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The Mathematics of Donald Gordon Higman

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1. Introduction

Donald Gordon Higman (born 20 September 1928 in Vancouver, B.C., Canada)—an architect of important theories in finite groups, representation theory, algebraic combinatorics, and geometry and a longtime faculty member at the University of Michigan (1960–1998)—died after a long illness on 13 February 2006.

Don left a significant legacy of mathematical work and personal impact on many mathematicians. A committee was formed in 2006 to work with the *Michigan Mathematical Journal* and create a memorial. The contributors have some mathematical closeness to Don. Several of Don's fifteen doctoral students are included in this group. The breadth of topics and quality of the writing is impressive. For example, the article of Broué is especially direct in examining the impact of one of Higman's basic results in representation theory (the "Higman criterion").

Don Higman was a serious intellectual who had the manner of a kind uncle or concerned friend. He worked broadly in algebra and combinatorics. He thought deeply about the ideas in his mathematical sphere, and his style was to seek the essence of a theory. His work had great influence on future developments. This is exemplified by one of his theorems in permutation groups, as related by Peter Neumann: Don's "fundamental observation that a permutation group is primitive if and only if all its nontrivial orbital graphs are connected changed the character of permutation group theory. It's a simple thing, but it introduces a point of view that allowed lovely simplifications and extensions of the proofs of many classical theorems due to Jordan, Manning, and Wielandt."

Len Scott relates Don's reaction to a John Thompson lecture, around 1968, at a conference at the University of Illinois. This was not long after the discovery of the *Higman–Sims sporadic simple group.* Thompson expressed agreement with Jacques Tits's "heliocentric view of the universe, with the general linear group as the sun, and these sporadic groups as just asteroids." Len happened to be on the same elevator with Don, shortly after the lecture, when one of the participants asked Don what he thought of the heliocentric model. Don's reply was, "Well, it hurts your eyes to look at the sun all the time."

The elevator passengers had a good laugh, and it really was a marvelous line. But, reflecting further, not only can we see a part of Don's personality and humor here, but also some of his identity as a mathematician and even some of his place in mathematical history.

The decade that followed saw a dramatically changed picture of the finite simple groups. At the beginning of this period, there were a number of well-defined infinite families of finite simple groups that contained all known finite simple groups, with only finitely many exceptions. At that time, these finitely many finite simple groups that did not fit were called "sporadic" (group theorists were unsure as to whether more infinite families might be found). The advance of the classification during that period suggested that there were, indeed, no more infinite families and there were only finitely many sporadic groups. Nevertheless, during the same period, many new sporadic groups were discovered, bringing the number of sporadics to 26 by 1975. In addition, these discoveries inspired new algebraic constructions and conjectures, as well as striking empirical observations, that starting in the late 1970s linked sporadic group theory with other areas of mathematics and physics (this body of phenomenon is generally called "moonshine"). These sporadic groups may well have as much impact outside group theory as within it. Don Higman's independent and skeptical opinions seem to have been vindicated by the end of the 1970s.

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2. A Brief Overview of Donald Higman's Impact on Mathematics

Donald Higman's beginnings were in group theory and representation theory, clearly reflecting the interests of his Ph.D. advisor Reinhold Baer. Don's focal subgroup theorem is logically at the center of basic local group theory (the study of finite groups by normalizers of their nonidentity p-subgroups, where p is a prime). The *focal subgroup theorem* has been desribed by Jon Alperin [1] as "a basic concept in the theory of transfer". Alperin also writes: "[Don] introduced *relatively projective modules,* which are the key tool for Green's whole theory of modules, and contributed to relative homological algebra. He proved, as a result, that if F is a field of prime characteristic p and G is a finite group then there are finitely many isomophism classes of indecomposable FG-modules if, and only if, the Sylow p-subgroups of G are cyclic."

Don's interest moved to groups and geometry, and with McLaughlin he established a characterization of rank-2 linear groups. This theory was later subsumed in the theory of BN-pairs developed by Jacques Tits. Higman's thinking became more abstractly combinatorial. His theory of rank-3 groups assisted in the discovery and construction of several sporadic simple groups. Indeed, Charles Sims and Don Higman used this theory in 1967 to make a rank-3 extension of the Mathieu group M_{22} to a then-new sporadic simple group, the Higman–Sims group.

A golden era followed, full of combinatorial theories and applications. One finds *coherent configurations, association schemes,* and other theories. Both combinatorial theorists and group theorists—especially those interested in permutation groups—realized that algebras each had been studying had a common framework, which could be extended and studied further with the insights provided by each discipline. The body of work created at this time continues to have an impact and be of interest today.

Don Higman's work on permutation groups transformed that area by introducing combinatorial techniques. Major contributions include his theory of rank-3 permutation groups and its impact on the theory of strongly regular graphs, his introduction of intersection matrices for permutation groups that led among other things to the study of *distance transitive graphs,* and his theory of coherent configurations. The influence this had on young researchers at the time cannot be overstated. For example, Peter Cameron refers to this as "the most exciting development" during his time as a research student, adding that, whereas "Sims's methods were graph-theoretic, based on the work of Tutte, Higman's were more representation-theoretic, and grew from the work of Wielandt"; Michael Aschbacher regards Higman's rank-3 theory as a "central tool" in his early research; to Cheryl Praeger, Higman's papers on rank-3 groups and intersection matrices seemed "like magic" and were "more dissected than read" by graduate students and faculty at Oxford. According to Eiichi Bannai, Don Higman "opened a new path" that led to the development of the area now called algebraic combinatorics, and he is regarded as one of its founders.

The class of rank-3 permutation groups contains many important families, including many classical groups acting on isotropic points of a bilinear form or singular points of a quadratic form. Higman's rank-3 theory was exploited by several mathematicians to find or analyze new rank-3 groups. In particular, Higman's table [H57] of numerical and group-theoretic information (of actual rank-3 groups as well as "possibilities") was recalled by Francis Buekenhout as "a document of great historical value", covering around 400 groups. Indeed, both the table and the rank-3 theory contributed to the triumph of the collaborators Charles Sims and Donald Higman in discovering and constructing, on 3 September 1967, the Higman–Sims group. The discovery of this sporadic simple group, jointly with Charles Sims, is one of the most famous results of Donald Higman (see Section 5.6 as well as a contemporary account in [7]; see also [H20], a preprint of which was handed out at the summer 1968 group theory conference in Ann Arbor).

3. Education, Influences, and Employment

Don Higman attended college at the University of British Columbia. Betty Higman explains that "there was a woman at U.B.C., Celia Kreiger, who urged [Don] to continue his interest in math and apply to grad school. She might have been instrumental in suggesting Reinhold Baer and the University of Illinois. Then there was Hans Zassenhaus [with whom he worked] at McGill." A group photo of a Canadian summer mathematics program, probably 1951 or 1952, includes Don Higman and Hans Zassenhaus. After Don's doctorate in 1952, he spent two years as a National Research Council Fellow at McGill University, then two years on the faculty of Montana State University. Thereafter, he accepted an assistant professorship at the University of Michigan and became professor in 1960.

The main mathematical influences on Donald Higman seemed to be his advisor Reinhold Baer and algebraist Jack E. McLaughlin, a colleague at the University of Michigan who was older by a few years. There was another significant contact during graduate school: Michio Suzuki and Don Higman overlapped at the University of Illinois for about a year. It is clear from their later work that mutual influence was likely. Both studied with Reinhold Baer. Coincidentally, Eiichi Bannai, an editor of this special issue, thought of Michio Suzuki as a mathematical father and Don Higman as a kindly mathematical uncle.

McLaughlin and Higman authored three joint articles [H10; H13; H16] and maintained an ongoing dialogue about mathematics for decades. Their common interests included general algebra, group theory, representation theory, and cohomology.

In 1968, McLaughlin discovered and constructed a sporadic simple group using the rank-3 theory of Higman and the embedding of $\mathrm{PSL}(3,4)$ in $\mathrm{PSU}(4,3)$ discovered by H. H. Mitchell [40]. See the book of Brauer and Sah [7] for how finite simple group theory looked in the late 1960s.

4. Details of Early Work: Group Theory and Representation Theory

Don Higman's focal subgroup theorem is an insightful result about intersections of normal subgroups of a finite group with a Sylow p-subgroup. In early work on homological aspects of group representation theory, Don established the important concept of a relatively projective module and gave a criterion, which bears his name, for relative projectivity. Higman proved that the finiteness of the number of isomorphism types of indecomposable modules in characteristic p for a finite group G is equivalent to cyclicity of the Sylow p-group of G. This is a basic result in the theory of modules for group algebras. He did some of the earliest computations of degree-1 cohomology of classical groups as part of his study of flag-transitive groups.

4.1. Focal Subgroup Theorem

Higman's theory of the focal subgroup of a Sylow subgroup was a basic tool in local analysis in finite group theory. It could be viewed as a contribution along the lines of Burnside, Frobenius, Grün, et al. to the determination of quotients that are p-groups (p is a prime number). It turns out that the focal subgroup theory is logically at the center of p-local group theory. Consider the recent words of George Glauberman: "When I was preparing some lectures on local analysis [22], I had to omit the details of the definition of the transfer mapping and all proofs using the explicit definition. I found that I could prove all the applications of transfer that I needed by assuming without proof only two applications: the focal subgroup theorem and the fact that for a Sylow subgroup S of a group G, $S \cap Z(G) \cap G' = S' \cap Z(G)$." (The priming here indicates commutator subgroup.)

In the focal subgroup paper [H3, p. 496], there is a final remark about a communication with Brauer who reports overlaps with results from his paper on the characterization of characters [6]. Higman states that those results follow from the focal subgroup theory. David Gluck brought to our attention the footnotes in Section 5 of [6], which indicate that Richard Brauer essentially agreed with Don's assertions. It seemed to Gluck that Brauer was impressed with Higman, and this may have influenced Don's success in getting a job at the University of Michigan (for Brauer had been a member of the University of Michigan faculty during 1948–1951 before moving to Harvard). There seemed to be logical connections between the characterization of characters and the focal subgroup theorem that are not yet fully explored.

The focal subgroup definition bears strong resemblance to the stable cocycle condition described by Cartan and Eilenberg [11], which (as Alperin has suggested) could be viewed as a way to extend the focal subgroup theory to higher cohomology groups. We have no evidence that Higman foresaw these connections.

4.2. Representation Theory

Donald Higman wrote eight papers on representation theory over a period of seven years (1954–1960). The most influential paper was the first [H5], in which he introduced the notions of a relatively projective (or injective) module for the group algebra of a finite group and gave the famous criterion for relative projectivity that bears his name, *Higman's criterion*. James A. (Sandy) Green writes [23]: "I am very sorry to learn that Don has died. Mathematically I owe him for the idea of relative projectivity of modules over finite groups. This was the starting point of work I did on the 'vertex' and 'source' of an indecomposable kG-module (G finite group, k field of characteristic $p > 0$)." One of the first results of any kind on indecomposable modules was Don's paper [H6], which showed that having a cyclic Sylow p-subgroup was a necessary and sufficient condition for a group to have just finitely many isomorphism classes of indecomposable modules over a given field of characteristic p. The theory has come a long way since then, and some of its main contributors have written in this very volume. In particular, Michel Broué [8] discusses modern generalizations of Higman's criterion, very much in the spirit of Don's own generalization [H7] beyond group algebras. Michel writes in the introduction of [8]: "It must be noted that ways of generalizing Higman's original criterion had been opened half a century ago by Higman himself". Again in a general algebraic context, rather than for group algebras alone, Don presented in [H9] a theory, independently obtained by G. Hochschild [33], of *relative homological algebra*.

Don was keenly aware that not all interesting algebras were group algebras, and he was especially attracted to integral representations of orders over an integral domain O in a semisimple or separable algebra (over the domain's quotient field). In the separable algebra case Don defined [H8] an ideal in O, now called the Higman ideal (see [42, pp. 253–258; 18, pp. 603–609]) as the (nonzero!) intersection of the annihilators of all first cohomology groups of bimodules for the order, an ideal that Don notes (see [H12]) to be contained in the intersection of the annihilators of all degree-1 Ext groups between lattices for the order. If O is a Dedekind domain then the 1-cohomology and degree-1 Ext groups used are the usual ones, but they require modifications otherwise [H12]. The intersection ideal defined with degree-1 Ext groups also makes sense in the somewhat weaker case of orders in semisimple, rather than separable, algebras. (All semisimple algebras are separable over fields of characteristic 0.) In the case of a finite group algebra over O, the principal ideal generated by the integer that is the group order is contained in all of these annihilator ideals. In this respect the Higman ideal and its variations provide a substitute in the general case for the group order. Higman used these ideals, and other variations defined for degree-1 Ext groups with a fixed lattice as one of the two module variables, to establish in [H8] and [H12] a series of results for general orders (in a separable algebra) over Dedekind domains that had been proved by Maranda [37; 38] for finite group algebras. Higman and Jack McLaughlin also studied orders in the function field case [H10].

Len Scott writes: "One of my favorites among Don's representation theory papers is [H11]. He proved that the issue of whether or not two orders over a complete principal ideal domain were isomorphic could be reduced to similar questions over a single finite length quotient of the domain, and he showed that there is a bound on such a length which is computable in terms of the Higman ideal. The proof is a convincing application of Don's homological ideas, and the result itself is an important landmark in the theory of group ring isomorphims, as well as of isomorphisms between more general orders."

4.3. Influence of the Work of Donald Higman, by Charles Curtis

Charles Curtis wrote to the editors as follows about Don's influence on his work, in particular regarding the widely used reference [17].

"The book of Reiner and myself was intended to be an introduction to the representation theory of finite groups and associative rings and algebras. One objective was to bring readers to the point where they could approach the papers in the area by Richard Brauer. While Brauer's work already contained important contributions to ordinary and modular representation theory and applications to the structure of finite groups, especially the finite simple groups, we believed that this area offered rich possibilities for the future.

"One thing that Brauer, Nakayama, and Nesbitt had begun to develop was the theory of nonsemisimple algebras, in particular group algebras of finite groups over fields whose characteristic divided the order of the group. Two chapters of our book were devoted to this subject. We tried to make full use of the new concepts of projective and injective modules, introduced a short time before by Cartan and Eilenberg. The adaptation of these ideas to the case of group algebras was done by Gaschütz in 1952, and extended to the notions of relative projective and injective modules by Higman in 1954. The main result obtained at this stage in the development was Higman's theorem, published in 1954, that a group algebra of a finite group G over a field of characteristic p has finite representation type (that is, has at most a finite number of isomorphism classes of indecomposable modules) if and only if the Sylow p-subgroups of G are cyclic. This result has been influential in the theory of integral representations of finite groups, in the representation theory of Hecke algebras, and in the modular representation theory of finite groups, up to the present time. Another important development was Green's theory of vertices and sources of indecomposable modules, which remains a central topic in the modular representation theory of finite groups, and is based on Higman's theory of relative projective modules. One of the chapters of our book was largely devoted to Higman's work in this area."

5. Details of Later Work: Algebraic Combinatorics, Groups, and Geometries

5.1. ABA Groups and Rank-2 Geometry Characterization

The "ABA groups" paper [H13] in 1961 with University of Michigan colleague Jack McLaughlin arose from an effort to characterize low-rank finite simple groups and their geometries. Apart from the famous theorem that a finite 2-transitive projective plane is Desarguesian, the paper contains a number of fundamental results that continue to have application in group theory and combinatorics. Shortly after it appeared, a result from the paper was used by Charles Curtis [14] to classify a family of finite Chevalley groups; and nearly thirty years after its publication, another of its results laid the basis for the classification [10] of flag-transitive linear spaces, proving that such spaces have a point primitive automorphism group. The paper remains influential, being cited nearly twenty times in the last decade in publications on linear spaces and symmetric designs.

5.2. Rank-3 Permutation Groups

Don Higman's theory of rank-3 permutation groups was initiated in 1964 in his paper [H15], in which he studied their parameters, incidence matrices, and character degrees. His approach was highly influential on the work of other group theorists, not least because several sporadic simple groups were discovered as rank-3 groups—in particular, the Higman–Sims simple group discussed in Section 5.3. Also in his 1964 paper Higman gave a pair of design constructions that are now standard constructions for symmetric designs. His other papers on this theme focused on characterizations proving, for example, that all rank-3 affine planes are translation planes. It is notable that Higman chose rank-3 groups as the topic for his invited lecture [H28] at the International Congress of Mathematicians in Nice in 1970.

5.3. The Higman–Sims Graph and the Higman–Sims Sporadic Simple Group of Order $44{,}352{,}000 = 2^9 3^2 5^3 7 \cdot 11$

The discovery by Donald Higman and Charles Sims of the Higman–Sims simple group is legendary. According to Sims's account in [30], Don and he had just heard Marshall Hall's description, at the 1967 "Computational Problems in Abstract Algebra" conference in Oxford, of the construction of the Hall–Janko sporadic simple group as a rank-3 permutation group on 100 points. Higman and Sims examined closely a list of possible rank-3 parameters that Higman had compiled using his rank-3 theory, and their attention quickly focused on a possible rank-3 group with point stabilizer the Mathieu group M_{22} or its automorphism group $M_{22}.2$. In discussion during an intermission in the formal conference dinner on the last day of the conference (2 September 1967), Higman and Sims realized that these groups had natural actions on both 22 points and 77 points. They continued their work on these actions after the dinner finished and eventually realized that they needed to construct a valency-22 graph on 100 vertices, now called the Higman–Sims graph. In the early hours of Sunday, 3 September 1967, using the uniqueness of the associated Witt design, they proved that the automorphism group of their graph was vertex-transitive and was a new simple group.

We re-publish with permission the 2002 account by Charles Sims [30] of this famous story.

"Prior to this conference, Don had been investigating rank-3 groups. He had discovered a number of conditions that the parameters [H57] of such a group have to satisfy and had used a computer to generate a list of parameters that satisfied all of his conditions. I was familiar with Don's work….

"At the Oxford conference, Marshall Hall announced the construction of Janko's second group. [This group is more correctly called the Hall–Janko group because both Hall and Janko discovered it independently; see [24; 27].] There is a long paper by Marshall in the conference proceedings that includes, among other things, a description of how this group was constructed. The group was given as a rank-3 group of degree 100 with subdegrees 1, 36, and 63.

"After Marshall's talk, Don and I discussed whether there might be other rank-3 groups of degree 100. If it were not the case that we use the decimal system and that $100 = 10^2$, I am not sure we would have asked this question.

"Don consulted his list of rank-3 parameters and found that the subdegrees 1, 18, 81 appeared on the list of degree 100. It did not take us long to realize that the wreath product of S_{10} and Z_2 has a rank-3 representation (on the Cartesian product of two copies of $\{1,\dots,10\}$) of degree 100 with these subdegrees.

"Encouraged, we looked at the list again. There was one other set of subdegrees for a possible rank-3 group of degree 100, namely 1, 22, 77. The number 22 certainly suggests that the stabilizer of a point should be M_{22} or its automorphism group. (S_{22} and A_{22} don't have representations of degree 77.) We did not immediately see how to construct such a group and agreed to continue our discussion later.

"The conference dinner was held on the last day of the conference, Saturday, September 2. This dinner was quite formal. After the main part of the meal, the participants were asked to leave the hall while the staff cleared the tables and prepared for dessert and coffee. As Don and I walked around the courtyard of the college in which the dinner was being held, we again talked about $100 = 1 + 22 + 77$. The first question to answer was whether M_{22} has a transitive representation of degree 77. Both Don and I were familiar with combinatorial designs and knew that M_{22} acts on $\mathfrak{S}(3,6,22)$. As we walked, we computed the number of blocks in this design. When we got the answer 77, we were sure we were on to something.

"At this point, it was time to go back in for dessert. After the dinner was finished, we went to Don's room ... to continue working. There were some false starts, but eventually we realized that we needed to construct a graph of degree or valence 22 with 100 vertices consisting of a point *, the 22 points of an $\mathfrak{S}(3,6,22)$ and the 77 blocks of that $\mathfrak{S}(3,6,22)$. The point * would be connected in the graph to the 22 points. One of the 22 points p would be connected to * and to the 21 blocks containing p. A block would be connected to the 6 points in the block and to 16 other blocks. We had to do some computations, but it was not hard to show that a block is disjoint from exactly 16 other blocks. Thus in the graph, two blocks should be connected if they are disjoint.

"At this point, we had the graph and we knew that $\mathrm{Aut}(M_{22})$ as the stabilizer of * in the automorphism group of the graph, but we did not know that the automorphism group had any elements that moved *. To get additional elements, we used the fact that Witt had proved that $\mathfrak{S}(3,6,22)$ was unique.

"I don't think we finished until the early morning hours of Sunday, September 3, 1967. There was one uncertainty. We knew that we had a new simple group, but we did not know whether the stabilizer of a point in the simple group was M_{22} or $\mathrm{Aut}(M_{22})$."

5.3.1. Mesner Thesis and Higman–Sims Graph

While researching this biography, the editors learned that the 1956 doctoral thesis of Dale Mesner [39] contains a construction of the Higman–Sims graph; see also [35]. Higman and Sims were unaware of the Mesner thesis. Jon Hall kindly gave us the following summary of its contents.

"This long thesis (291 pages) explores several topics, including integrality conditions for strongly regular graphs (association schemes with two classes) related to Latin squares. One case of particular interest is that of graphs with the parameters of the Higman–Sims graph. Mesner notes that the existence of such a graph is equivalent, via a graph construction essentially the same as that of Higman and Sims, to the existence of a balanced incomplete block design with the parameters of the $S(3,6,22)$ design (including having 16 blocks disjoint from each block). He proceeds to construct such a design. At a certain point in this construction he is faced with four choices, among which he cannnot distinguish. He makes one choice and completes his construction, saying that the other three cases give similar results. At the end he then has four constructions of graphs on 100 points having the same parameters as the Higman–Sims graph. Of course, what he has is four slightly different constructions of the same graph, but he does not know or investigate that. On page 147 he finishes this part of the thesis by saying, 'This completes the construction of scheme #94 [the graph]. There are at most four solutions to the association scheme, corresponding to the four choices for the structure of the blocks of [the design]. It is not known whether any of the four solutions are equivalent under some permutation of treatments.' There is no further determination of isomorphisms or the automorphism group, nor mention of the groups of Mathieu or the work of Witt on Steiner systems."

5.4. Strongly Regular Graphs and Association Schemes

Don's original motivation for studying the combinatorics underlying permutation groups was purely group theoretic. For example, in his magnificent "intersection matrices" paper [H18] of 1967, Don proved his famous primitivity criterion for finite transitive permutation groups mentioned in the Introduction. In that same paper he introduced (what are now called) *intersection arrays* of finite distance-regular graphs from a group-theoretic perspective and began the study of permutation groups of maximal diameter. Gradually it was realized that rank-3 permutation groups correspond to strongly regular graphs; that Don's intersection matrices correspond to association schemes; and that permutation groups of maximal diameter correspond to distance-transitive graphs, a class of graphs studied intensively over the succeeding decades. The concept of a strongly regular graph already existed in the combinatorial literature, and association schemes (i.e., symmetric association schemes) were known in statistics and experimental design theory, in particular in the work of R. C. Bose and his school.

It seems that these connections were not known to group theorists at the time of Don's paper [H18]. Several years later in 1970, in an important paper [H26] with Hestenes, Don explored the connection between rank-3 groups and strongly regular graphs, introducing and studying the adjacency algebras of such graphs and giving a combinatorial form of primitivity. The theories of strongly regular graphs and association schemes were given real mathematical depth by this fruitful encounter with group theory.

Moreover, it was natural for Don to study these objects also from the combinatorial viewpoint: in his 1970 paper [H26] he introduced the "4-vertex condition" on a graph, a combinatorial alternative to the group-theoretic rank-3 condition. He used this to obtain a new proof of a result of Seidel determining the strongly regular graphs with smallest eigenvalue -2.

5.5. Coherent Configurations

One of Don's most notable contributions in the area of combinatorics is his introduction of the concept of a *coherent configuration* in his 1970 paper [H23]. A coherent configuration is an axiomatization, in a purely combinatorial setting, of the structure of an arbitrary permutation group. It generalizes the notion of an association scheme, which may be regarded as a combinatorial axiomatization for a transitive permutation group. Don arrived at this general concept as soon as he was aware of the combinatorial interpretation of his early work on permutation groups. He developed the theory of coherent configurations in full generality: first in [H23] and in more polished form in other papers, most notably [H35], [H37], and his paper [H41] on coherent algebras in 1987. The main content of these papers is a study of the algebraic conditions arising from association schemes and coherent configurations. Don studied the adjacency algebras (centralizer algebras) of these combinatorial structures, obtaining many important properties such as *orthogonality relations, Schur relations,* and *Krein conditions.*

A major reason why Don studied the representation theory of association schemes and coherent configurations was to apply the algebraic conditions he obtained to the study of many geometric objects. In his 1974 paper [H32] he illustrated how the theory of coherent configurations could be applied by studying *generalized polygons* and, in particular, gave alternative proofs of the famous theorem of W. Feit and G. Higman and of the Krein condition (to be discussed shortly). These results, in turn, were applied in [H32] to prove the following theorem: If a generalized quadrangle or octagon has $s + 1$ points on each line and $t + 1$ lines through each point with $t > 1$, then $s \leq t^2$. Don also applied his theory more generally to study, among other things, *partial geometries,* a concept related to that of a strongly regular graph.

Don Higman's representation theory of coherent configurations is thorough and useful and is unequalled by any other author. On the other hand, he did not treat many concrete examples in his papers, leaving this to others such as [4] and [9] (for symmetric or commutative association schemes).

The nonnegativity of the so-called Krein parameters is very important in the theory of association schemes—in particular, in the Delsarte theory of codes and designs in association schemes [4; 9; 19]. A version of this condition was first noticed in 1973 in the context of permutation groups by Leonard Scott and his colleague Charles Dunkl, translating from earlier work of Krein; see [44] and [45]. Using work of Schur dating even earlier than Krein's work, Don (and independently N. Biggs [5]) proved the Krein condition in full generality in the context of association schemes. Don's new approach to the development and exposition of the Krein condition theory has been most influential [13; 21; 29; 31]. Don often lectured on this to new audiences.

5.6. Geometric Applications of Coherent Configurations

Donald Higman's research in the 1980s and 1990s focused mainly on the application of theories of association schemes and coherent configurations to geometry. Many of his later works may be characterized as follows. Using the algebraic properties of adjacency algebras of association schemes and coherent configurations, he determined their feasible parameter sets under various conditions. He then characterized the known examples, and for many of the other small parameter sets he proved the nonexistence of examples. He was interested in a variety of geometric structures, especially those admitting interesting group actions. In particular, he studied special kinds of buildings (in the sense of Jacques Tits) as imprimitive association schemes. We discuss in what follows other notable geometric structures that he studied.

A transitive permutation representation can be regarded as the induced representation of the identity representation of a subgoup H of G. A similar theory is available for the induced representation of a linear representation of a subgoup H of G. Don's general theory of weights on coherent configurations (and association schemes) is a combinatorial analogue of this. Using this theory, Don gave combinatorial proofs in [H33; H37] of several group theoretic results due to Frame, Wielandt, Curtis–Fossum, and Keller (see [16; 36; 51]). An especially nice result of this type is his calculation in [H36] of the degrees of the irreducible representations appearing in the induced representation of the alternating character of the subgroup $2 \cdot {}^2E_6(2) \cdot 2$ of the "Baby Monster" sporadic simple group.

In [H41], Don introduced several geometric structures associated with small-rank coherent configurations and proposed that they be studied systematically. Typical examples are those with the following four "Higman parameters":

$$\text{(i) } \binom{2\,2}{\;\,2}, \qquad \text{(ii) } \binom{2\,2}{\;\,3}, \qquad \text{(iii) } \binom{3\,2}{\;\,3}, \qquad \text{(iv) } \binom{3\,3}{\;\,3},$$

where the corresponding geometric structures are (i) symmetric designs, (ii) quasi-symmetric designs, (iii) strongly regular designs of the first kind, and (iv) strongly regular designs of the second kind. Much work had already been done in cases (i) and (ii), and Don studied cases (iii) and (iv) in [H42; H48]. In the meantime, Hobart [31] studied the case $\binom{2\,2}{\;\,4}$. Again, the idea was to study algebraic properties and obtain a list of feasible parameters of small sizes.

Motivated by examples associated with classical triality and some other related group-theoretic examples, Higman [H47] studied imprimitive rank-5 permutation groups. He divided them into three cases according to the rank and corank of a parabolic subgroup and then studied algebraic conditions of the associated imprimitive association schemes. His student Yaotsu Chang had earlier dealt with the imprimitive rank-4 case in his Ph.D. thesis. In an unpublished preprint [H55] Don extended his research to include uniform (t, p, m)-schemes, in particular, such structures afforded by a transitive action of a group of the form $G \cdot S_{t+1}$ with G simple and S_{t+1} acting faithfully on G. He focused on the cases $t = 2, 3$ and eliminated those with small sizes by studying algebraic conditions of their associated coherent configurations. Again, group-theoretic assumptions seem necessary to obtain complete classifications.

In [H45] and an unpublished preprint [H52], Don studied regular t-graphs as a generalization of regular 2-graphs in the sense of Seidel. In particular, a regular 3-graph is naturally associated with a coherent configuration of rank 6. By examining their algebraic properties, he obtained a list of feasible small parameters for such coherent configurations and then studied whether such configurations exist. However, a complete classification of regular t-graphs for large t, or even for $t = 3$, seems infeasible. The Ph.D. thesis of Don's student Alyssa Sankey in 1992 discusses regular weights on strongly regular graphs—a different generalization of 2-graphs, but still in the spirit of [H45].

Table I summarizes the correspondence between groups and combinatorics.

In passing, we would like to mention the following two streams of similar ideas to study combinatorial objects from a group-theoretic viewpoint.

(1) Essentially the same concept as coherent configuration was studied in Russia under the name of "cellular rings", independently and simultaneously, as there was not much scientific communication between East and West at that time (see e.g. [50]).

Table I Correspondence between group-theoretic and graph-theoretic terminology

Groups	Graphs
rank-3 group	strongly regular graph
transitive permutation group	association scheme
arbitrary permutation group	coherent configuration
distance-transitive group (graph)	distance-regular graph
permutation group of maximal diameter	distance-regular graph

(2) The theory of Schur rings that was developed by Wielandt [51] and furthered by Tamaschke [49] also treats a connection between groups and combinatorics (note that the concept of a Schur ring falls somewhere between a permutation group and an association scheme).

5.7. Connections with Statistics, Topology, and Mathematical Physics

There are several connections that the work of Donald Higman makes outside algebra. Association schemes occur in statistics (this article mentions the work of Bose and Bailey). The thesis of Mesner [39] on Latin squares turned up a special set of graphs on 100 points, one of which is the Higman–Sims graph. François Jaeger constructed the spin model (in the sense of V. H. F. Jones) on the Higman–Sims graph, and it plays an important role in the theory of spin models (more broadly in topology and mathematical physics); see [28; 34].

6. Younger Mathematicians and Donald Higman

Don Higman's international engagement in the group theory and combinatorics communities made a strong impression on younger mathematicians. He made regular visits to England, Holland, Belgium, Germany, Australia, Italy, and Japan, and he organized meetings in Oberwolfach starting with the "Die Geometrie der Gruppen und die Gruppen der Geometrie under besondere Berücksichtigung endliche Strukturen" (18–23 May 1964).

6.1. Michael Aschbacher

"I went to graduate school to become a combinatorist, and wrote my thesis in design theory. My approach was to study designs from the point of view of their automorphism groups, so I became interested in finite permutation groups, to the point that in my last year in graduate school I decided to work in finite group theory rather than combinatorics.

"In one part of my thesis, I studied certain designs whose automorphism group was rank 3 on points of the design. Thus I was led to read Donald Higman's fundamental papers on rank-3 permutation groups, and rank-3 groups have remained one of my interests ever since.

"In particular in one of my early papers [2], I showed that there is no Moore graph on 3250 vertices with a rank-3 group of automorphisms. This was motivated by Higman's paper [H15], where in Section 6 he shows that in a rank-3 group with $\lambda = 0$ and $\mu = 1$, k is 2, 3, 7, or 57, and he determines the possible groups in all but the last case, which he leaves open. In the graph-theoretic literature, the corresponding strongly regular graphs are called *Moore graphs,* and of course a Moore graph of valence 57 is on 3250 vertices.

"A year later, reading Bernd Fischer's wonderful preprint on groups generated by 3-transpositions, I decided to attempt to extend Fischer's point of view to what I called 'odd transpositions'. The base case in this analysis involves almost simple groups G, and after handling some small degenerate cases, an extension of Fischer's fundamental lemma shows that G acts as a rank-3 group on a certain system of imprimitivity for the action of G on its odd transpositions. Thus, once again, Higman's theory of rank-3 groups became a central tool in my early work.

"I probably first met Donald Higman at a meeting in Gainesville, Florida, in 1972, organized by Ernie Shult. However, my strongest early memories of Higman are from a two-week meeting in Japan in 1974 organized by Michio Suzuki. This was only my second trip abroad, and one of my favorites. A number of wives also took part, including my wife Pam. Both Donald and Graham Higman attended the meeting, and I recall Pam referring to them as "the Higmen". As I recall the two were connected in our minds, not just because of their names, but because both had extremely luxuriant eyebrows....

"He was an excellent mathematician who will be missed."

6.2. R. A. Bailey

Rosemary Bailey relates three small anecdotes about Don, whom she—as did many of her fellow research students in Oxford—called DGH.

"I started as a Ph.D. student at Oxford in October 1969, under the supervision of Graham Higman. In summer 1970 I attended the International Congress of Mathematicians in Nice. Of course, much of it was above my head at that stage. The talk that most impressed me was the one by DGH. I was even brave enough to go up and talk to him about it afterwards, and I remember him being very kind.

"Some time in 1971, DGH visited Oxford for a few months. He gave a course of lectures called 'Combinatorial Considerations about Permutation Groups'. At the time, not only was I working on permutation groups but Graham Higman had given us a series of advanced classes on strongly regular graphs, so we were well prepared. I was absolutely fascinated by DGH's clear lectures. Well, I went away and became a statistician, but something must have stuck in my mind, because when I came back into association schemes via the original Bose–Nair approach I was able to put it all together, and this eventually led to my 2004 book [3] on association schemes. I mentioned these lectures of DGH's in the Acknowledgments page to that book.

"Most combinatorialists know what a t-design is, or at least what a 2-design is. Dan Hughes introduced this terminology into the literature. However, he says that it was DGH who suggested it to him: see pages 344–345 of [3]."

6.3. Eiichi Bannai

"I knew Donald G. Higman for many years, and I was mathematically very much influenced by him. This was at first through Suzuki, whom I viewed as a kind of mathematical father. In an article written in Japanese, Suzuki wrote as follows: 'When I [Suzuki] went to Illinois in 1952, Donald Higman was there and just completed his Ph.D. thesis on focal subgroups. Partly because I knew the name of Higman already, by his paper on homomorphism correspondence of subgroup lattices, soon we became very good friends.' I also remember that Suzuki described Higman as a mathematician doing very fundamental and essential work with his own viewpoint.

"Sometime in the 1960s, Higman visited Japan and stayed with the family of Suzuki (who had settled at Illinois, but regularly returned in the summers—Suzuki and Higman were sufficiently close that I came to regard Higman as a kind of uncle). I found an official three-page note published in *Sugaku* (the official journal of Mathematical Society of Japan) that summarizes Higman's talk on 3 June 1966 at the University of Tokyo. This note was prepared by Koichiro Harada, then a young graduate student. The title of his lecture was 'Remarks on Finite Permutation Groups'. According to the note, Higman talked on the discovery of Janko's new simple group and discussed D. Livingstone's new construction of it as the rank-5 permutation group of degree 266 with subdegrees 1, 11, 110, 12, and 132. Then he [Higman] discussed several attempts of his own and of McLaughlin to try to construct (new) rank-3 permutation groups. In particular, he mentioned his result that if there exists a rank-3 permutation group with subdegrees 1, k, and $k(k-1)$, then $k = 2, 3, 7$, and possibly 57. (At the time, it seems that he was not aware of the connection with strongly regular graphs and the work of Hoffman–Singleton.) In addition, he discussed some of the work of Tutte, Wielandt, Sims, etc. Interestingly enough, the Higman–Sims group HS was not mentioned at all, as this was before its discovery.

"The content of this note tells us that Higman had several years of serious preparations before his discovery of the group HS. The discovery was clearly not just a matter of luck. On the other hand, there was also some element of good fortune. See the account of Sims in Section 5.3 of this article.

"Higman also visited Japan for the Sapporo Conference in 1974 (where I first met him in person). Since then I had many chances to meet him in the United States and in Europe, and he was very generous and helpful to others. He contributed to a good research environment in the field and greatly helped other researchers in many ways, for example, by organizing several Oberwolfach conferences, etc. When Suzuki visited Japan during summers, he always gave a series of lectures on hot new developments of group theory.... I was able to attend Suzuki's lectures for the first time in 1968. In his lectures, Suzuki covered many topics, but the main topic was the discoveries of new finite simple groups, including Hall–Janko, Higman–Sims, and several others.... I learned the charm of rank-3 permutation groups from Suzuki's lectures. At that time, I was studying representation theory of finite groups (through the book of Curtis–Reiner and also from Iwahori, my advisor) and also finite permutation groups, in particular multiply transitive permutation groups (through the book of Wielandt [51]). After Suzuki's lectures, I started to study rank-3 permutation groups along the line of Donald Higman. Also, I was interested in the combinatorial aspect of finite permutation groups, such as strongly regular graphs. The connection between strongly regular graphs and rank-3 permutation groups was well known by that time. I read many of Higman's papers on rank-3 permutation groups, those related to classical geometries, and the one on intersection matrices for finite permutation groups. Although Higman's earlier papers were in the context of finite permutation groups, their combinatorial aspects were evident. In early 1970s, I tried to follow Higman's research direction in various ways. Let me comment on some of these directions of Higman.

"(i) (Small-rank subgroups of classical groups.) The first thing I tried in my research was to find (and classify) small-rank (in particular rank-2, i.e., 2-transitive) subgroups of classical groups, or of simple groups of Lie type. During that study, I determined rank-2 subgroups H of $\mathrm{PSL}(n,q)$, that is, 2-transitive permutation representations of $\mathrm{PSL}(n,q)$, by determining the candidates of possible irreducible characters χ that might possibly appear in 1_H^G; I used Green's theory of the character theory of $\mathrm{GL}(n,q)$. Later, I realized that this approach is considerably simplified by using the work of Higman [H14] on flag-transitive subgroups, since it holds that χ must appear in 1_B^G if H is not flag transitive. (Here B is the Borel subgroup, i.e., the upper triangular subgroup.) Also, I was very much interested in the paper [H13], which, before the Tits general theorem on spherical buildings, characterized the group $\mathrm{PSL}(3,q)$ by essentially giving the classification of A_2-type buildings (in the context of groups). This was used in many situations later by many authors.

"(ii) (Rank-3 transitive extensions.) Another direction of my research was to try to find new simple groups: starting with a known group H, to find G that contains H as a rank-3 (or small-rank) subgroup. To be more precise, starting from a permutation group H, finding a transitive group G whose stabilizer of a point is isomorphic to H is called a transitive extension problem. The discovery of the Higman–Sims group was one such beautiful example. Various sporadic groups—such as Hall–Janko, McLaughlin, Suzuki, Rudvalis, and the Fischer groups—were discovered this way. I think I started this project too late, since it turned out that all were already discovered as a consequence of the classification of finite simple groups.

"(iii) (Intersection matrices for finite permutation groups.) The paper I read most carefully was … 'Intersection Matrices for Finite Permutation Groups' [H18]. That paper gave a systematic study of finite permutation groups with some good properties. The paper also gave essentially a new proof of the Feit–Higman theorem on generalized polygons (in the context of groups). This new proof, which was to look at 1_P^G rather than 1_B^G in the original proof, was very transparent for me, and I was much impressed. (Here, P is a maximal parabolic subgroup.) I noticed that many of the algebraic properties of association schemes were also in this paper. Actually, I can say that I first learned association schemes mainly from this paper of Higman. In that paper he used the notation for intersection numbers as $A_iA_j = \sum p_{j,k}^{(i)} A_k$ (actually, such an explicit product is not written down in the paper), which was somewhat different from the traditional one $A_iA_j = \sum p_{i,j}^{(k)} A_k$

used in the theory of association schemes (apparently unknown to Higman at the time). I was so accustomed to using Higman's notation that I first insisted on using his notation when I later wrote a book [4] with Tatsuro Ito, though we later returned to the traditional notation (as did Higman). Anyway, the influence of Higman's intersection matrix paper on me was very great.

"There are many other areas of influence on my research, and I have described them in contributing to Section 5.4 (strongly regular graphs and association schemes) and Section 5.5 (coherent configurations) of this article. Let me add here that Higman's development of the general theory of coherent configurations in his papers [H35; H37] is very thorough and beautiful. Though not many concrete examples are treated, his papers are also useful as well as complete. There is nothing comparable to the work of Higman in the study of coherent configurations.

"Later, Higman did study many concrete examples of coherent configurations with special types, in papers [H41; H42; H45; H47; H52; H53; H54; H55], etc. They are related to some kinds of designs and finite geometries. While these studies may not be breakthroughs themselves, they indicate many possible future research directions. Some of the papers in this special issue will reflect Higman's strong influence along this line of study. Coherent configurations will be studied more in the future, as they are a very natural concept. Yet it is difficult to know what is the most important general research direction for their study (beyond association schemes). I personally feel that the motivation for Higman's later research was to discover some new possibilities by trying several directions.

"In conclusion, let me say Higman's work since 1960 has been extremely influential on those of us working on groups and combinatorics. Higman opened a new path that led to the development of so-called algebraic combinatorics, and we regard him as one of the founders of this approach. Higman continued his research in a more combinatorial context by giving solid foundations for the theory of coherent configurations, and he also studied geometric (design-theoretic) aspects of the theory of coherent configurations. He succeeded in combining group theory and combinatorics very nicely, and he will be remembered by the mathematical community for his fundamental contributions to these areas. He will be remembered also for his kind and generous personality by those of us who knew him personally."

6.4. Peter J. Cameron

"I arrived in Oxford to do my D.Phil. [Oxford term for Ph.D.] in 1968 and worked under the supervision of Peter Neumann. I worked on permutation groups. At the time, the most exciting development was the introduction of combinatorial techniques into the study of permutation groups, due to Charles Sims and Donald Higman. Sims's methods were graph-theoretic, based on the work of Tutte; Higman's were more representation-theoretic, and grew from the work of Wielandt. All my subsequent work has grown out of my initial exposure to this material.

"It was very important to me that Don Higman spent part of the year 1970/71 in Oxford, for several reasons. First, he gave a course of lectures. I was one of two students (the other was Susannah Howard) responsible for producing printed notes after each week's lectures and then compiling them into a volume of lecture notes published by the Mathematical Institute, Oxford, under the title ‘Combinatorial Considerations about Permutation Groups’. These notes contained a number of ideas that appeared only later in conventional publications; I was in the very fortunate position of having a preview of the development of the theory of coherent configurations....

“Fourth, and most important for me, Don arranged for me to spend a semester at the University of Michigan as a visiting assistant professor. This was a very productive time for me. I was allowed an hour a week in which I could lecture on anything I liked, to anyone who wanted to come along. I did a vast amount of mathematics there, and made many good friends.”

6.5. Robert L. Griess, Jr.

“When I joined the University of Michigan Mathematics Department faculty in September, 1971, I met Don Higman for the first time. I knew his name because the simple group constructed by Don Higman and Charles Sims had been well-studied by members of the group theory community for a few years. About two or three years before at the University of Chicago, where I was a graduate student, I met Doris and Jack McLaughlin during Jack’s sabbatical in Chicago, where he presented his new construction of the McLaughlin sporadic simple group. The rank-3 theory of Higman was prominently featured in McLaughlin’s lectures. Roger Lyndon’s achievements in combinatorial group theory, homological algebra, and logic were well known. For these reasons, I felt attraction to the University of Michigan.

“When I met Don, his era of algebraic combinatorics had begun. An important undercurrent was representation theory of groups and algebras. His productivity during the 1970s and the attention it drew from experts worldwide were quite impressive.

“Don and I together attended hundreds of classes and seminars. Although we never collaborated on any research, we conversed for decades about algebra, especially representation theory, and aspects of finite groups and combinatorics. During this time, I had many opportunities to see how his mind operated.

“A lot of Don’s research involved long-term methodical work. He collected data that he studied as he developed his theories. He was repeatedly attracted to the idea of elegant, simple explanations and to finding axiom systems. His publications may not have clearly indicated the lengthy efforts he made.

“About a year after arriving, I heard Don talk about his short result on transitive extensions of permutation groups [H30]. It was presented in Don’s characteristic modest and humorous style. It struck my young mind as quite original and fresh, a real piece of mathematical wit. While I was a graduate student, I had gained the impression that his work in finite group theory was fundamental. Finite-group theorists valued the focal subgroup theorem, basic results on indecomposable modules, the idea of relative projectivity, and work on rank-3 permutation groups.

“Don regularly had mathematical visitors, including many who stayed at his home. Betty and Don were generous hosts on many occasions and made me, in particular, feel very welcome during dinners and parties. I appreciated the chance to get acquainted with so many mathematicians from Don’s world.

"Betty and Don took friendly interest in my personal life, and Don was always keen to hear about my mathematical activities. He made helpful suggestions and on many occasions he indicated publications or preprints I did not know about that might lead to interesting connections. I learned new viewpoints and details about the genesis of ideas in groups and geometries.

"Donald Higman was a model of perseverance, clarity, and elegance in mathematics. His death in 2006 was a complete surprise. Fortunately, I had dedicated an article to him a few years before [25; 26]. When I told him, he was happy and thanked me quietly. I shall never forget him."

6.6. Willem Haemers

"When I met Don (probably in 1976), I was a Ph.D. student at the Technical University in Eindhoven (supervised by Jaap Seidel). Then Don gave a beautiful course on 'classical groups', and I had the pleasure of participating in several conversations with Don and Jaap.

"Already in those days Don had the habit of writing unfinished manuscripts. He often gave such manuscripts away, hoping that something would come out of it. Sometimes this worked. But probably many such manuscripts remained unpublished. [The unpublished manuscripts and other material known to the editors are listed in Section 9 of this article.] It is how my two [joint] papers with Don came to existence. ... He always showed great interest in me, in my mathematics and private life. I liked him very much."

6.7. Cheryl E. Praeger

"I was in Oxford for the course given by DGH and alluded to in the preceding contributions of Ro Bailey and Peter Cameron. The course made a big impression on me, and the papers by DGH on rank-3 groups and intersection matrices were ones that I dissected more than read. They seemed like magic—and those papers, together with papers by Peter Cameron and Charles Sims that built on the notion of orbitals for transitive permutation groups as well as Norman Biggs's book on algebraic graph theory, were formative in my thinking and development in work on automorphism groups of graphs....

"I visited Michigan for exactly one day in 1974. I remember meeting DGH then (and also meeting Bob [Griess] for the first time). DGH asked me to send him copies of my preprints. I cannot explain how overwhelmed I felt by his asking me to do this. I really felt like I was unworthy, that I should presume to send this great man something that I had written. I know that sounds bizarre, but that's how it felt to me in 1974."

6.8. C. C. Sims

Charles Sims's first course in abstract algebra, when he was an undergraduate at the University of Michigan, had been taught by Don Higman. Thus, Don was a teacher who became a colleague. Charles writes as follows about his first consultation with Don.

"I think the first research interaction with Don occurred shortly after I got to Rutgers [about 1966]. I read one of Don's papers on rank-3 groups and as I remember it, Don asked about a group of degree 50 with subdegrees 1, 7, and 42. I wrote to him to point out that two graph theorists, Hoffman and Singleton, had proved in 1960 the uniqueness of the associated strongly regular graph without the assumption that the graph had an automorphism group that was edge transitive. This started the contacts that ultimately [led] to the discovery of Higman–Sims."

6.9. Stephen D. Smith

Steve Smith writes: "In the mid-70s, conversations with Don and Bruce Cooperstein (I think at Ann Arbor) showed me the relevance of Dynkin diagrams to the geometry of the Lie type groups (this was before I understood much about Tits buildings). That in turn was the background for my input into the joint work with Mark Ronan on the 2-local geometries for sporadic groups [43]. (And hence for my later long-term interest in geometries for groups.)"

6.10. John G. Thompson

In the 1990s, Don Higman told Bob Griess that young John Thompson spoke to Don about some of his insecurities during his struggle to prove the Frobenius conjecture. Thompson's eventual solution resulted in his celebrated 1959 thesis and in energizing the classification work on finite simple groups. Don also reported that at one Oberwolfach meeting he observed John Thompson pacing for hours, pondering a step in the classification of finite simple groups. After the pacing, Thompson was pleased to tell Don that he saw what to do. The date of this event is unknown. However it probably refers to work on the N-group theorem, which places it in the mid- to late 1960s. Don's role as confidant confirms the general impression he gave as a concerned and friendly authority figure in the groups and geometry community.

John Thompson confirms that the preceding paragraph "touches the right bases", adding: "One of my abiding memories of Don is that he came up with a simple group on 100 letters [Don Higman and Charles Sims are co-discoverers] which was not the one found by Hall and Janko. This really made me scratch my head and wonder if the list of sporadic simple groups would terminate. The equations $100 = 1 + 36 + 63 = 1 + 22 + 77$ were really scary."

6.10.1. About the Odd-Order Theorem

This is the famous and important theorem by Walter Feit and John Thompson [20], which states that finite groups of odd order are solvable. It is technically very difficult and took up an entire issue of the *Pacific Journal of Mathematics* in 1963. Don Higman told Bob Griess that he (DH) had been one of a team of referees for that paper of 255 pages. Don said that his responsibility was mainly the generators and relations section at the end.

Bob Griess acquired two copies of that issue from Don's collection. One includes a note, in Walter Feit's handwriting, on Yale University stationery. It says only "You are incorrigible." The note is not addressed to anyone. It is in Don's possession and so was probably inserted in an offprint that Walter gave Don.

6.11. Students and Associates

The fifteen Ph.D. students of Don Higman are listed at the end of this section. Many found jobs at research universities and published actively. During the years of Higman and McLaughlin at the Unversity of Michigan, these two often had many students and there was a lot of interaction between them. The students of Roger Lyndon and Bob Griess—though in group theory and interactive with the Higman and McLaughlin circle—were less numerous and mathematically a bit distant.

Three of Don's students contributed articles to this memorial issue: Robert Liebler (1970), Bruce Cooperstein (1975), and Sylvia Hobart (1987).

A few students testified that Don gently nudged and engaged them more and more until they were doing original mathematics. Manley Perkel wrote about his realization, during one of Don's advanced courses on permutation groups, that Don was in the midst of working out parts of his new theory on coherent configurations even while simultaneously lecturing on them. Perkel realized only later what a special opportunity Don gave him, as a student, to read early drafts of many of his papers on this subject, especially one component ("Part IV") of the coherent configuration theory.

Perkel also wrote that "the mathematics department at the University of Michigan in the 1970s was an amazing place to meet distinguished visitors who were there primarily due to Don's influence. Among the long-term visitors during this period were Peter Neumann, Bill Kantor, and Peter Cameron, and short-term visitors included Ernie Schult, John Conway, Charles Sims, Bernd Fischer, and Jaap Seidel, among others (including Cheryl Praeger's one-day visit), all there no doubt because of Don. For a graduate student to have the opportunity to meet such distinguished mathematicians was inspiring, to say the least." A complete list of Don's visitors would be much longer.

Bob Liebler relates this impression from his grad student years in the late 1960s: "For many years Higman and Roger [Lyndon] shared an office on the 3rd floor of Angel hall. [Lyndon] was a chain smoker and Higman something of an [athlete]. Therefore Higman minimized the time he spent in the office. Somehow I discovered that he would hang out in the basement of the student union, often occupying a large table with stacks of papers and computer printouts. I developed a habit of approaching him there, and he was always happy to talk about math or even politics."

Don spent a lot of time working with other mathematicians. Betty Higman mentions Charles Sims and adds that "Michio Suzuki spent a lot of time with Don when they were both working on their Ph.D. at Illinois, and Hans Zassenhaus was whom he worked with when he had the National Science Fellowhip at McGill. He also had contact with Jim Lambek (McGill) and Jean Maranda [in Montréal].

"Since that time Karl Gruenberg, Bernd Fischer, Christoph Hering,.... Jaap Seidel was his main contact at Eindhoven. Most recently he mentioned Willem

Haemers, also from Einhoven; Don Taylor from Australia was also at Eindhoven when we were there. As I wrote before I don't have any real idea of how much they influenced him but I do know he spent a lot of time with each one.

"As far as Jack McLaughlin goes I remember Don saying that they thought alike and could almost anticipate what the other would come up with, they were on the same wavelength."

Finally, we mention that Graham Higman and Don Higman both worked on finite groups and combinatorics. Both had quite heavy eyebrows, a point often remarked on by observers. Both men had ancestors from the same area in Cornwall, England, though no familial relationship between them has been established. See [12].

6.11.1. List of Doctoral Students

James Brooks, University of Michigan, 1964
Yaotsu Chang, University of Michigan, 1994 (ytchang@isu.edu.tw)
Bruce Cooperstein, University of Michigan, 1975 (coop@ucsc.edu)
Raymond Czerwinski, University of Michigan, 1966
David Foulser, University of Michigan, 1963
Robert Gill, University of Michigan, 1998
Marshall Hestenes, University of Michigan, 1967 (hestenes@msu.edu)
Sylvia Hobart, University of Michigan, 1987 (SAHobart@uwyo.edu)
Alan Hoffer, University of Michigan, 1969
Robert Liebler, University of Michigan, 1970 (liebler@math.colostate.edu)
Roger Needham, University of Michigan, 1992 (needham@rio.sci.ccny.cuny.edu)
Manley Perkel, University of Michigan, 1977 (manley.perkel@wright.edu)
Alyssa Sankey, University of Michigan, 1992 (asankey@unb.ca)
Betty Stark, University of Michigan, 1971 (salzberg@ccs.neu.edu)
Andrew Vince, University of Michigan, 1981 (vince@math.ufl.edu)

7. Honors

Donald Higman's academic honors include giving an invited lecture to the 1970 International Congress of Mathematicians in Nice, where he presented his theory of rank-3 groups. Don received the 1975 Alexander von Humboldt Stiftung Prize. He spent sabbatical and academic leaves in Eindhoven and Giessen, was a visiting professor at Frankfurt, a visiting senior scientist at Birmingham and Oxford, and a visiting fellow at the Institute for Advanced Study in Canberra, Australia.

8. Life in Ann Arbor

The atmosphere in the University of Michigan Mathematics Department in the 1950s and 1960s was especially cordial. There were many parties and frequent expressions of hospitality.

Hirosi Nagao, who visited the University of Michigan in 1958/59, spoke of many interactions involving Raoul Bott, Donald Higman, Donald Livingstone, and Roger Lyndon. He says “At Ann Arbor, Higman, McLaughlin and Raoul Bott used to have lunch together at the University Union [now called the Michigan Union]. I often joined them and enjoyed listening to their conversations. I was invited many times to [parties] held at the home of Higman.”

Donald Higman, wife Betty, and their five children were generous hosts on many social occasions. Betty worked at the Library for the Blind and Physically Handicapped. Don was an active member of the Flounders and the Ann Arbor Track Club, and he frequently rode his bicycle. Don played water polo well into his later years. Bob Griess remembers many times being at home, noticing Don jogging past as he expelled his breath in loud bursts. Regular jogging continued for years past Don’s 1998 retirement.

9. Donald Higman Publications and Preprints

We found 48 items listed on MathSciNet and 10 additional items, including a historically significant table [H57]. A citation made in this article is listed here if Donald Higman was an author; other citations are listed in the References.

[H1] Higman, D. G., *Lattice homomorphisms induced by group homomorphisms.* Proc. Amer. Math. Soc. 2 (1951), 467–478. [MR0041138]

[H2] Higman, D. G., *Focal series in finite groups,* Thesis abstract, Univ. Illinois, 1952. [MR0047644]

[H3] Higman, D. G., *Focal series in finite groups,* Canad. J. Math. 5 (1953), 477–497. [MR0058597]

[H4] Higman, D. G., *Remarks on splitting extensions,* Pacific J. Math. 4 (1954), 545–555. [MR0066379]

[H5] Higman, D. G., *Modules with a group of operators,* Duke Math. J. 21 (1954), 369–376. [MR0067895]

[H6] Higman, D. G., *Indecomposable representations at characteristic p*, Duke Math. J. 21 (1954), 377–381. [MR0067896]

[H7] Higman, D. G., *Induced and produced modules,* Canad. J. Math. 7 (1955), 490–508. [MR0087671]

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The items listed below are not referred to in MathScinet. Except for [H50], they have not been published as far as we know. The first three have dates but the others may not. Some physical characteristics are noted.

[H49] Preprint: "Part __, Homogeneous Configurations of Rank 4" (36 pages, handwritten) [Don Higman told Manley Perkel that this was supposed to be part 4 of a series]. Don Higman gave this to Manley Perkel in 1974 to proofread; it is related to notes from a course in 1972/73 at the University of Michigan.

[H50] "Monomial Representations"; this was published in *Proceedings of the Taniguchi International Symposium on Finite Groups* (N. Iwahori, ed.), pp. 55–68, Japan Society for the Promotion of Science, 1976.

[H51] Preprint: "Some Highly Symmetric Chamber Systems" (8 pages); from Don Higman's collection, obtained by Bob Griess.

[H52] Preprint: "A Note on Regular 3-graphs" (7 pages); contributed by Alyssa Sankey.

[H53] Preprint: "The Parabolics of a Semi-coherent Configuration" (18 pages, seems to be dot-matrix printed); contributed by Alyssa Sankey.

[H54] Preprint: "Relation Configurations and Relation Algebras" (27 pages, signature under title); contributed by Alyssa Sankey.

[H55] Preprint: "Uniform Association Schemes" (23 pages); contributed by Alyssa Sankey.

[H56] Preprint: "Untitled" (16 pages, typed, with handwriting); contributed by Alyssa Sankey. (The table of contents starts with color schemes and morphisms and ends with homology and weights.)

[H57] "This is a List of the Known Rank Three Groups for Degrees up to 10000"; copy provided by Francis Buekenhout, who gives the date 17 July 1968. The print had faded and was difficult to read; a transcription of it (done in 2007), consisting of most of the information on the table, is on the Web site of Robert Griess. We thank Ching Hung Lam for arranging the typing.

[H58] Table: "Parameter Values for Rank 3 Groups." In the mid-1990s, Don Higman gave a table of parameter values to Bob Griess for inclusion in his book [24, p. 125]; this is a smaller set of data than [H57].

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