Hokkaido Mathematical Journal Vol. 39 (2010) p. 115–126

Some weak-type estimates for singular integral operators on CMO spaces

Dedicated to Professor Yuichi Kanjin on his sixtieth birthday

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(Received December 15, 2008; Revised May 28, 2009)

Abstract. We introduce a "weak CMO" space $WCMO$ and consider the boundedness of the singular integral operators from CMO to $WCMO$. Our result is the best possible.

Key words: B^p space, CMO space, weak CMO space, standard singular integral operator.

1. Introduction

Since A. Beurling [B] introduced the Beurling algebras and their dual spaces $B^p(\mathbb{R}^n)$, many studies have been done for these spaces (see, for example, [AGL], [CL], [FW], [G], [GH], [H], [K], [KM], [LY] and [M]). The B^p space $B^p(\mathbb{R}^n)$ is a special case of the non-homogeneous Herz spaces $K_p^{\alpha,r}(\mathbb{R}^n)$, introduced by C. Herz [H], precisely speaking $B^p(\mathbb{R}^n) = K_p^{-n/p,\infty}(\mathbb{R}^n)$ (see [GH]). However the singular integral operators are bounded on $K_p^{\alpha,r}(\mathbb{R}^n)$, where $-n/p < \alpha < n(1-1/p)$, $1 < p < \infty$ and $0 < r \le \infty$ (see [LY]). Therefore, when we consider the boundedness of the singular integral operators on $B^p(\mathbb{R}^n)$, we can't use the general theory about the Herz spaces.

Y. Chen and K. Lau [CL] and J. García-Cuerva [G] introduced the CMO spaces $CMO^p(\mathbb{R}^n)$, which are the dual spaces of the Beurling-type Hardy spaces, and proved that the singular integral operators are bounded from $B^p(\mathbb{R}^n)$ to $CMO^p(\mathbb{R}^n)$, where $1 < p < \infty$ (see also [AGL]). They considered the case $1 < p < \infty$, only.

In this paper, we will consider the boundedness of the singular integral operators on $B^1(\mathbb{R}^n)$. But, the singular integral operators are not bounded

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2000 Mathematics Subject Classification : 42B20, 42B35.

*The author has been partially supported by grant 2007/2008 Overseas Research of College of Economics, Nihon University, Japan.

from $B^1(\mathbb{R}^n)$ to $CMO^1(\mathbb{R}^n)$ (see Counterexample 1 in Section 5). Therefore, for our purpose, we will introduce a new function space "weak CMO", i.e. $WCMO^1(\mathbb{R}^n)$, and prove the boundedness from $B^1(\mathbb{R}^n)$ to $WCMO^1(\mathbb{R}^n)$.

Note that for the homogeneous spaces, we have the corresponding results to those obtained in this paper. But, since their proofs are similar to those of the results in this paper, we omit the details.

2. Definitions and results

First, following Y. Chen and K. Lau [CL] and J. García-Cuerva [G], we define a B^p space $B^p(\mathbb{R}^n)$ and a CMO space $CMO^p(\mathbb{R}^n)$.

Definition 1 For $1 \le p < \infty$,

$$B^p(\mathbb{R}^n) = \{ f \in L^p_{loc}(\mathbb{R}^n) : \|f\|_{B^p} < \infty \},$$

where

$$\|f\|_{B^p} = \sup_{R\ge 1} \left(\frac{1}{|B(0,R)|} \int_{B(0,R)} |f(x)|^p dx \right)^{1/p}.$$

Here (and in the following), we denote the open ball in $\mathbb{R}^n$, having center 0 and radius $R > 0$, by $B(0,R)$ and the Lebesgue measure of a measurable set E by $|E|$.

Definition 2 For $1 \le p < \infty$,

$$CMO^p(\mathbb{R}^n) = \{ f \in L^p_{loc}(\mathbb{R}^n) : \|f\|_{CMO^p} < \infty \},$$

where

$$\|f\|_{CMO^p} = \sup_{R\ge 1} \inf_{c} \left(\frac{1}{|B(0,R)|} \int_{B(0,R)} |f(x) - c|^p dx \right)^{1/p}.$$

Note that for $1 \le p < \infty$,

$$L^\infty(\mathbb{R}^n) \subset B^p(\mathbb{R}^n) \subset CMO^p(\mathbb{R}^n).$$

Secondly, we define a new function space, i.e. a "weak CMO" space $WCMO^1(\mathbb{R}^n)$.

Definition 3

$$WCMO^1(\mathbb{R}^n) = \big\{ f \in L^1_{loc}(\mathbb{R}^n) : \|f\|_{WCMO^1} < \infty \big\},$$

where

$$\|f\|_{WCMO^1} = \sup_{R\geq 1} \frac{1}{|B(0,R)|} \inf_c \sup_{\lambda>0} \lambda \big| \{ x \in B(0,R) : |f(x) - c| > \lambda \} \big|.$$

Next, we define a standard singular integral operator T and its modified singular integral operator $\widetilde{T}$.

Definition 4 We say that T is a standard singular integral operator, if there exists a function K which satisfies the following conditions:

$$Tf(x) = \text{p.v.} \int_{\mathbb{R}^n} K(x-y) f(y) dy$$

exists almost everywhere, where $f \in L^2(\mathbb{R}^n)$;

$$|K(x)| \leq \frac{C_K}{|x|^n} \quad \text{and} \quad |\nabla K(x)| \leq \frac{C_K}{|x|^{n+1}}, \quad x \neq 0;$$

$$\int_{\epsilon<|x|<N} K(x) dx = 0 \ \text{ for all } \ 0 < \epsilon < N.$$

Remark We can weaken the conditions of Definition 4, but we assume these conditions for the simplicity.

Definition 5 For a standard singular integral operator T, we define the modified singular integral operator $\widetilde{T}$ as follows:

$$\widetilde{T} f(x) = \text{p.v.} \int_{\mathbb{R}^n} \big\{ K(x-y) - K(-y) \chi_{\{y \in \mathbb{R}^n : |y| \geq 1\}} \big\} f(y) dy,$$

where χ_E is the characteristic function of a set E.

Note that for every $f \in \bigcup_{1 \leq p < \infty} CMO^p(\mathbb{R}^n)$, the modified operator $\widetilde{T}$ of the standard singular integral operator T is well-defined, because by Lemma 2 of Section 3,

$$\int_{|y|\geq 1} \frac{|f(y)|}{|y|^{n+1}} dy < \infty$$

holds. And also note that for $f \in L^2(\mathbb{R}^n) \cap L^\infty(\mathbb{R}^n)$, $\widetilde{T}f(x) = Tf(x) + C_f$ a.e., where C_f is a constant.

Now, concerning the boundedness of the Hardy-Littlewood maximal operator M, we will recall that Y. Chen and K. Lau [CL] and J. García-Cuerva [G] showed the following theorem.

Definition 6 The operator M is called the Hardy-Littlewood maximal operator, if for any measurable function f on $\mathbb{R}^n$, $Mf(x)$ is defined by

$$Mf(x) = \sup_{x \in B} \frac{1}{|B|} \int_B |f(y)| dy,$$

where the supremum is taken over all open balls $B \subset \mathbb{R}^n$ containing x.

Theorem A *If $1 < p < \infty$, then the Hardy-Littlewood maximal operator M is bounded on $B^p(\mathbb{R}^n)$, that is for $f \in B^p(\mathbb{R}^n)$,*

$$\|Mf\|_{B^p} \leq C\|f\|_{B^p}.$$

On the contrary, when $1 < p < \infty$, for a standard singular integral operator T, $\widetilde{T}$ is not bounded on $B^p(\mathbb{R}^n)$, but Y. Chen and K. Lau [CL] and J. García-Cuerva [G] proved the following theorem.

Theorem B *If $1 < p < \infty$ and T is a standard singular integral operator, then the modified singular integral operator $\widetilde{T}$ is bounded from $B^p(\mathbb{R}^n)$ to $CMO^p(\mathbb{R}^n)$, that is for $f \in B^p(\mathbb{R}^n)$,*

$$\|\widetilde{T}f\|_{CMO^p} \leq C\|f\|_{B^p}.$$

On the other hand, for a standard singular integral operator T, in general, $\widetilde{T}$ is not bounded from $B^1(\mathbb{R}^n)$ to $CMO^1(\mathbb{R}^n)$ (see Counterexample 1 in Section 5). But we can show the boundedness of $\widetilde{T}$ from $B^1(\mathbb{R}^n)$ to $WCMO^1(\mathbb{R}^n)$. Moreover, we can obtain the following weak-type estimate of $\widetilde{T}$ on $CMO^1(\mathbb{R}^n)$. This is the most important reason why we introduce a new function space $WCMO^1(\mathbb{R}^n)$.

Theorem 1 *If T is a standard singular integral operator, then the modified* *singular integral operator $\widetilde{T}$ is bounded from $CMO^1(\mathbb{R}^n)$ to $WCMO^1(\mathbb{R}^n)$, that is for $f \in CMO^1(\mathbb{R}^n)$,*

$$\|\widetilde{T}f\|_{WCMO^1} \leq C\|f\|_{CMO^1}.$$

Furthermore, we can generalize Theorem 1. For this purpose, we define new function spaces $CMO^1_q(\mathbb{R}^n)$ and $WCMO^1_q(\mathbb{R}^n)$ (cf. [GH]).

Definition 7 For $0 < q \leq 1$,

$$CMO^1_q(\mathbb{R}^n) = \{f \in L^1_{loc}(\mathbb{R}^n) : \|f\|_{CMO^1_q} < \infty\},$$

where

$$\|f\|_{CMO^1_q} = \sup_{R\geq 1} \inf_c |B(0,R)|^{-1/q} \int_{B(0,R)} |f(x) - c|dx.$$

Definition 8 For $0 < q \leq 1$,

$$WCMO^1_q(\mathbb{R}^n) = \{f \in L^1_{loc}(\mathbb{R}^n) : \|f\|_{CMO^1_q} < \infty\},$$

where

$$\|f\|_{WCMO^1_q} = \sup_{R\geq 1} |B(0,R)|^{-1/q} \inf_c \sup_{\lambda>0} \lambda \big|\{x \in B(0,R) : |f(x) - c| > \lambda\}\big|.$$

Note that, in particular, when $q = 1$,

$$CMO^1_1(\mathbb{R}^n) = CMO^1(\mathbb{R}^n) \quad \text{and} \quad WCMO^1_1(\mathbb{R}^n) = WCMO^1(\mathbb{R}^n).$$

Then, we can show the following weak-type estimate of the boundedness of the modified singular integral operator $\widetilde{T}$ on $CMO^1_q(\mathbb{R}^n)$.

Theorem 2 *If $n/(n+1) < q \leq 1$ and T is a standard singular integral operator, then the modified singular integral operator $\widetilde{T}$ is bounded from $CMO^1_q(\mathbb{R}^n)$ to $WCMO^1_q(\mathbb{R}^n)$, that is for $f \in CMO^1_q(\mathbb{R}^n)$,*

$$\|\widetilde{T}f\|_{WCMO^1_q} \leq C\|f\|_{CMO^1_q}.$$

Remark In Theorem 2, the condition $n/(n+1) < q$ is natural. See

Counterexample 2 in Section 5.

3. Proof of Theorem 2

Throughout this section, we set for $j \in \mathbb{N}$, $2^j B = B(0, 2^j R)$ and for any open ball $Q \subset \mathbb{R}^n$,

$$f_Q = \frac{1}{|Q|} \int_Q f(x)dx.$$

In order to prove Theorem 2, we need the following two lemmas.

Lemma 1 *If T is a standard singular integral operator, then T is bounded from $L^1(\mathbb{R}^n)$ to $L^{1,\infty}(\mathbb{R}^n)$.*

This lemma is well-known (see, for example, [T, p. 290]).

Lemma 2 *Let $0 < q \le 1$ and $f \in CMO^1_q$. Then,*

$$\int_{2^{j+1}B} |f(y) - f_{2B}|\, dy \le Cj|2^{j+1}B|^{1/q} \|f\|_{CMO^1_q} \quad (j = 1, 2, 3, \dots).$$

Proof. First, note that

$$|2^j B|^{-1/q} \int_{2^j B} |f(x) - f_{2^j B}| dx \le 2\|f\|_{CMO^1_q}.$$

Thus, we have

$$\begin{aligned} &\int_{2^{j+1}B} |f(y) - f_{2B}| dy \\ &\quad \le \int_{2^{j+1}B} |f(y) - f_{2^{j+1}B}| dy + |2^{j+1}B| \sum_{k=1}^{j} |f_{2^k B} - f_{2^{k+1}B}| \\ &\quad \le 2 \cdot |2^{j+1}B|^{1/q} \|f\|_{CMO^1_q} + C\,|2^{j+1}B| \sum_{k=1}^{j} |2^{k+1}B|^{1/q-1} \|f\|_{CMO^1_q} \\ &\quad \le \begin{cases} C\,j\,|2^{j+1}B|^{1/q} \|f\|_{CMO^1_q}, & \text{if} \quad q = 1, \\ C\,|2^{j+1}B|^{1/q} \|f\|_{CMO^1_q}, & \text{if} \quad 0 < q < 1. \end{cases} \end{aligned}$$

□

Now we are ready to prove Theorem 2.

Proof of Theorem 2. Let $n/(n+1) < q \le 1$, and take $R \ge 1$.

First of all, we shall show that for $x \in B(0,R)$,

$$\text{p.v.} \int_{\mathbb{R}^n} \big\{K(x-y) - K(-y)\chi_{|y|\ge 1}\big\} dy = 0. \tag{$*$}$$

Now, take N sufficiently large. Then,

$$\begin{aligned}
&\text{p.v.} \int_{\mathbb{R}^n} \big\{K(x-y) - K(-y)\chi_{|y|\ge 1}\big\} dy \\
&\quad = \text{p.v.} \int_{|x-y|\le N} \big\{K(x-y) - K(-y)\chi_{|y|\ge 1}\big\} dy \\
&\qquad + \int_{|x-y|>N} \{K(x-y) - K(-y)\} dy \\
&\quad = -\int_{|x-y|\le N} K(-y)\chi_{|y|\ge 1} dy + \int_{|x-y|>N} \{K(x-y) - K(-y)\} dy.
\end{aligned}$$

Moreover, since

$$\begin{aligned}
&\left| \int_{|x-y|\le N} K(-y)\chi_{|y|\ge 1} dy \right| \\
&\quad = \left| \int_{|x-y|\le N} K(-y)\chi_{|y|\ge 1} dy - \int_{|y|\le N-|x|} K(-y)\chi_{|y|\ge 1} dy \right| \\
&\quad \le \int_{N-|x|\le |y|\le N+|x|} \frac{C}{|y|^n} dy \to 0, \quad \text{when} \quad N \to \infty,
\end{aligned}$$

and

$$\begin{aligned}
&\int_{|x-y|>N} |K(x-y) - K(-y)| dy \\
&\qquad \le C \int_{|x-y|>N} \frac{|x|}{|x-y|^{n+1}} dy \to 0, \quad \text{when} \quad N \to \infty,
\end{aligned}$$

we can get $(*)$.

Applying $(*)$, we have for $x \in B(0,R)$,

$$
\begin{aligned}
\widetilde{T}f(x) &= \text{p.v.}\int_{\mathbb{R}^n} \left\{K(x-y) - K(-y)\chi_{|y|\geq 1}\right\} f(y)dy \\
&= \text{p.v.}\int_{\mathbb{R}^n} \left\{K(x-y) - K(-y)\chi_{|y|\geq 1}\right\} (f(y)-f_{2B})dy \\
&= \text{p.v.}\int_{\mathbb{R}^n} \left\{K(x-y) - K(-y)\chi_{|y|\geq 1}\right\} (f(y)-f_{2B})\chi_{2B}(y)dy \\
&\quad + \text{p.v.}\int_{\mathbb{R}^n} \left\{K(x-y) - K(-y)\chi_{|y|\geq 1}\right\} (f(y)-f_{2B})\chi_{\mathbb{R}^n\setminus 2B}(y)dy \\
&= T\big((f-f_{2B})\chi_{2B}\big)(x) - \int_{\mathbb{R}^n} K(-y)\chi_{|y|\geq 1}(f(y)-f_{2B})\chi_{2B}(y)dy. \\
&\quad + \int_{|y|\geq 2R} \{K(x-y)-K(-y)\}(f(y)-f_{2B})dy.
\end{aligned}
$$

Hence, putting

$$
C_R = -\int_{\mathbb{R}^n} K(-y)\chi_{|y|\geq 1}\left(f(y)-f_{2B}\right)\chi_{2B}(y)dy,
$$

it follows that

$$
\begin{aligned}
&\sup_{\lambda>0} 2\lambda \left|\left\{x \in B(0,R) : |\widetilde{T}f(x) - C_R| > 2\lambda\right\}\right| \\
&\leq 2\Bigg\{\sup_{\lambda>0} \lambda \left|\left\{x \in B(0,R) : |T((f-f_{2B})\chi_{2B})(x)| > \lambda\right\}\right| \\
&\qquad + \sup_{\lambda>0} \lambda \Bigg|\Bigg\{x \in B(0,R) : \\
&\qquad\qquad \left|\int_{|y|\geq 2R} \{K(x-y)-K(-y)\}(f(y)-f_{2B})dy\right| > \lambda\Bigg\}\Bigg|\Bigg\} \\
&= 2(I_1 + I_2), \quad \text{say.}
\end{aligned}
$$

First, to estimate I_1, we apply Lemma 1. Then,

$$I_1 \le C\big\|(f - f_{2B})\chi_{2B}\big\|_{L^1} = C\int_{2B} |f(x) - f_{2B}|dx$$

$$\le C|B(0,R)|^{1/q}\|f\|_{CMO_q^1}.$$

Therefore, we obtain

$$|B(0,R)|^{-1/q} I_1 \le C\|f\|_{CMO_q^1}.$$

Next, we estimate I_2. Using the condition of K, Lemma 2 and the assumption $n/(n+1) < q$, it follows that for $x \in B(0,R)$,

$$\Big|\int_{|y|>2R} \{K(x-y) - K(-y)\}(f(y) - f_{2B})dy\Big|$$

$$\le \sum_{j=1}^{\infty} \int_{2^{j+1}B\setminus 2^jB} |K(x-y) - K(-y)||f(y) - f_{2B}|dy$$

$$\le C\sum_{j=1}^{\infty} \int_{2^{j+1}B\setminus 2^jB} \frac{|x|}{|y|^{n+1}}|f(y) - f_{2B}|dy$$

$$\le C\sum_{j=1}^{\infty} \frac{R}{(2^jR)^{n+1}} \int_{2^{j+1}B} |f(y) - f_{2B}|dy$$

$$\le C\sum_{j=1}^{\infty} \frac{Rj}{(2^jR)^{n+1}} |2^{j+1}B|^{1/q}\|f\|_{CMO_q^1}$$

$$\le C\cdot R^{n(1/q-1)}\|f\|_{CMO_q^1}.$$

Therefore,

$$|B(0,R)|^{-1/q} I_2 \le C|B(0,R)|^{-1/q} R^{n(1/q-1)}\|f\|_{CMO_q^1}|B(0,R)| \le C\|f\|_{CMO_q^1}.$$

Thus,

$$|B(0,R)|^{-1/q}\sup_{\lambda>0}\lambda\big|\{x\in B(0,R) : |\widetilde{T}f(x) - C_R| > \lambda\}\big| \le C\|f\|_{CMO_q^1}.$$

This proves the theorem. □

4. Remark

We can also prove the strong boundedness of the modified singular integral operator $\widetilde{T}$ on the following function space $CMO_q^p(\mathbb{R}^n)$.

Definition 9 For $1 \le p < \infty$ and $0 < q \le 1$,

$$CMO_q^p(\mathbb{R}^n) = \left\{ f \in L_{loc}^p(\mathbb{R}^n) : \|f\|_{CMO_q^p} < \infty \right\},$$

where

$$\|f\|_{CMO_q^p} = \sup_{R \ge 1} \inf_c |B(0,R)|^{1-1/q} \left(\frac{1}{|B(0,R)|} \int_{B(0,R)} |f(x) - c|^p dx \right)^{1/p}.$$

Theorem 3 *If $1 < p < \infty$, $n/(n+1) < q \le 1$ and T is a standard singular integral operator, then the modified singular integral operator $\widetilde{T}$ is bounded on $CMO_q^p(\mathbb{R}^n)$, that is for $f \in CMO_q^p(\mathbb{R}^n)$,*

$$\|\widetilde{T} f\|_{CMO_q^p} \le C \|f\|_{CMO_q^p}.$$

Proof. In the proof of Theorem 2, use the L^p boundedness of T to estimate I_1. □

5. Counterexamples

By virtue of some counterexamples, we shall show that our results are the best possible.

For the simplicity, we consider the Hilbert transform H, i.e.

$$Hf(x) = \text{p.v.} \int_{-\infty}^{\infty} \frac{f(y)}{x-y} dy.$$

Counterexample 1 *The modified Hilbert transform $\widetilde{H}$ is not bounded from $B^1(\mathbb{R}^1)$ to $CMO^1(\mathbb{R}^1)$.*

Proof. Let

$$f(x) = \frac{1}{(1-x)(\log(1-x))^2} \chi_{\{x \in \mathbb{R}^1 : 1/2 < x < 1\}}(x).$$

Then, $f \in B^1(\mathbb{R}^1)$.

Now, for $1 < x < 3/2$, putting $x = 1 + h$ (i.e. $0 < h < 1/2$), then

$$|\widetilde{H}f(x)| \geq \frac{1}{2h}\int_{1-h}^{1} f(y)dy = \frac{1}{2h|\log h|}.$$

Thus, we obtain

$$\int_1^{3/2} |\widetilde{H}f(x)|dx = \infty,$$

i.e. $\widetilde{H}f \notin CMO^1(\mathbb{R}^1)$. □

Counterexample 2 *When $0 < q \leq 1/2$, the modified Hilbert transform $\widetilde{H}$ is not defined on $CMO^1_q(\mathbb{R}^1)$.*

Proof. Let

$$f(x) = \sum_{k=1}^{\infty} 2^{k(-1+1/q)} \chi_{\{x \in \mathbb{R}^1 : 2^{k-1} < x \leq 2^k\}}(x).$$

Then, $f \in CMO^1_q(\mathbb{R}^1)$.

Now, for $1/2 < x < 1$,

$$|\widetilde{H}f(x)| \geq \int_1^{\infty} \frac{x}{y^2} f(y)dy \geq C\sum_{k=1}^{\infty} 2^{k(-2+1/q)}.$$

Thus, by the assumption $q \leq 1/2$, we have

$$|\widetilde{H}f(x)| = \infty.$$ □

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