_{l,l'}=\bigcap_m\bigcap_{k=l,l'}\{x+l\xi_{m,l_k}(x):x\in U_{m,l_k},\ l\geqq 0\}.$$

At this time, from the hypotheses (1.3) and (1.4) it holds that

$$\bigcup_{j\neq j_l,j_{l'}}\mathcal{O}_j\subset M_{l,l'} \text{ for any } l,\ l',$$

and

$$\omega=(0,0,\cdots,0,1)\in\bigcap_{l\neq l'}S_{l,l'}. \tag{2.2}$$

We consider any broken ray $L(x_0,\omega)$ with $x_0\in P$. Assume that the first reflection point x_1 of $L(x_0,\omega)$ belongs to $\partial\mathcal{O}_{j_1}$. We see that $\omega\in S_{1,2}$ by (2.2). From lemma 2.2 in [7] it follows that there exists U_{2,j_2} satisfying (i)~(ii) in (2.1) with $l=1$ and $l'=2$ when x_0 moves some open set in P. Repeating this process inductively, we have U_{m,j_m} satisfying (i)

$\sim$(ii) in (2.1) with $l=m-1$, $l'=m$. Hence the existence of the broken ray with $\xi_{m,j_m}=-\omega$ is proved. The proof is complete.

REMARK 2.1. When each $\mathcal{O}_j$ is strictly convex, Lemma 2.2 in [7] shows that the Gaussian curvature of the m^{th} reflected wave front never vanish, and so we have the uniqueness of broken ray stated in Theorem 2.1.

Before proving Theorem 2, we explain a key lemma. From now on we assume that each $\mathcal{O}_j$ is strictly convex. Let $\{y_l\}_{l=1}^k$ $(y_l\in\partial\mathcal{O}_{j_l})$ be the reflection points of the periodic ray (showed in [2]) stated in Introduction. Set

$$\mathscr{L}_j=\{ty_j+(1-t)y_{j+1}: t\in\boldsymbol{R}\} \text{ for } j=1,\cdots,k-1,$$
$$U(\delta)=\{y\in\partial\Omega\,;\text{dist }(y, \bigcup_{j=1}^{k-1}\mathscr{L}_j)\leqq\delta.$$

We take points $x_1\in\partial\mathcal{O}_{j_1}$ and $x_2\in\partial\mathcal{O}_{j_2}$ such that

$$\delta<\text{dist }(x_1, \mathscr{L}_1)\leqq\text{dist }(x_2, \mathscr{L}_1),$$

and consider the broken ray $L(x_1, \xi_1)$ $(\xi_1=(x_2-x_1)/|x_2-x_1|)$.

LEMMA 2.1. *Assume that*

$$L(x_1, \xi_1)\cap U(\delta)=\phi.$$

Then there exist $N\in\boldsymbol{N}$ *and* l $(0\leqq l<k)$ *such that*

$$x_{Nk+l}\notin\mathcal{O}_{j_l} \text{ and } x_{Nk+l-1}\in\mathcal{O}_{j_{l-1}}.$$

PROOF. It is enough to consider the case

$$\overline{x_1x_2}\cap\{z\in\partial\Omega: \text{dist }(z, \mathscr{L}_1)<|x_1-y_1|\}=\phi \qquad (2.3)$$
$$(\overline{x_1x_2}=\{tx_1+(1-t)x_2: 0\leqq t\leqq 1\}).$$

The proof in the case that $\overline{x_1x_2}\cap\{z\in\partial\Omega: \text{dist }(z, \mathscr{L}_1)<|x_1-y_1|\}\neq\phi$ can be treated in the same way as in the case (2.3), because it holds that

$$\delta<\text{dist }(x_2, \mathscr{L}_2)\leqq\text{dist }(x_3, \mathscr{L}_3)$$

and

$$\overline{x_2x_3}\cap\{z\in\partial\Omega: \text{dist }(z, \mathscr{L}_2)<|x_2-y_2|\}=\phi.$$

It is easily seen that there exists a constant $c_0\geqq 0$ such that

$$\text{dist }(x_2, y_2)\geqq(1+\min_{i\neq j}\text{dist }(\mathcal{O}_i, \mathcal{O}_j)c_0)\text{dist }(x_1, y_1).$$

If we use the estimates of the Gaussian curvature of the wave front obtained in [7], there exists a constant $c_1(>c_0)$ such that

$$\text{dist }(x_3, y_3)\geqq 1+\min_{i\neq j}\text{dist }(\mathscr{O}_i, \mathscr{O}_j)c_1)\text{dist }(x_2, y_2).$$

Repeating this process inductively, we get

$$\text{dist }(x_{Nk+l}, y_{Nk+l})\geqq(1+\min_{i\neq j}\text{dist }(\mathscr{O}_i, \mathscr{O}_j)c_1)^{Nk+l-2}\delta.$$

Since $y_{Nk+l}\in\partial\mathscr{O}_{j_l}$, there exist $N\in\boldsymbol{N}$ and $l(0\leqq l<k)$ such that $x_{Nk+l}\notin\partial\mathscr{O}_{j_l}$ and $x_{Nk+1-l}\in\partial\mathscr{O}_{j_{l-1}}$ which proves our lemma.

REMARK 2. 2. Lemma 2.1 is also true when $(x_1, x_2)\in\partial\mathscr{O}_{j_l}\times\partial\mathscr{O}_{j_{l+1}}$. From Lemma 2.1 we can choose $\tilde{N}$ such that

$$\tilde{N}=\max\{N:\ (x_{Nk+1}, \mathrm{y}_1)\in\partial\mathscr{O}_{j_1}\times\partial\mathscr{O}_{j_1}\}.$$

We are going to estimate the difference $\sum_{j=1}^{\tilde{N}k}|x_j-x_{j-1}|-\tilde{N}\tilde{s}_{k,j_1,\cdots,j_k}$, where x_j denote the reflection point stated in Lemma 2.1 and $\tilde{s}_{k,j_1,\cdots,j_k}$ is the length of the periodic ray stated in Introduction. We have

$$\begin{aligned}&\textstyle\sum_{j=1}^{\tilde{N}k}|x_j-x_{j+1}|-\tilde{N}\tilde{s}_{k,j_1,\cdots,j_k}\\&=\textstyle\sum_{j=1}^{\tilde{N}k}|x_j-x_{j+1}|-\sum_{j=1}^{\tilde{N}k}|y_j-y_{j+1}|\\&\leqq 2\textstyle\sum_{j=1}^{\tilde{N}k}|x_j-y_j|.\end{aligned}$$

From lemma 2.1, there exists a constant $\rho(0<\rho<1)$ such that

$$|x_j-y_j|\leqq\rho|x_{j+1}-y_{j+1}|\ \text{ for } j\geqq 3.$$

Hence we get

$$\begin{aligned}&\textstyle\sum_{j=1}^{\tilde{N}k}|x_j-x_{j+1}|-\tilde{N}\tilde{s}_{k,j_1,\cdots,j_k}\\&\leqq 2\textstyle\sum_{j=1}^{\tilde{N}k}|x_j-y_j|\\&\leqq 6\left(\textstyle\sum_{j=0}^{\infty}\rho^j\right)|\mathrm{x}_{\tilde{N}\mathrm{k}+1}-y_{\tilde{N}k+1}|.\end{aligned}$$

If $x_{\tilde{N}k+1}$ belongs to $U(\varepsilon)$, then there exists a positive constant C such that

$$\textstyle\sum_{j=1}^{\tilde{N}k}|x_j-x_{j+1}|-\tilde{N}\tilde{s}_{k,j_1,\cdots,j_k}\leqq C\varepsilon. \tag{2.4}$$

By the above consideration, we can estimate a difference between the length of the part of a broken ray which has the reflection points belonging to $U(\varepsilon)$. Using these fact, let us prove Theorem 2 in Introduction.

PROOF OF THEOREM 2. Let $\{x_j\}$ and $\{z_j\}$be the points defining $s_{m+k,j_1,\cdots,j_{h+k-1}}$ and $s_{m,j_1,\cdots,j_{h+k-1}}$ respectively. We show for any $\varepsilon>0$

$$|s_{m+k,j_1,\cdots,j_{h+k-1}}-s_{m,j_1,\cdots,j_{h+k-1}}-\tilde{s}_{k,j_1,\cdots,j_{h+k-1}}|<\varepsilon$$

if m is large enough. Lemma 2.1 shows that, for sufficently large m, the number of the reflection points defining $s_{m,j_1,\cdots,j_{h+k-1}}$ ($s_{m+k,j_1,\cdots,j_{h+k-1}}$) which do not belong to $U(\varepsilon)$ is finite. And the above consideration (below Remark 2.2) shows that we can choose a part of $\{x_j\}$ and $\{z_j\}$ such that

$$^{\#}\{j : j\geqq h,\ (x_j, x_{j+1})\in U(\varepsilon)^2\}-{}^{\#}\{j : j\geqq h\ (z_j, z_{j+1})\in U(\varepsilon)^2\}=k,$$

and then the following estimate holds (cf. (2. 4)) :

$$(2.5)\qquad |\textstyle\sum_{\substack{(x_j,\, x_{j+1})\in U(\varepsilon)^2\\ j\geqq h}}|x_j-x_{j+1}|-$$
$$^{\#}\{j : j\geqq h,\ (x_j, x_{j+1})\in U(\varepsilon)^2\}\,\tilde{s}_{k,j_h,\cdots,j_{h+k-1}}|\leqq C_2\varepsilon$$

and

$$|\textstyle\sum_{\substack{(z_j,\, z_{j+1})\in U(\varepsilon)^2\\ j\geqq h}}|z_j-z_{j-1}|-$$
$$^{\#}\{j : j\geqq h,\ (z_j, z_{j+1})\in U(\varepsilon)^2\}\,\tilde{s}_{k,j_h,\cdots,j_{h+k-1}}|\leqq C_3\varepsilon,$$

where $^{\#}\{\cdot\}$ denotes the number of elements of $\{\cdot\}$.
We have

$$\begin{aligned}
&s_{m+k,j_1,\cdots,j_{h+k-1}}-s_{m,j_1,\cdots,j_{h+k-1}}-\tilde{s}_{k,j_h,\cdots,j_{h+k-1}}\\
&=\textstyle\sum_{(x_j,z_j,x_{j+1},z_{j+1})\notin U(\varepsilon)^4}(|x_j-x_{j+1}|-|z_j-z_{j+1}|)\\
&+\textstyle\sum_{\substack{(x_j,\, x_{j+1})\in U(\varepsilon)^2\\ j\geqq h}}|x_j-x_{j+1}|-\sum_{\substack{(z_j,\, z_{j+1})\in U(\varepsilon)^2\\ j\geqq h}}|z_j-z_{j+1}|\\
&-\tilde{s}_{k,j_h,\cdots,j_{h+k-1}}\\
&\equiv I_1+I_2-I_3-\tilde{s}_{k,j_h,\cdots,j_{h+k-1}}.
\end{aligned}$$

Estimate (2. 5) gives the following:

$$(2.6)\qquad |I_2-I_3-\tilde{s}_{k,j_h,\cdots,j_{h+k-1}}|\leqq C_4\varepsilon.$$

On the other hands, by using the same argument in the last part of § 2 in [6] or [7], we see that the points x_j and z_j with $(x_j, z_j)\notin U(\varepsilon)^2$ tend to the same point as $m\to+\infty$. Hence we get

$$|I_1|\leqq C_5\varepsilon$$

for sufficently large m. Combining this with (2. 6) yields the required inequality. This completes the proof.

§ 3. Sketch of the procedures in the proof of Theorem 1.

Taking Theorem 2.1 in § 2 into consideration, it remains only to prove (1. 5) in Theorem 1. To analyze the singularities of $S(\cdot, -\omega, \omega)$, we use the following representation (cf. MAJDA [5] and SOGA [10]) :

$$(3.1)\qquad S(s, \theta, \omega)=\int_{\partial\Omega}\{\nu\cdot\theta\partial_t^{n-1}v(x\cdot\theta-s, x ; \omega)-\partial_t^{n-2}\partial_\nu v(x\cdot\theta-s, x ; \omega)\}dS_x$$

$(\theta \neq \omega)$.

Here, ν denotes the unit inner normal of $\partial\Omega$, and $v(t, x, ; \omega)$ is the solution of the mixed problem

$$(3.2)\qquad \begin{cases} \square\, v=0 & \text{in } \boldsymbol{R}^1\times\Omega \\ \partial_\nu[v+2^{-1}(-2\pi i)^{1-n}\delta(t-x\cdot\omega)]=0 & \text{on } \boldsymbol{R}^1\times\partial\Omega, \\ v=0 & \text{for } t\leqq \min_{1\leqq i\leqq J} r_i(\omega). \end{cases}$$

We expect that sing supp $S(\cdot, -\omega, \omega)$ is determined by only the broken rays with the property $(*)$. It has been proved in [6] and [7] this is true when $\mathcal{O}$ consists of two disjoint convex obstacles. The procedures in the proof of (1.5) are the same as those used in [6] (for details, see § 3 in [6]), so we only sketch them. We can construct asymptotic solution of the equation (3.2) near the broken rays with the property $(*)$ in the same way as in § 7 in [1]. Using this asymptotic solution and the representation (3.1), we can reduce the proof of (1.5) in Theorem 1 to showing that the following integral does not decrease rapidly as $|\sigma|\to+\infty$.

$$(3.3)\qquad \int_{\partial\Omega} e^{i\sigma(x\cdot\omega+\phi_m^{\vec{J}}(x))}\alpha(-x\cdot\omega-\phi_m^{\vec{J}}(x)+2\min_{1\leqq i\leqq J} r_i(\omega)+s_m^{\vec{J}})\beta^{\vec{J}}(x)\,dS_x$$

where $\beta^{\vec{J}}(x)$ is a C^∞ function, $\alpha(s)$ is a cutt-off C^∞ function with small support with $\alpha(0)\neq 0$ and $\phi_m^{\vec{J}}$ is a phase function associated with the broken ray defining $s_m^{\vec{J}}$. Generally it may happen that there exists another pair $(\tilde{m}, \vec{\tilde{J}})$ which is different from $(m, \vec{J})$ but $s_m^{\vec{J}}=s_{\tilde{m}}^{\vec{\tilde{J}}}$. By the Neumann boundary condition, $\beta^{\vec{J}}(x)$ and $\beta^{\vec{\tilde{J}}}(x)$ have same sign. Hence the integrals associated with $(\tilde{m}, \vec{\tilde{J}})$ and $(m, \vec{J})$ do not have influence to each other. It is known that (3.3) does not decrease rapidly as $|\sigma|\to\infty$ alternatively using the stationary phase methods when $\mathcal{O}_i$ is strictly convex or using the results in [11] when $\mathcal{O}_i$ is convex.

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