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Weakly complex manifolds with semi-free S^1-action whose fixed point set has complex codimension 2

Dedicated to Professor Haruo Suzuki on his sixtieth birthday

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1. Introduction

A weakly complex manifold means a smooth manifold whose tangent bundle is stably equivalent to a complex vector bundle. Let M^{2n} be a $2n$-dimensional closed weakly complex manifold and let $\varphi: S^1\times M^{2n}\to M^{2n}$ be a smooth semi-free S^1-action which preserves the complex structure. We denote this manifold by the pair (M^{2n}, φ). Let $F(M^{2n}, \varphi)=F_1\cup F_2\cup\cdots\cup F_s$, where $F_i(i=1,2,\cdots,s)$ is a fixed point set component. Each F_i has an S^1-invariant weakly complex structure. Then we have the following theorem by the Kamata's formula [2].

THEOREM 1. *Let k be a positive integer and let (M^{2n}, φ) be a weakly complex semi-free S^1-manifold. Let $dim_C F_i=n-2k$ $(i=1, \cdots, s)$. Then the Chern number $c_1^n[M^{2n}]\equiv 0$ mod $(2k)^{2k}$.*

Next in this paper we study, up to mod 2 bordism, those manifolds with semi-free S^1-action with the property that all the components of the fixed point set have the same complex codimension 2.

Let $\mathscr{U}_*$ be the bordism ring of closed weakly complex smooth manifolds. It is known that the bordism ring $\mathscr{U}_*$ is generated by a set of bordism classes $\{[CP(k)], [H_{m,n}(C)]; k\geq 1, n\geq m>1\}$, where $CP(k)$ is the k dimensional complex projective space and $H_{m,n}(C)$ is the Milnor hypersurface in $CP(m)\times CP(n)$. For our purpose, we calculate a base of the mod 2 weakly complex bordism ring $\mathscr{U}_*\otimes Z_2$. Let $(n_1, n_2, \cdots, n_k)$ be a k-tuple of non negative integers. We denote by $CP(n_1, n_2, \cdots, n_k)$ the complex projective space bundle $CP(\lambda_1\oplus\lambda_2\oplus\cdots\oplus\lambda_k)$ associated to the bundle $\lambda_1\oplus\lambda_2\oplus\cdots\oplus\lambda_k$ over $CP(n_1)\times CP(n_2)\times\cdots\times CP(n_k)$, where $\lambda_i(i=1,2,\cdots, k)$ is the pullback of the canonical line bundle over the ith factor.

Now we define an ideal $\mathscr{T}$ in $\mathscr{U}_*\otimes \boldsymbol{Z}_2$ as follows.

$$\mathscr{T}=\{[M^{2n}]\in \mathscr{U}_*\otimes \boldsymbol{Z}_2 \mid c_1^n[M^{2n}]\equiv 0 \ mod\ 2\}.$$

Then we have the following

THEOREM 2. *$\mathscr{T}$ is the ideal generated by the set*

$$\{[CP(1)], [CP(2)]^2, [H_{2,2}(C)], [CP(n_1, n_2, n_3, n_4)]\},$$

where $n_1+n_2+n_3+n_4\neq 1$.

The bordism ring $\mathscr{U}_*$ is a polynomial ring with a generator in each dimension $2k$, $k>0$. We take $x_{2^j}=[CP(2^j)]$ as a ring generator of $\mathscr{U}_*\otimes \boldsymbol{Z}_2$ in dimension 2^{j+1}. We consider suitable semi-free S^1-actions on $CP(1)\times CP(1)$, $H_{2,2}(C)$ and $CP(n_1, n_2, n_3, n_4)$, and then from above two theorems we obtain the following

THEOREM 3. *Suppose that the bordism class of a weakly complex manifold M^{2n} is represented by a polynomial in $\mathscr{U}_*\otimes \boldsymbol{Z}_2$ which does not involve any type of monomial factorized with $(x_{2^{j_1}})^{\varepsilon_1}(x_{2^{j_2}})^{\varepsilon_2}\cdots(x_{2^{j_r}})^{\varepsilon_r}(x_1)^{\delta}$, $\varepsilon_i\geq 2$, $\delta=0$ or $1, j_1>j_2\cdots>j_r\geq 1$. Then there exists a weakly complex semi-free S^1-manifold (N^{2n}, φ) wich satisfies $F(N^{2n}, \varphi)=F_1\cup F_2\cup\cdots\cup F_t$, $dim_C F_i=n-2$ and $[N^{2n}]=[M^{2n}]$ in $\mathscr{U}_*\otimes \boldsymbol{Z}_2$ if and only if*

$$c_1^n[M^{2n}]\equiv 0 \ mod\ 2.$$

REMARK. Let $M^{2n}\doteq CP(2^{j+1}-3, 0, 0, 0)$ $(j\geq 2, n=2^{j+1})$. M has such a semi-free S^1-action as our thinking and $c_1^n[M^{2n}]\equiv 0$ mod 2.

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2. An application of Kamata's formula and some Chern numbers

Let (M^{2n}, φ) be a weakly complex manifold with semi-free S^1-action. Let $F(M^{2n}, \varphi)=F_1\cup F_2\cup\cdots\cup F_s$, where $F_i(i=1, 2, \cdots, s)$ is a fixed point set component. Let τ' be the complex n'-dimensional vector bundle which is stably equivalent to the tangent bundle of M^{2n} and let ν_i be the normal bundle of F_i and let τ_i' be the stable tangent bundle of F_i. Then the total Chern classes are expressed in the factored form as follows.

$$c(\tau')=\prod_{i=1}^{n'}(1+\gamma_i)$$
$$c(\nu_i)=(1+\alpha_1^{(i)})(1+\alpha_2^{(i)})\cdots(1+\alpha_{l_i}^{(i)})$$
$$c(\tau_i')=(1+\beta_1^{(i)})(1+\beta_2^{(i)})\cdots(1+\beta_{m_i}^{(i)}),$$

where $l_i=dim_C\nu_i$ and $m_i=dim_C\tau_i'$. Then we have the following

PROPOSITION 1 (M. Kamata [2]). *Let $f(z_1, \cdots, z_{n'})$ be a symmetric* *polynomial of degree n and let (M^{2n}, φ) be a weakly complex semi-free S^1-manifold. Then*

$$\langle f(\gamma_1, \cdots, \gamma_{n'}), \sigma(M)\rangle = \sum_{i=1}^{s} \langle \frac{f(1+\alpha_1^{(i)}, 1+\alpha_2^{(i)}, \cdots, 1+\alpha_{l_i}^{(i)}, \beta_1^{(i)}, \cdots, \beta_{m_i'}^{(i)})}{(1+\alpha_1^{(i)})(1+\alpha_2^{(i)})\cdots(1+\alpha_{l_i}^{(i)})}, \ \sigma(F_i)\rangle,$$

where $\sigma(M)$ and $\sigma(F_i)$ are fundamental homology classes of M^{2n} and F_i respectively.

Now we apply this formula to a weakly complex semi-free S^1-manifold whose every fixed point set component has same codimension.

PROOF OF THEOREM 1.

PROOF. $c_1(M)=\gamma_1+\cdots+\gamma_{n'}(n'\geq n)$. Applying Proposition 1 to $f(z_1, \cdots, z_{n'})=(z_1+\cdots+z_{n'})^n$, we have

$$\begin{aligned} c_1^n[M] &= \langle f(\gamma_1, \cdots, \gamma_{n'}), \sigma(M)\rangle \\ &= \sum_{i=1}^{s} \langle \frac{f(1+\alpha_1^{(i)}, 1+\alpha_2^{(i)}, \cdots, 1+\alpha_{2k}^{(i)}, \beta_1^{(i)}, \cdots, \beta_{n'_{2k}}^{(i)})}{(1+\alpha_1^{(i)})(1+\alpha_2^{(i)})\cdots(1+\alpha_{2k}^{(i)})}, \ \sigma(F_i)\rangle \\ &= \sum_{i=1}^{s} \langle \frac{(2k+\alpha_1^{(i)}+\cdots+\alpha_{2k}^{(i)}+\beta_1^{(i)}+\cdots+\beta_{n'_{2k}}^{(i)})^n}{(1+\alpha_1^{(i)})(1+\alpha_2^{(i)})\cdots(1+\alpha_{2k}^{(i)})}, \ \sigma(F_i)\rangle \\ &= \sum_{i=1}^{s} \langle (2k+c_1(\nu_i)+c_1(\tau_i'))^n \sum_{j=0}^{2k}(-1)^j c_j(\nu_i), \ \sigma(F_i)\rangle \\ &\equiv 0 \ mod \ (2k)^{2k}, \end{aligned}$$

because $dim_{C}F_i=n-2k(i=1, \cdots, s)$. *q. e. d.*

Next we calculate some Chern numbers. Let M^{2n} be a weakly complex manifold and let the total Chern class $c(M)$ be expressed in the factored form $\prod_{i=1}^{n'}(1+\gamma_i)$ as mentioned above. We denote $s_k(c_1(M), \cdots, c_n(M))=\sum_{i=1}^{n'}\gamma_i^k$, and then we define the Chern number

$$s_n[M]=\langle s_n(c_1(M), \cdots, c_n(M)), \sigma(M)\rangle.$$

We call this number s-number, and simply often denote by $s[M]$. This is a weakly complex bordism invariant and we have

PROPOSITION 2 (J. Milnor[6]). *A weakly complex manifold M^{2n} may be taken to be the $2n$-dimensional generator in $\mathscr{U}_*$ if and only if*

$$s[M]=\begin{cases} \pm 1 & \text{if } n+1\neq p^j \text{ for any prime } p \\ \pm p & \text{if } n+1=p^j \text{ for some prime } p \text{ and } j>0 \end{cases}$$

Now we obtain the following lemma (cf. Stong [5, p. 434, Lemma 3. 4]).

LEMMA 1. *For $k\geq 2$*

$$s[CP(n_1, n_2, \cdots, n_k)]=\pm\left\{(-1)^{n-n_1}\binom{n+k-2}{n_1}+\cdots+(-1)^{n-n_k}\binom{n+k-2}{n_k}\right\},$$

where $n=n_1+\cdots+n_k$.

PROOF. We put $X=CP(n_1, n_2, \cdots, n_k)$, $Y=CP(n_1)\times CP(n_2)\times\cdots\times CP(n_k)$, and $\lambda=\lambda_1\oplus\lambda_2\oplus\cdots\oplus\lambda_k$. Let $p: X\to Y$ be the projection and ξ the canonical complex line bundle over X. We shall denote by $a\in H^2(X; Z)$ the characteristic class of ξ. The total Chern class of λ can be expressed in the factored form $\prod_{i=1}^{k}(1+t_i)$. We set $u_i=p^*(t_i)$ for $i=1, \cdots, k$ and let v_j be the jth Chern class of λ, so v_j is the jth elementary symmetric function of $u_1, \cdots, u_k$. Then the total Chern class of X is given by

$$(2.1)\qquad \begin{aligned} c(X)&=p^*((c(Y))\left(\sum_{j=0}^{k}(1-a)^{k-j}p^*(v_j)\right)\\ &=\prod_{i=1}^{k}(1+u_i)^{n_i+1}\prod_{i=1}^{k}(1+u_i-a)\end{aligned}$$

with relation

$$(2.2)\qquad \sum_{j=0}^{k}(-1)^{k-j}p^*(v_j)a^{k-j}=0.$$

Now, we denote the ith dual Chern class of λ by $\bar{c}_i(\lambda)$ and we put $s_j(\lambda)=\sum_{i=1}^{k}t_i^j$. Then, from Conner's theorem [1, p. 293, (4.1)], we obtain the s-number of X as follows.

$$(2.3)\qquad s_{n+k-1}[X]=\pm(-1)^{k-1}\langle k\bar{c}_n(\lambda)+\sum_{j=1}^{n}\binom{n+k-1}{j}s_j(\lambda)\bar{c}_{n-j}(\lambda), \sigma(Y)\rangle,$$

where $\sigma(Y)$ is the fundamental homology class of Y. From this formula, we obtain the desired result. *q.e.d.*

LEMMA 2.

$$(2.4)\qquad c_1^n[CP(n_1, n_2, n_3, n_4)]=2^6 d\,(d\in Z, d\neq 0) \textit{ where } n=n_1+\cdots+n_4+3.$$

$$(2.5)\qquad c_1^{m+n-1}[H_{m,n}(C)]=\frac{2(m+n-1)!}{(m-1)!(n-1)!}m^{m-1}n^{n-1}.$$

$$(2.6)\qquad c_1^n[CP(2^{j_1})\times\cdots\times CP(2^{j_r})]=\frac{n!}{(2^{j_1})!\cdots(2^{j_r})!}(2^{j_1}+1)^{2^{j_1}}\cdots(2^{j_r}+1)^{2^{j_r}},$$

where $n=2^{j_1}+\cdots+2^{j_r}$ and $j_1\geq j_2\geq\cdots\geq j_r\geq 0$.

PROOF OF (2.4). Let $X=CP(n_1, n_2, n_3, n_4)=CP(\lambda_1\oplus\lambda_2\oplus\lambda_3\oplus\lambda_4)$, $Y=CP(n_1)\times CP(n_2)\times CP(n_3)\times CP(n_4)$ and let $\lambda=\lambda_1\oplus\lambda_2\oplus\lambda_3\oplus\lambda_4$. Let $p: X\to Y$ be the projection and let $a\in H^2(X; \boldsymbol{Z})$ be the characterisitic class of the canonical complex line bundle over X. Let $v_j\in H^{2j}(Y; \boldsymbol{Z})$ be the *jth*

Chern class of λ. Then by the formula (2.1)

$$c(X)=p^*(c(Y))\Big(\sum_{j=0}^{4}(1-a)^{4-j}p^*(v_j)\Big).$$

Now $c_1(X)=p^*(c_1(Y)+v_1)-4a$. Put $b=c_1(Y)+v_1$. Then

$$\begin{aligned} c_1^n[X]&=\langle(p^*(b)-4a)^n,\ \sigma(X)\rangle\\ &=\sum_{i=0}^{n}(-1)^{n-i}\binom{n}{i}4^{n-i}\langle(p^*(b^i)a^{n-i},\ \sigma(X)\rangle\\ &=\sum_{i=0}^{n-3}(-1)^{n-i}\binom{n}{i}4^{n-i}\langle b^i,\ p_*(a^{n-i}\cap\sigma(X))\rangle\\ &=2^6d. \end{aligned}$$

q. e. d.

Proof of (2.5). Let ξ_m and ξ_n be the canonical line budles over $CP(m)$ and $CP(n)$ respectively. Let $i: H_{m,n}(C)\to CP(m)\times CP(n)$ be the inclusion map and ν be the normal bundle. Then $c(\nu)=i^*(c(\xi_m\hat{\otimes}\xi_n))$, where $\xi_m\hat{\otimes}\xi_n$ is the outer tensor product of ξ_m and ξ_n. Since $H^*(CP(m);\boldsymbol{Z})\otimes H^*(CP(n);\boldsymbol{Z})\cong H^*(CP(m)\times CP(n);\boldsymbol{Z})$, we may identify $c_1(\xi_m\hat{\otimes}\xi_n)=\alpha+\beta$, where $\alpha=x_m\times 1$ and $\beta=1\times x_n$, $x_k=c_1(\xi_k)$: the generator of $H^2(CP(k);\boldsymbol{Z})$. On the other hand, $i^*(\tau(CP(m)\times CP(n)))=\tau(H_{m,n}(C))\oplus\nu$, therefore $c_1(H_{m,n}(C))=i^*(c_1(CP(m)\times CP(n))-c_1(\xi_m\hat{\otimes}\xi_n))=i^*((m+1)\alpha+(n+1)\beta-(\alpha+\beta))=i^*(m\alpha+n\beta)$. Let $\sigma_1=\sigma(CP(m))$ and $\sigma_2=\sigma(CP(n))$, then

$$\begin{aligned} c_1^{m+n-1}[H_{m,n}(C)]&=\langle(i^*(m\alpha+n\beta)^{m+n-1},\ \sigma(H_{m,n}(C))\rangle\\ &=\langle(m\alpha+n\beta)^{m+n-1}\cup c_1(\xi_m\hat{\otimes}\xi_n),\ \sigma(CP(m)\times CP(n))\rangle\\ &=\langle(m\alpha+n\beta)^{m+n-1}(\alpha+\beta),\ \sigma(CP(m))\times\sigma(CP(n))\rangle\\ &=\langle\{\binom{m+n-1}{m-1}(m\alpha)^{m-1}(n\beta)^{n}\\ &\qquad+\binom{m+n-1}{m}(m\alpha)^{m}(n\beta)^{n-1}\}(\alpha+\beta),\ \sigma_1\times\sigma_2\rangle\\ &=m^{m-1}n^{n-1}\{\binom{m+n-1}{m-1}n+\binom{m+n-1}{m}m\}\langle\alpha^m,\ \sigma_1\rangle\langle\beta^n,\ \sigma_2\rangle\\ &=\frac{2(m+n-1)\,!}{(m-1)\,!\,(n-1)\,!}m^{m-1}n^{n-1}. \end{aligned}$$

q. e. d.

Proof of (2.6). Let $M=CP(2^{j_1})\times\cdots\times CP(2^{j_r})$ then the total Chern class $c(M)=c(CP(2^{j_1})\times\cdots\times CP(2^{j_r}))=c(CP(2^{j_1}))\cdots c(CP(2^{j_r}))=(1+\alpha_1)^{2^{j_1}+1}\cdots(1+\alpha_r)^{2^{j_r}+1}$, where $\alpha_i=1\times\cdots\times 1\times x_{l(i)}\times 1\times\cdots\times 1$, $l(i)=2^{j_i}(1\leq i\leq r)$. Therefore $c_1(M)=(2^{j_1}+1)\alpha_1+\cdots+(2^{j_r}+1)\alpha_r$. So we have the $c_1^n[M]$ by the multinomial theorem. *q. e. d.*

3. A ring structure for $\mathscr{U}_*\otimes \boldsymbol{Z}_2$ and proofs of Theorem 2 and 3.

LEMMA 3. *The following manifolds represent the indecomposable bordism classes in the polynomial ring* $\mathscr{U}_*\otimes \boldsymbol{Z}_2$.

(1) $H_{2,2}(C)$,

(2) $CP(2^j), j\geq 0$,

(3) $CP(2^j-4, 0, 0, 0)$, $j\geq 3$,

(4) $CP(n-3, 0, 0, 0)$, $n\equiv 2$ *mod* 4,

(5) $CP(2^{p_{r-1}}, 2^{p_{r-1}}, n-2^{p_r}-3, 0)$, *where* $n=2^{p_1}+\cdots+2^{p_{r-1}}+2^{p_r}$, $r>1$, *and* $p_1>\cdots>p_r\geq 2$.

(6) $CP(2, 2q-2, 2q-2, 0)$, $q\geq 1$.

(7) $CP(2^{2+j}, 2(q-2^j), 2(q-2^j), 0)$, $q=a_0+a_1 2+\cdots+a_s 2^s$ *with* $a_j=0$ *for some* j.

PROOF. We denote such a manifold as described above by M. It is known that $s[CP(n)]=n+1$ for $n\geq 1$ and $s[H_{m,n}(C)]=-\binom{m+n}{m}$ for $1<m\leq n$ [6]. By these facts, Lemma 1 and [4, Chapter, 1, 2.6. Lemma.], we obtain $s[M]\equiv 1$ *mod* 2 for (2), (4), (5), (6) and (7). For (1) and (3), $s[M]\equiv 2$ *mod* 4. Here we apply the Milnor theorem to the mod 2 weakly complex bordism ring $\mathscr{U}_*\otimes \boldsymbol{Z}_2$, and we obtain the results. *q. e. d.*

It is known that $\mathscr{U}_*\otimes \boldsymbol{Z}_2$ is a polynomial ring over $\boldsymbol{Z}_2$ with one generator in each even dimension. Let x_{2^j} be the class $[CP(2^j)]$ for $j\geq 0$ and let x_3 be the class $[H_{2,2}(C)]$. Denote y_n be the class $[CP(2^j-4, 0, 0, 0)]$ for $n=2^j-1, j\geq 3$ and let z_n be the class $[CP(n_1, n_2, n_3, n_4)]$ for $n=n_1+n_2+n_3+n_4+3\neq 2^j, 2^j-1$ whose types are (5), (6) or (7) of Lemma 3. Then we have the following proposition by Lemma 3.

LEMMA 4. $\mathscr{U}_*\otimes \boldsymbol{Z}_2$ *is a polynomial ring over* $\boldsymbol{Z}_2$ *with the system of generators*

$$\{x_3, x_{2^j}(j\geq 0), y_n(n=2^j-1, j\geq 3), z_n(n\neq 2^j, 2^j-1)\}.$$

PROOF OF THEOREM 2.

ASSERTION 1. *We define ideal* $\mathscr{T}_1$ *in* $\mathscr{U}_*\otimes \boldsymbol{Z}_2$ *is generated by the set*

$$\{x_1, x_3, (x_{2^j})^2(j\geq 1), y_n(n=2^j-1, j\geq 3), z_n(n\neq 2^j, 2^j-1)\}.$$

Then $\mathscr{T}=\mathscr{T}_1$.

PROOF. We have $\mathscr{T}\supset\mathscr{T}_1$ from Lemma 2. If an element $[M]$ is chosen from $\mathscr{T}$, then we express $[M^{2n}]=\sum a_{i_1\cdots i_r}u_{i_1}\cdots u_{i_r}$, where $a_{i_1\cdots i_r}\in \boldsymbol{Z}_2$ and u_{i_k} is a generator of $\mathscr{U}_*\otimes \boldsymbol{Z}_2$ as described in Lemma 4. As $c_1^n[M]\equiv 0$ *mod* 2, the coefficients of $x_{2^{j_1}}x_{2^{j_2}}\cdots x_{2^{j_r}}(j_1>j_2>\cdots>j_r\geq 1)$ are equal to 0 *mod* 2.

Therefore $[M]\in\mathcal{T}_1$, hence $\mathcal{T}\subset\mathcal{T}_1$. Thus $\mathcal{T}=\mathcal{T}_1$. *q. e. d.*

ASSERTION 2. *We can turn the generator* $(x_{2^j})^2$ *of* $\mathcal{T}_1$ *into* $y'_{2^{j+1}}=[CP(2^{j+1}-3, 0, 0, 0)]$ for $j\geq 2$.

PROOF. The characteristic number $s_{(2^j,2^j)}[CP(2^j)\times CP(2^j)]=s_{2^j}[CP(2^j)]\times s_{2^j}[CP(2^j)]=(2^j+1)(2^j+1)\equiv 1$ *mod* 2. On the other hand $s_{(2^j,2^j)}[CP(2^{j+1}-3, 0, 0, 0)]\equiv c_2^{2^j}[CP(2^{j+1}-3, 0, 0, 0)]$ *mod* 2. We set $X=CP(2^{j+1}-3, 0, 0, 0)]$. By (2.1), $c_2(X)\equiv u^2+au$ *mod* 2, where $u=p^*(t_1)$. For $j\geq 2$, $c_2^{2^j}(X)\equiv a^{2^j}u^{2^j}$ *mod* $2=a^3u^{2^{j+1}-3}$ because by (2.2) $a^4=a^3u$. Then $S_{(2^j,2^j)}[X]\equiv\langle a^3u^{2^{j+1}-3}, \sigma(X)\rangle$ *mod* $2=\langle p^*(t_1^{2^{j+1}-3}), a^3\cap\sigma(X)\rangle=\langle t_1^{2^{j+1}-3}, p_*(a^3\cap\sigma(X)\rangle\equiv\langle t_1^{2^{j+1}-3}, \sigma(Y)\rangle$ *mod* $2=1$, where $Y=CP(2^{j+1}-3)$. Hence we can turn the generator $(x_{2^j})^2$ into $y'_{2^{j+1}}$ for $j\geq 2$. Therefore we obtain the Theorem 2 from these assertions. *q. e. d.*

PROOF OF THEOREM 3.
Let $\varphi_1: S^1\times CP(n_1, n_2, n_3, n_4)\rightarrow CP(n_1, n_2, n_3, n_4)$ be $\varphi_1(\zeta, [u_1, u_2, u_3, u_4])=[u_1, u_2, \zeta u_3, \zeta_{u4}]$ for any $\zeta\in S^1$ and $[u_1, u_2, u_3, u_4]\in CP(n_1, n_2, n_3, n_4)$. Let $\varphi_2: S^1\times H_{2,2}(C)\rightarrow H_{2,2}(C)$ be $\varphi_2(\zeta, ([z_0:z_1:z_2], [w_0:w_1:w_2]))=([z_0:z_1:\zeta z_2], [w_0:w_1:\bar{\zeta}w_2])$ for any $\zeta\in S^1$ and $([z_0:z_1:z_2], [w_0:w_1:w_2])\in H_{2,2}(C)$, where $\bar{\zeta}$ is conjugate of ζ. Then φ_1 and φ_2 are semi-free S^1-actions whose fixed point sets are $CP(\lambda_1\oplus\lambda_2)\cup CP(\lambda_3\oplus\lambda_4)$ and $CP(1)\cup CP(1)\cup H_{1,1}(C)$ respectively. The dimension of those fixed point sets are complex codimension 2. Moreover $(CP(1))^2$ has also natural diagonal semi-free S^1-action whose fixed point set has complex codimension 2. So we obtain the result from Theorem 1 and 2. *q. e. d.*

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