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Domains with a Contracting Automorphism at a Boundary Point

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1. Introduction

The aim of this paper is to classify smoothly bounded pseudoconvex domains with a contracting automorphism at a boundary point. According to Kim–Yoccoz [10], this is the same as to study the smoothly bounded realization of weighted homogeneous models. Let us consider the complex Euclidean space $\mathbb{C}^{n+1}$ with the standard coordinates $(w, z) = (w, z_1, \dots, z_n)$. By a *weight* to the vector z, we mean an n-tuple $\delta = (\delta_1, \dots, \delta_n)$ of nonnegative real numbers. Given the weight δ, the *total degree* of the monomial $z^\alpha z^{\bar\beta} = z_1^{\alpha_1} \cdots z_n^{\alpha_n} \bar z_1^{\beta_1} \cdots \bar z_n^{\beta_n}$ of multi-indices $\alpha = (\alpha_1, \dots, \alpha_n)$ and $\beta = (\beta_1, \dots, \beta_n)$ is defined by $\delta(\alpha + \beta) = \sum_{j=1}^n \delta_j(\alpha_j + \beta_j)$. We say that a polynomial Q in z, $\bar z$ is *weighted homogeneous* if each monomial of Q has the same total degree for the given weight δ, that is, Q can be written as $Q(z, \bar z) = \sum_{\delta(\alpha+\beta)=\mu} Q_{\alpha\bar\beta} z^\alpha z^{\bar\beta}$ for some complex numbers $Q_{\alpha\bar\beta}$. A weighted homogeneous polynomial Q is said to be *balanced* if $Q_{\alpha\bar\beta} \neq 0$ only for (α, β) with $\delta(\alpha) = \delta(\beta)$.

A *weighted homogeneous model* is a domain in $\mathbb{C}^{n+1}$ defined by

$$M_P = \{(w, z) \in \mathbb{C} \times \mathbb{C}^n : \ \operatorname{Re} w + P(z, \bar z) < 0\}, \tag{1.1}$$

where P is a weighted homogeneous polynomial of total degree 1 for a weight $\delta = (\delta_1, \dots, \delta_n)$. Each weighted homogeneous model M_P admits the *dilation,*

$$\mathcal{D}_t(w, z) = (e^t w, e^{\delta_1 t} z_1, \dots, e^{\delta_n t} z_n) \quad (t \in \mathbb{R}),$$

and the *translation,*

$$\mathcal{T}_t(w, z) = (w + it, z) \quad (t \in \mathbb{R}),$$

as its automorphisms. Thus M_P has a noncompact automorphism group. Simultaneously, the dilation $\mathcal{D}_t$ with $t \neq 0$ extends to the CR automorphism of the boundary ∂M_P which is contracting or dilating at the origin.

In this paper, we shall prove the following theorem.

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THEOREM 1.1. *Let Ω be a smoothly bounded pseudoconvex domain in $\mathbb{C}^{n+1}$. Suppose that there exists $f \in \mathrm{Aut}(\Omega) \cap \mathrm{Diff}(\overline{\Omega})$ such that $f|_{\partial\Omega}$ and $f^{-1}|_{\partial\Omega}$ are CR contractions of $\partial\Omega$. Then Ω is biholomorphic to a weighted homogeneous model M_P with a weight $\delta = (\delta_1, \dots, \delta_n)$ such that*

(i) *P is plurisubharmonic and balanced,*
(ii) *$P(z, \bar{z}) \geq 0$ for any $z \in \mathbb{C}^n$ and $P(z, \bar{z}) = 0$ only if $z = 0$,*
(iii) *there is a positive integer m_j with $\delta_j = 1/2m_j$ for each $j = 1, \dots, n$.*

Here a CR contraction of a CR manifold H means a CR mapping $f : H \to H$ leaving a point $p \in H$ fixed such that each eigenvalue of $df_p : \mathbb{C} \otimes T_pH \to \mathbb{C} \otimes T_pH$ has a modulus strictly smaller than 1. Note that each M_P in Theorem 1.1 admits the *Cayley transform,*

$$(w, z) \mapsto \left(\frac{1+w}{1-w}, \frac{2^{1/m_1} z_1}{(1-w)^{1/m_1}}, \dots, \frac{2^{1/m_n} z_n}{(1-w)^{1/m_n}} \right),$$

which is a biholomorphism from M_P to the smoothly bounded domain $E_P = \{(w, z) \in \mathbb{C} \times \mathbb{C}^n : \ |w|^2 + P(z, \bar{z}) < 1\}$. For the biholomorphic equivalence between weighted homogeneous models, see Coupet–Pinchuk [4].

According to a result of S.-Y. Kim [12], we also have the following.

COROLLARY 1.2. *Let Ω be a smoothly bounded pseudoconvex domain in $\mathbb{C}^{n+1}$. Suppose that the Bergman kernel of Ω extends to $\overline{\Omega} \times \overline{\Omega}$ minus the boundary diagonal set as a locally bounded function. If there is an automorphism orbit $\varphi_\nu(q)$ with $\varphi_\nu(q) \to p_1$ and $\varphi_\nu^{-1}(q) \to p_2$ as $\nu \to \infty$ for some different $p_1, p_2 \in \partial\Omega$, then Ω is biholomorphic to a weighted homogeneous model M_P as in Theorem 1.1.*

The research of this paper pertains to the classification program of smoothly bounded domains with a noncompact automorphism group. For the general description of the program, see [6; 9]. The general scheme for the classification which has been developed by E. Bedford and S. Pinchuk as in [1; 2; 3] involves two steps. The first step is to construct a biholomorphism by the scaling method from the initial domain Ω with noncompact $\mathrm{Aut}(\Omega)$ to a certain unbounded model domain invariant under the action by the translation $\{\mathcal{T}_t\}$. However, the unbounded model after scaling is not unique and generically has no smoothly bounded realization. The second step is to analyze the geometry of the parabolic fixed point of Ω under the holomorphic vector field corresponding to the infinitesimal generator of $\{\mathcal{T}_t\}$. Then one can determine which model is biholomorphic to the initial Ω. In [3], the analysis of the parabolic fixed point is systematically generalized in terms of tangential polynomial vector fields on the weighted homogeneous model.

However, the scaling method cannot be applied to general smoothly bounded domains. An alternative approach is to classify smooth CR manifolds with a contracting CR automorphism as studied in [8; 11; 10]. In their paper [10], K.-T. Kim and J.-P. Yoccoz show that a smoothly bounded domain with an automorphism inducing a CR contraction at a boundary point is biholomorphic to a weighted homogeneous model M_P. Theorem 1.1 partly completes their classification.

In order to prove Theorem 1.1 in Section 3, we shall follow the Bedford–Pinchuk scheme. As the first step, we use Kim–Yoccoz [10] in place of the scaling method. Then, in order to complete the proof, we apply the Bedford–Pinchuk theory [3] which is summarized as Theorem 2.1 in Section 2.

2. Holomorphic Vector Fields on Weighted Homogeneous Models

For a domain Ω, we denote by $\mathfrak{aut}(\Omega)$ the algebra of complete holomorphic vector fields on Ω, which corresponds to the Lie algebra of the automorphism group $\mathrm{Aut}(\Omega)$. In this section, we introduce the graded Lie algebra structure of $\mathfrak{aut}(M_P)$ for the weighted homogeneous model M_P as in Bedford–Pinchuk [3].

For convenience, we especially set $\delta_0 = 1$, the weight to w. Then we can also consider a weighted homogeneous polynomial of variables w, z, $\bar{w}$, $\bar{z}$. For instance, the defining function $\mathrm{Re}\, w + P(z, \bar{z})$ in (1.1) is a weighted homogeneous polynomial of total degree 1.

2.1. Infinitesimal Automorphisms

A holomorphic vector field X locally defined on $\mathbb{C}^{n+1}$ is called a *polynomial vector field* if in the expression $X = X_0 \partial/\partial w + \sum_{j=1}^{n} X_j \partial/\partial z_j$, each holomorphic function X_a $(a = 0, \ldots, n)$ is a polynomial in w and z. It is a global vector field on $\mathbb{C}^{n+1}$. For a weighted homogeneous model M_P with the weight $\delta = (\delta_1, \ldots, \delta_n)$, the polynomial vector fields

$$\mathcal{D} = w\frac{\partial}{\partial w} + \sum_{j=1}^{n} \delta_j z_j \frac{\partial}{\partial z_j}, \qquad \mathcal{T} = i\frac{\partial}{\partial w}$$

belong to $\mathfrak{aut}(M_P)$ which infinitesimally generate $\mathcal{D}_t$ and $\mathcal{T}_t$, respectively. Given $\mathcal{D}$ associated by the weight δ, we say that a polynomial vector field X on $\mathbb{C}^{n+1}$ is of *degree* μ if it satisfies $[\mathcal{D}, X] = \mu X$. Equivalently, each X_a in $X = X_0 \partial/\partial w + \sum_{j=1}^{n} X_j \partial/\partial z_j$ is identically vanishing or a weighted homogeneous polynomial of total degree $\mu + \delta_a$ for $a = 0, \ldots, n$.

We denote by $\mathfrak{g}(M_P)$ the set of all polynomial vector fields in $\mathfrak{aut}(M_P)$ and by $\mathfrak{g}_\mu = \mathfrak{g}_\mu(M_P)$ the set of polynomial vector fields of degree μ in $\mathfrak{aut}(M_P)$.

For the model M_P, we consider two kinds of nondegeneracy conditions to P (see (2.7), (2.8) in [3]). We say that a real-valued polynomial P on $\mathbb{C}^n$ is *nondegenerate* if $\{z \in \mathbb{C}^n : \; P(z, \bar{z}) = 0\}$ contains no nontrivial analytic set, and *weakly nondegenerate* if there is no nontrivial holomorphic vector field Y on $\mathbb{C}^n$ with $YP \equiv 0$. The nondegeneracy of a weighted homogeneous polynomial implies its weak nondegeneracy (Lemma 2.2 in [3]). Denote by $P^{(0)}$ the summation of all balanced monomials of the weighted homogeneous polynomial P.

THEOREM 2.1 [3]. *Let $\delta = (\delta_1, \ldots, \delta_n)$ be a weight with rational $0 < \delta_j \leq 1/2$, and let M_P be a weighted homogeneous model. If P is nondegenerate and $P^{(0)}$ is weakly nondegenerate, then $\mathfrak{g}(M_P)$ can be decomposed by*

$$\mathfrak{g}(M_P) = \mathfrak{g}_{-1} + \mathfrak{g}_{-\delta_1} + \cdots + \mathfrak{g}_{-\delta_n} + \mathfrak{g}_0 + \mathfrak{g}_{1/2} + \mathfrak{g}_1 + \mathfrak{g}_{3/2} + \cdots .$$

Moreover,

(i) $[\mathfrak{g}_\mu, \mathfrak{g}_\nu] \subset \mathfrak{g}_{\mu+\nu}$.
(ii) $\mathfrak{g}_{-1} = \{c\mathcal{T} : \ c \in \mathbb{R}\}$.
(iii) *If* $X \in \mathfrak{g}_0$, *then* $X = c\mathcal{D} + \mathcal{L}$ *for some* $c \in \mathbb{R}$ *and* $\mathcal{L}$ *is a holomorphic vector field on* $\mathbb{C}^n$ *with variable* z, *that is,* $\mathcal{L} = \sum_{j=1}^n \mathcal{L}_j(z)\partial/\partial z_j$.
(iv) *If* $X \in \mathfrak{g}_{-1} + \mathfrak{g}_{-\delta_1} + \cdots + \mathfrak{g}_{-\delta_n}$ *is nontrivial, then* $X(0) \neq 0$.
(v) *If* $\mathfrak{g}_\mu \neq \{0\}$ *for some* $\mu > 0$, *then* P *is balanced.*

The original theorem has the restriction to the weight δ by $\delta_j = 1/2m_j$ for a positive integer m_j. However, E. Bedford and S. Pinchuk (and also the authors of this paper) confirmed that the same argument is also valid for any weight δ with rational $0 < \delta_j \leq 1/2$.

At this juncture, we shall introduce a similar property of weighted homogeneous models with the Siegel domains (see Theorem 1 in [7]).

PROPOSITION 2.2. *Let* M_P *be a weighted homogeneous model. If the automorphism group* $\mathrm{Aut}(M_P)$ *is a finite-dimensional Lie group, then all complete holomorphic vector fields of* M_P *are polynomial vector fields, that is,* $\mathfrak{aut}(M_P) = \mathfrak{g}(M_P)$.

Proof. The proof is based on Section 2 of [7]. Consider the translation $(w, z) \mapsto (w + 1, z)$ and its image D of M_P. Then D contains the origin, and the corresponding vector fields of $\mathcal{D}$, $\mathcal{T}$ are $\mathcal{D}_* = (w-1)\partial/\partial w + \sum_{j=1}^n \delta_j z_j \partial/\partial z_j$, $\mathcal{T}_* = i\partial/\partial w$, respectively. Thus the complex Lie algebra $\mathfrak{aut}^{\mathbb{C}}(D)$ generated by $\mathfrak{aut}(D)$ contains $\mathcal{D} = \mathcal{D}_* - i\mathcal{T}_*$. Since the translation is affine, it suffices to show that every element of $\mathfrak{aut}^{\mathbb{C}}(D)$ is a polynomial vector field.

Consider any $X \in \mathfrak{aut}^{\mathbb{C}}(D)$ that should be a holomorphic vector field. From the Taylor expansion of each coefficient of X at the origin, we can write $X = \sum_{\mu \geq -1} X_\mu$, where X_μ is a polynomial vector field of degree μ. It follows that $[\mathcal{D}, X] = \mathrm{ad}\,\mathcal{D}(X) = \sum_\mu \mu X_\mu$. Since $\mathfrak{aut}^{\mathbb{C}}(D)$ is a finite-dimensional complex vector space, we can consider the minimal polynomial ϕ of the operator $\mathrm{ad}\,\mathcal{D}$ in $\mathfrak{aut}^{\mathbb{C}}(D)$. Then $\phi(\mathrm{ad}\,\mathcal{D})X = \sum_\mu \phi(\mu) X_\mu$ so that $\phi(\mu)X_\mu = 0$ for any μ. That means that $X_\mu \equiv 0$ for all but finitely many μ. This completes the proof. □

2.2. *Note on the Nondegeneracy*

In [3], the weak nondegeneracy of $(P^2)^{(0)}$ is additionally assumed for Theorem 2.1. But it is indeed given by the nondegeneracy of P as follows.

PROPOSITION 2.3 (proof of Theorem 1 in [3]). *Let* P *be a real-valued weighted homogeneous polynomial. If* P *is nondegenerate and* $P(z, \bar{z}) \geq 0$ *for any* $z \in \mathbb{C}^n$, *then* $P^{(0)}$ *is strictly plurisubharmonic at some point, so weakly nondegenerate.*

In this section, we shall show that the weak nondegeneracy of $P^{(0)}$ is also given by the strict plurisubharmonicity of the plurisubharmonic P at a point.

LEMMA 2.4. *If a real-valued weighted homogeneous polynomial P is nondegenerate and plurisubharmonic on $\mathbb{C}^n$ and strictly plurisubharmonic at $z \in \mathbb{C}^n$, then $P^{(0)}$ is also strictly plurisubharmonic at z, so weakly nondegenerate.*

Proof. Let P be a nondegenerate, weighted homogeneous polynomial on $\mathbb{C}^n$ of the weight $\delta = (\delta_1, \dots, \delta_n)$. The nondegeneracy of P means that each δ_j is rational. Choose an integer m, for which each $m\delta_j$ is also an integer, and denote by $e^{im\delta\theta}z = (e^{im\delta_1\theta}z_1, \dots, e^{im\delta_n\theta}z_n)$ for the vector $z = (z_1, \dots, z_n) \in \mathbb{C}^n$. Then we have

$$P^{(0)}(z, \bar{z}) = \frac{1}{2\pi}\int_0^{2\pi} P(e^{im\delta\theta}z, \overline{e^{im\delta\theta}z})\, d\theta.$$

When we simply denote by $P_{j\bar{k}} = \partial^2 P/\partial z_j \partial \bar{z}_k$,

$$P^{(0)}_{j\bar{k}}(z, \bar{z}) = \frac{1}{2\pi}\int_0^{2\pi} e^{im\delta_j\theta}e^{-im\delta_k\theta} P_{j\bar{k}}(e^{im\delta\theta}z, \overline{e^{im\delta\theta}z})\, d\theta.$$

For any complex vector $v = (v_1, \dots, v_n)$, it follows that

$$\begin{aligned}\sum_{j,k=1}^{n} P^{(0)}_{j\bar{k}}(z, \bar{z}) v_j v_{\bar{k}} &= \sum_{j,k=1}^{n} \frac{1}{2\pi}\int_0^{2\pi} e^{im\delta_j\theta}e^{-im\delta_k\theta} P_{j\bar{k}}(e^{im\delta\theta}z, \overline{e^{im\delta\theta}z}) v_j v_{\bar{k}}\, d\theta \\ &= \sum_{j,k=1}^{n} \frac{1}{2\pi}\int_0^{2\pi} P_{j\bar{k}}(e^{im\delta\theta}z, \overline{e^{im\delta\theta}z})(e^{im\delta\theta}v)_j (e^{im\delta\theta}v)_{\bar{k}}\, d\theta.\end{aligned}$$

Suppose that P is plurisubharmonic on $\mathbb{C}^n$ and strictly plurisubharmonic at $z \in \mathbb{C}^n$. Then the term $\sum_{j,k=1}^n P_{j\bar{k}}(e^{im\delta\theta}z, \overline{e^{im\delta\theta}z})(e^{im\delta\theta}v)_j (e^{im\delta\theta}v)_{\bar{k}}$ is always nonnegative and positive near $\theta = 0$ when $v \neq 0$. That means $\sum_{j,k=1}^n P^{(0)}_{j\bar{k}}(z, \bar{z}) \times v_j v_{\bar{k}} > 0$ for any nonzero vector v so that $P^{(0)}$ is strictly plurisubharmonic at z. □

3. Proof of Theorem 1.1

Let Ω be a bounded pseudoconvex domain in $\mathbb{C}^{n+1}$ with smooth boundary, and let $f \in \mathrm{Aut}(\Omega) \cap \mathrm{Diff}(\overline{\Omega})$ such that f and f^{-1} are contracting at different boundary points q_1 and q_2, respectively. By the results in Section 4.3 of [10], there are two weighted homogeneous models M_{P_1} and M_{P_2}, both of which are biholomorphic to Ω by $\psi_1 : \Omega \to M_{P_1}$ and $\psi_2 : \Omega \to M_{P_2}$. Simultaneously, $\psi_1 : \partial\Omega \setminus \{q_2\} \to \partial M_{P_1}$ and $\psi_2 : \partial\Omega \setminus \{q_1\} \to \partial M_{P_2}$ are CR diffeomorphisms with $\psi_1(q_1) = 0$ and $\psi_2(q_2) = 0$. Moreover, we have

$$\psi_1 \circ f \circ \psi_1^{-1} = \mathcal{D}_{t_1}, \qquad \psi_2 \circ f^{-1} \circ \psi_2^{-1} = \mathcal{D}_{t_2} \tag{3.1}$$

for some negative real numbers t_1, t_2 (see also [11]). Since M_{P_1} is biholomorphic to M_{P_2}, we may assume that the weights for P_1 and P_2 are the same from Main Theorem in [4]. We denote this by $\delta = (\delta_1, \dots, \delta_n)$.

Since Ω is a bounded pseudoconvex domain and ψ_1, ψ_2 are CR diffeomorphisms, each M_{P_k} $(k = 1, 2)$ is pseudoconvex and also strongly pseudoconvex on an open subset of ∂M_{P_k}. Equivalently, each $P_k : \mathbb{C}^n \to \mathbb{R}$ is plurisubharmonic on

$\mathbb{C}^n$ and should be strictly plurisubharmonic on an open subset of $\mathbb{C}^n$. By Corollary 4.4 in [10], $\partial\Omega$ is of finite type at q_1 and q_2, so is ∂M_{P_k} near the origin. Since ∂M_{P_k} is of real-analytic type, there is no nontrivial complex analytic set in ∂M_{P_k} passing through a point of finite type (see [5]). Because of $\mathcal{D}_t \in \mathrm{Aut}(M_{P_k})$, there is no nontrivial complex analytic set in ∂M_{P_k}; thus each P_k should be nondegenerate. Simultaneously, each $P_k^{(0)}$ is weakly nondegenerate by Lemma 2.4 and each δ_j is a rational number with $0 < \delta_j \le 1/2$. Now we can apply Theorem 2.1 to M_{P_k}.

Let us consider the biholomorphism $\varphi = \psi_2 \circ \psi_1^{-1} : M_{P_1} \to M_{P_2}$ and the holomorphic vector field $\mathcal{D} \in \mathfrak{g}(M_{P_1})$. Since Ω is bounded, the automorphism group $\mathrm{Aut}(\Omega) \simeq \mathrm{Aut}(M_{P_k})$ is a finite-dimensional Lie group. Proposition 2.2 says that $\mathfrak{aut}(M_{P_k}) = \mathfrak{g}(M_{P_k})$ for $k = 1, 2$. Hence $\varphi_*\mathcal{D}$ should be a polynomial vector field in M_{P_2}. It is also achieved by Lemma 3.3 in [4].

STEP 1. Expecting a contradiction, we suppose that P_1 is not balanced. By (v) of Theorem 2.1, we can write

$$\varphi_*\mathcal{D} = c\mathcal{D} + \mathcal{L} + \mathcal{E},$$

where $c\mathcal{D} + \mathcal{L} \in \mathfrak{g}_0(M_{P_2})$ as in (iii) of Theorem 2.1 and $\mathcal{E}$ is the summation of all components of negative degree in $\varphi_*\mathcal{D}$.

The dynamics of $-\mathcal{D}$ on each M_{P_k} is a contraction at the origin, so it coincides with the dynamics of $\mathcal{D}_t$ for $t < 0$. Since $\varphi \circ \mathcal{D}_{t_1} \circ \varphi^{-1} = \mathcal{D}_{t_2}^{-1} = \mathcal{D}_{-t_2}$ for negative t_1, t_2 by (3.1), the vector field $\varphi_*\mathcal{D} = -\varphi_*(-\mathcal{D}) \in \mathfrak{g}(M_{P_2})$ should be a contraction at the origin. Therefore we get $c < 0$ and $\mathcal{E} \equiv 0$ simultaneously. See (iv) of Theorem 2.1.

Since $\mathcal{L}$ is independent of variable w, we have $[\mathcal{L}, \mathcal{T}] = 0$, where $\mathcal{T} = i\partial/\partial w \in \mathfrak{g}_{-1}(M_{P_2})$. This means $[\varphi_*\mathcal{D}, \mathcal{T}] = [c\mathcal{D}, \mathcal{T}] = -c\mathcal{T}$. Pushing forward by φ^{-1}, it follows that $[\mathcal{D}, \varphi_*^{-1}\mathcal{T}] = -c(\varphi_*^{-1}\mathcal{T})$. Therefore the nontrivial vector field $\varphi_*^{-1}\mathcal{T} \in \mathfrak{g}(M_{P_1})$ has the positive number $-c$ as its degree. This is a contradiction to (v) of Theorem 2.1 under the unbalanced assumption. Hence P_1 is balanced.

STEP 2. From here on, we let $P_k = P$. In order to show that $\delta_1 = 1/2m_1$ for some positive integer m_1, we consider the complex plain $L = \{z_2 = \cdots = z_n = 0\}$. Since P is now balanced, we have a pseudoconvex domain $M_P \cap L = \{(w, z_1) : \mathrm{Re}\, w + r|z_1|^{1/\delta_1} < 0\}$ in $L \simeq \mathbb{C}^2$ for some real constant r. Being pseudoconvex at every boundary point, $1/\delta_1$ should be a positive even integer and $r > 0$. Similarly, there is a positive integer m_j with $\delta_j = 1/2m_j$ for each $j = 1, \ldots, n$.

Now, for any $p = (p_1, \ldots, p_n) \in \mathbb{C}^n \setminus \{0\}$, we consider the holomorphic mapping $\iota : \mathbb{C}^2 \to \mathbb{C}^{n+1}$ defined by $\iota(\zeta, \xi) = (\zeta, \xi^{2m\delta_1} p_1, \ldots, \xi^{2m\delta_n} p_n)$, where $m = m_1 \cdots m_n$. Then it is a proper mapping from $\iota^{-1}(M_P) = \{(\zeta, \xi) \in \mathbb{C}^2 : \mathrm{Re}\,\zeta + P(p, \bar{p})|\xi|^{2m} < 0\}$ to M_P. Since $\iota^{-1}(M_P)$ should be pseudoconvex and hyperbolic, we have $P(p, \bar{p}) > 0$. This completes the proof of Theorem 1.1.

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