# On the Structure of Some Lie Algebras of Kuznetsov

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## 1. Introduction

In [3] Frank showed the existence of two simple graded Lie algebras $L=\sum_{i=-1}^{r}\oplus L_i$ of characteristic 3 for which $L_0$ is solvable. In [4] Kuznetsov has given a classification of simple graded Lie algebras $L=\sum_{i=-1}^{r}\oplus L_i$ with $L_0$ having a noncentral radical. In this investigation he encountered algebras of characteristic 3 of what he called series $T$ and $\mathcal{R}$, which are described in terms of Cartan prolongations of certain solvable Lie algebras.

It is our purpose here to give (in Section 2) a simple description of Kuznetsov's algebras that will reveal many of their structural features, and then in Section 3 to prove the isomorphism of the algebras of series $T$ with those described in [2] and to point out structural differences between those of series $\mathcal{R}$ and contact algebras of the same dimension.

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## 2. Description of the Algebras

We shall regard the notation and terminology of [1] as standard and use it without necessarily redefining it here.

Let $F$ be a field of characteristic 3. Let $R=W(2)\oplus\mathfrak{A}(2)$, where $\mathfrak{A}(2)$ is the completed free divided power algebra in $x_1, x_2$ over $F$, and $W(2)$ is the Lie algebra of special derivations of $\mathfrak{A}(2)$. $R$ is an algebra under a product $[\ ,\ ]$ such that for $D\in W(2)$ and $f,g\in\mathfrak{A}(2)$, $[D,f]=-[f,D]=\operatorname{div}(fD)$ (where, as usual, $\operatorname{div}(u_1D_1+u_2D_2)=D_1u_1+D_2u_2$); $[f,g]=f\mathfrak{D}_g-g\mathfrak{D}_f$ where $\mathfrak{D}_f=(D_2f)D_1-(D_1f)D_2$; and on $W(2)$ $[\ ,\ ]$ is the usual Lie product.

Let $\mathfrak{A}^{\#}$ be the subset of $\mathfrak{A}(2)$ consisting of all formal sums

$$\sum_{0\le i\le 1}\sum_{0\le j} a_{ij}x_1^{(i)}x_2^{(j)} \quad \text{with } a_{ij}\in F,$$

and let $W^{\#}$ be the subalgebra of $W(2)$ consisting of all formal sums

$$\sum_{0\le i\le 2}\sum_{0\le j} b_{ij}x_1^{(i)}x_2^{(j)}D_1+\sum_{0\le k} c_kx_2^{(k)}D_2 \quad \text{with } b_{ij}, c_k\in F.$$

$T^{\#}=W^{\#}\oplus\mathfrak{A}^{\#}$ is a subalgebra of $R$ under the product $[\ ,\ ]$ defined above.

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THEOREM 2.1. *$R$ and $T^\#$ are Lie algebras over $F$.*

*Proof.* Anticommutativity is clear from the definition of [ , ]. For $f, g \in \mathfrak{A}(2)$ and $D, E \in W(2)$, it readily follows from the definition of [ , ] and the properties of derivations that $[D, fE] = f[D,E] + (Df)E$, $\operatorname{div}(fD) = f(\operatorname{div} D) + (Df)$, $\operatorname{div}[D,E] = D(\operatorname{div} E) - E(\operatorname{div} D)$, $\mathfrak{D}_{fg} = f\mathfrak{D}_g + g\mathfrak{D}_f$, and $[D, \mathfrak{D}_f] = \mathfrak{D}_{Df} - (\operatorname{div} D)\mathfrak{D}_f$. From these it follows that

$$[[D,E],f] = fD(\operatorname{div} E) - fE(\operatorname{div} D) + [D,E]f,$$

$$[D,[E,f]] = f(\operatorname{div} E)(\operatorname{div} D) + (Df)(\operatorname{div} E) + fD(\operatorname{div} E) + (Ef)(\operatorname{div} D) + DEf,$$

$$[D,[f,g]] = f\mathfrak{D}_{Dg} - f(\operatorname{div} D)\mathfrak{D}_g + (Df)\mathfrak{D}_g - g\mathfrak{D}_{Df} + g(\operatorname{div} D)\mathfrak{D}_f - (Dg)\mathfrak{D}_f,$$

$$[[D,f],g] = f(\operatorname{div} D)\mathfrak{D}_g - fg\mathfrak{D}_{\operatorname{div} D} - g(\operatorname{div} D)\mathfrak{D}_f + (Df)\mathfrak{D}_g - g\mathfrak{D}_{Df},$$

and

$$[[f,g],h] = h(f \operatorname{div} \mathfrak{D}_g - g \operatorname{div} \mathfrak{D}_f + \mathfrak{D}_g f - \mathfrak{D}_f g) + (f\mathfrak{D}_g - g\mathfrak{D}_f)(h).$$

Therefore

$$[[D,f],E] + [D,[E,f]] = [[D,E],f],$$

$$[D,[f,g]] - [[D,f],g] - [f,[D,g]] = 3(\operatorname{div} D)(-f\mathfrak{D}_g + g\mathfrak{D}_f),$$

and

$$[[f,g],h] + [[g,h],f] + [[h,f],g] = 0.$$

Thus, since $W(2)$ is a Lie algebra and since $F$ has characteristic 3, the Jacobi identity is satisfied and both $R$ and its subalgebra $T^\#$ are Lie algebras. □

In order to describe the algebras of [4] we consider a vector space $V$ over $F$ and a Lie algebra $\mathfrak{L} \subset \mathfrak{gl}(V)$. Let $\mathfrak{L}^{(-1)} = V$ and $\mathfrak{L}^{(0)} = \mathfrak{L}$, and inductively define $\mathfrak{L}^{(i)} = \{\phi \in \operatorname{Hom}(V, \mathfrak{L}^{(i-1)}) \mid \phi(x)(y) = \phi(y)(x),\ x, y \in V\}$ for $i > 0$. The space $L$ of all formal sums $\sum_{i=-1}^{\infty} \ell_i$, where $\ell_i \in \mathfrak{L}^{(i)}$, becomes a Lie algebra when supplied with the product determined by $[\mathfrak{L}^{(-1)}, \mathfrak{L}^{(-1)}] = 0$, $[\phi, y] = -[y, \phi] = \phi(y)$ for $y \in V$, $\phi \in \mathfrak{L}^{(i)}$, $i > 0$, and (inductively) for $\phi \in \mathfrak{L}^{(i)}$, $\psi \in \mathfrak{L}^{(j)}$ $(i, j \geq 0)$ by $[\phi, \psi](y) = [\phi, \psi(y)] - [\psi, \phi(y)]$ for all $y \in V$. Following Kuznetsov, let $\mathfrak{R}_{-1} = T_{-1} = F[x]/(x^3)$,

$$\mathfrak{R}_0 = \left\langle \frac{d^2}{dx^2}, x\frac{d}{dx}, x^2\frac{d}{dx}, x, 1 \right\rangle \subset \mathfrak{gl}(\mathfrak{R}_{-1}),$$

and

$$T_0 = \left\langle x\frac{d}{dx}, x^2\frac{d}{dx}, x, 1 \right\rangle \subset \mathfrak{R}_0.$$

For $\mathfrak{L}^{(-1)} = \mathfrak{R}_{-1}(T_{-1})$ and $\mathfrak{L}^{(0)} = \mathfrak{R}_0(T_0)$ we denote the corresponding $L$ by $\mathfrak{R}(T)$ and $\mathfrak{L}^{(i)}$ by $\mathfrak{R}_i(T_i)$. We shall show that $\mathfrak{R}(T)$ is isomorphic to $R(T^\#)$.

Let a topological grading be defined on $\mathfrak{A}(2)$ and $W(2)$ in which $\mathfrak{A}(2)_{[i]}$ is spanned by monomials $u$ with $\deg u = i+1$ and $W(2)_{[i]}$ by elements $u_1D_1 + u_2D_2$ for monomials $u_1$, $u_2$ with $\deg u_1 = \deg u_2 = i+1$, where $\deg x_1 = \deg x_2 = 1$. Then $R$ has a topological grading in which $R_{[i]} = W(2)_{[i]} \oplus \mathfrak{A}(2)_{[i]}$.

The mapping $\sigma$ taking $1+(x^3), x+(x^3), x^2+(x^3)$ into $D_2, -1, D_1$, respectively, and $d^2/dx^2, x(d/dx), x^2, d/dx, x, 1$ into $x_1D_2, x_1D_1, -x_2D_1, -x_1, x_2, -x_1D_1-x_2D_2$, respectively, determines an isomorphism from $\mathcal{R}_{-1}\oplus\mathcal{R}_0$ onto $R_{[-1]}\oplus R_{[0]}$ preserving the grading indicated by the subscripts.

Let $\Phi_i=\mathcal{R}_i$ for $i=-1,0$, and for $i>0$ inductively define $\Phi_i$ to be the set of $\phi_\ell\in\mathrm{Hom}(\mathcal{R}_{-1}, \Phi_{i-1})$ for $\ell\in R_{[i]}$, where $\phi_\ell(v)=\phi_{[\ell,\sigma(v)]}$ for $v\in\mathcal{R}_{-1}$ and $\phi_\ell=\sigma^{-1}(\ell)$ for $\ell\in R_{[-1]}\oplus R_{[0]}$. We readily observe that $\Phi_i$ is a subspace of $\mathcal{R}_i$ and that the correspondence $\ell\to\phi_\ell$ is one-to-one since $[\ell, R_{[-1]}]=0$ only for $\ell\in R_{[-1]}$.

LEMMA 2.2. $\Phi_j=\mathcal{R}_j$ *for all* $j\geq -1$.

*Proof.* By definition, $\Phi_j=\mathcal{R}_j$ for $j\leq 0$. Therefore suppose $j>0$, and assume that $\Phi_i=\mathcal{R}_i$ for all $i<j$. $\Phi_i=\Phi_{i,W}\oplus\Phi_{i,\mathfrak{H}}$, where $\Phi_{i,W}=\{\phi_\ell\in\Phi_i \mid \ell\in W(2)\}$ and $\Phi_{i,\mathfrak{H}}=\{\phi_\ell\in\Phi_i \mid \ell\in\mathfrak{H}(2)\}$. For $\phi\in\mathcal{R}_j$ let $\phi\sigma^{-1}(G)=\psi_1\sigma^{-1}(G)+\psi_2\sigma^{-1}(G)$ for $G\in W(2)_{[-1]}$, $\phi\sigma^{-1}(1)=\psi_2\sigma^{-1}(1)+\psi_1\sigma^{-1}(1)$ where $\psi_1\sigma^{-1}(G), \psi_2\sigma^{-1}(1)\in\Phi_{j-1,W}$ and $\psi_1\sigma^{-1}(1), \psi_2\sigma^{-1}(G)\in\Phi_{j-1,\mathfrak{H}}$. Hence the condition $\phi(x)(y)=\phi(y)(x)$ implies that $\psi_1,\psi_2\in\mathcal{R}_j$. Therefore, for $k=1$, 2, $\psi_1\sigma^{-1}(D_k)=\phi_{a_{k1}D_1+a_{k2}D_2}$ for some $a_{k1}D_1+a_{k2}D_2\in W(2)_{[j-1]}$ and $\psi_1\sigma^{-1}(1)=\phi_f$ for some $f\in\mathfrak{H}(2)_{[j-1]}$. The definition of $\mathcal{R}_j$ requires $D_2a_{1k}=D_1a_{2k}$ and $D_1a_{k1}+D_2a_{k2}=-D_kf$ for $k=1,2$. For $a_{k\ell}$ $(1\leq k,\ell\leq 2)$ and $f$ satisfying these equations, there exists a unique $g_1D_1+g_2D_2\in R_{[j]}$ such that $a_{k\ell}=-D_kg_\ell$ and $f=\mathrm{div}(g_1D_1+g_2D_2)$. It is easily observed that $\psi_1=\phi_{g_1D_1+g_2D_2}$. For $k=1,2$, $\psi_2\sigma^{-1}(1)=\phi_{h_1D_1+h_2D_2}$ and $\psi_2\sigma^{-1}(D_k)=\phi_{f_k}$ for some $h_1D_1+h_2D_2\in W(2)_{[j-1]}$ and $f_k\in\mathfrak{H}(2)_{[j-1]}$. The definition of $\mathcal{R}_j$ requires $D_2f_1=D_1f_2$, $D_1h_1=D_2f_1$, $D_1h_2=-D_1f_1$, $D_2h_1=D_2f_2$, and $D_2h_2=-D_1f_2$. Because $j>0$, these are equivalent to $f_1=-h_2$, $f_2=h_1$, and $\mathrm{div}(h_1D_1+h_2D_2)=0$. For such $h_1, h_2$ there exists a unique $g\in R_{[j]}$ such that $D_1g=h_2$ and $D_2g=-h_1$. It is easily observed that $\psi_2=\phi_g$. Thus $\mathcal{R}_j=\Phi_{j,W}\oplus\Phi_{j,\mathfrak{H}}=\Phi_j$, and the lemma follows by induction. □

Let the map $\sigma$ defined above be extended to the linear map from $\mathcal{R}$ onto $R$ determined by defining $\sigma(\phi_\ell)=\ell$ for $\ell\in R_{[i]}$, $i>0$.

$$\sigma([\phi_{\ell_1}, \phi_{\ell_2}])=[\sigma(\phi_{\ell_1}), \sigma(\phi_{\ell_2})] \quad \text{with } \ell_1\in R_{[j]},\ \ell_2\in R_{[k]}$$

has already been shown to hold if $j$ or $k$ is $-1$ or if $j=k=0$. Assume that it holds whenever $-2\leq j+k<n$, and suppose that $j+k=n$. For $v\in R_{-1}$,

$$[\phi_{\ell_1}, \phi_{\ell_2}](v)=[\phi_{\ell_1}, \phi_{\ell_2}(v)]-[\phi_{\ell_2}, \phi_{\ell_1}(v)]=[\phi_{\ell_1}, \phi_{[\ell_2,\sigma(v)]}]-[\phi_{\ell_2}, \phi_{[\ell_1,\sigma(v)]}].$$

Thus, by induction and Theorem 2.1,

$$\begin{aligned}[\phi_{\ell_1}, \phi_{\ell_2}](v)&=\sigma^{-1}([\ell_1,[\ell_2,\sigma(v)]]-[\ell_2,[\ell_1,\sigma(v)]])\\ &=\sigma^{-1}([[\ell_1,\ell_2],\sigma(v)])=\phi_{[[\ell_1,\ell_2]\sigma(v)]}=\phi_{[\ell_1,\ell_2]}(v).\end{aligned}$$

Therefore $[\phi_{\ell_1}, \phi_{\ell_2}]=\phi_{[\ell_1,\ell_2]}$, and so by induction

$$\sigma([\phi_{\ell_1}, \phi_{\ell_2}])=[\sigma(\phi_{\ell_1}), \sigma(\phi_{\ell_2})]$$

holds for all $\ell_1, \ell_2\in R$, and the following result has been established.

THEOREM 2.3. *$R$ is a Lie algebra isomorphic to $\mathfrak{R}$.*

THEOREM 2.4. *$T^{\#}$ is a Lie algebra isomorphic to $T$.*

*Proof.* Simply modify the proof of Theorem 2.3 by replacing $R$ and $\mathfrak{R}$ by $T^{\#}$ and $T$, respectively, and subspaces of $R$ and $\mathfrak{R}$ by their intersections with $T^{\#}$ and $T$, respectively. Note from the restrictions on $\deg x_1$ that

$$g_1 D_1 + g_2 D_2 \in T^{\#} \quad \text{and} \quad g \in T^{\#}$$

in the proof of Lemma 2.2. Hence it follows that $\sigma(T) = T^{\#}$. □

We now consider isomorphic copies of the algebras whose existence was implied in [4]. Let $R(2 : (n_1, n_2))$ be the subalgebra of $R$ spanned by $W(2:(n_1,n_2))$ and $\mathfrak{A}(2:(n_1,n_2))$, and let $T(n) = R(2:(1,n)) \cap T^{\#}$.

Clearly the derived algebra $R(2:(n_1,n_2))^{(1)}$ is spanned by $W(2:(n_1,n_2))$ and $\{x_1^{(i)}x_2^{(j)} \in \mathfrak{A}(2:(n_1,n_2)) \mid i+j < 3^{n_1}+3^{n_2}-2\}$, and hence has dimension $3^{n_1+n_2+1}-1$; $T(n)$ has dimension $2\cdot 3^{n+1}$.

## 3. Some Properties of $R(2:(n_1,n_2))$ and $T(n)$

We first show that the algebras $T(n)$ are isomorphic to the algebras studied in [2]. Thus they are simple and also have gradings for which their 0-component is isomorphic to $\mathfrak{gl}(2)$.

THEOREM 3.1. *$T(n)$ is isomorphic to the algebra $T(3:n)$ of* [2].

*Proof.* The linear mapping sending $x_2^{(i+1)}D_1$, $-x_1^{(2)}x_2^{(i-1)}D_1$, $x_1x_2^{(i)}D_1$, $x_2^{(i+1)}D_2$, $x_2^{(i+1)}$, $-x_1x_2^{(i)}$ into $a_{2(i+1)}$, $b_{2(i-1)}$, $c_{2i}$, $d_{2i}$, $e_{2i+1}$, $f_{2i-1}$ respectively, for the values of $i$ for which these elements are defined, is the required isomorphism. □

THEOREM 3.2. *$R(2:(n_1,n_2))^{(1)}$ is simple for all positive integers $n_1, n_2$.*

*Proof.* A nonzero ideal $I$ of this algebra contains a nonzero element $\ell \in R_{[-1]}$. Either $\ell \in \mathfrak{A}(2)$ or $[x_1D_1 - x_2D_2, \ell] \in W(2)_{[-1]} \backslash 0$. If $\ell \in \mathfrak{A}(2)_{[-1]}$, then $[x_1, \ell] \in W(2)_{[-1]} \backslash 0$. Therefore $I$ contains a nonzero element of $W(2)_{[-1]}$, and so the simplicity of $W(2:(n_1,n_2))$ implies its inclusion in $I$. Because the Cartan subalgebra $\langle x_1D_1, x_2D_2\rangle \subset I$, so are the root-space elements

$$x_1^{(3^{n_1}-1)}x_2^{(3^{n_2}-2)} \quad \text{and} \quad x_1^{(3^{n_1}-2)}x_2^{(3^{n_2}-1)},$$

whence also all of $\mathfrak{A}(2) \cap R(2:(n_1,n_2))^{(1)} \subset I$. Thus $I = R(2:(n_1,n_2))^{(1)}$. □

THEOREM 3.3. *$R(2:(n_1,n_2))^{(1)}$ is restricted if and only if $n_1 = n_2 = 1$.*

*Proof.* Because $W(2)$ is invariant under $\operatorname{ad}\ell$ if and only if $\ell \in W(2)$, $R(2:(n_1,n_2))^{(1)}$ cannot be restricted unless $W(2:(n_1,n_2))$ is; that is, $n_1 = n_2 = 1$. Conversely, for $f \in \mathfrak{A}(2)$, $(\operatorname{ad} f)^3 = 0$, and if $n_1 = n_2 = 1$ and $g \in \mathfrak{A}(2 : (1, 1))$ then $(gD_k)^3 = (g(D_kg)^2 + g^2(D_k^2g))D_k$ for $k = 1, 2$, and so

$(\operatorname{ad} gD_k)^3 = \operatorname{ad}((g(D_k g)^2 + g^2(D_k^2 g))D_k)$ on $W(2:(1,1))$; this is also seen to hold when applied to $\mathfrak{R}(2) \cap R(2:(1,1))^{(1)}$. □

THEOREM 3.4. *There exists a toral element* $t \in R(2:(n_1, n_2))^{(1)}$ *such that the kernel of* $\operatorname{ad} t$ *is solvable of dimension* $3^{n_1+n_2}-1$.

*Proof.* $t = x_1 D_1 - x_2 D_2$ is toral. The kernel of $\operatorname{ad} t$ is $N = \langle x_1^{(i)} x_2^{(j)} D_1,\ i-j \equiv 1 \pmod 3;\ x_1^{(i)} x_2^{(j)} D_2,\ i-j \equiv -1 \pmod 3;\ x_1^{(i)} x_2^{(j)},\ i-j \equiv 0 \pmod 3 \rangle$. Thus

$$N^{(1)} \subseteq \langle 1, x_1 D_1 - x_2 D_2 \rangle + N \cap \sum_{i>0} R_{[i]}, \quad N^{(2)} \subseteq \langle x_1 D_1 - x_2 D_2 \rangle + N \cap \sum_{i>0} R_{[i]},$$

and $N^{(3)} \subseteq \sum_{i>0} R_{[i]}$. Hence $N$ must be solvable, and $t$ has the properties stated in the theorem. □

Theorem 3.4 will enable us to show that $R(2:(n_1, n_2))^{(1)}$ is not isomorphic to any of the contact algebras of the same dimension, that is, $K(m:\mathbf{q})^{(1)}$ where $m = 2r+1$, 3 divides $m$, and $\sum_{i=1}^{m} q_i = n_1 + n_2 + 1$.

THEOREM 3.5. $R(2:(n_1, n_2))^{(1)}$ *is not isomorphic to any of the contact algebras* $K(m:\mathbf{q})^{(1)}$.

*Proof.* For any Lie algebra $M$ of prime characteristic $p$, define $\tau(M) = \{D \in M \mid (\operatorname{ad} E)^p D \in [M, D]\ \forall E \in M\}$. Since the formulas for $(\operatorname{ad} f)^3$ and $(\operatorname{ad} gD_k)^3$ used in the proof of Theorem 3.3 are valid in $R(2:(n_1, n_2))^{(1)}$ when applied to $R(2:(1,1))$, it is readily observed that

$$\tau(R(2:(n_1, n_2))^{(1)}) = R(2:(1,1)) \quad \text{if} \quad (n_1, n_2) \neq (1,1),$$

while clearly $\tau(R(2:(1,1))^{(1)}) = R(2:(1,1))^{(1)}$. By similar reasoning we have $\tau(K(m:\mathbf{q})^{(1)}) = K(m:\mathbf{1})$ if $\mathbf{q} \neq \mathbf{1}$, and $\tau(K(m:\mathbf{1})^{(1)}) = K(m:\mathbf{1})^{(1)}$. Therefore any isomorphism between $R(2:(n_1, n_2))^{(1)}$ and $K(m:\mathbf{q})^{(1)}$ must induce an ismorphism between $R(2:(1,1))^{(1)}$ and $K(m:\mathbf{1})^{(1)}$. Thus by comparison of dimensions this requires $m = 3$. By Theorem 1.3.1 of [1] every toral element of $K(3:\mathbf{1})^{(1)}$ is a conjugate of one of the following: $t_1 = \mathfrak{D}_K(x_1 x_2)$, $t_2 = \mathfrak{D}_K(x_3)$, $t_3 = \mathfrak{D}_K(x_1 x_2 + x_3)$, $t_4 = \mathfrak{D}_K(x_1 x_2 - x_3)$, $t_5 = \mathfrak{D}_K(x_3 + 1)$, or $t_6 = \mathfrak{D}_K((x_1 + 1)x_2)$. It is readily verified that the algebra $A_i$ $(1 \leq i \leq 6)$ is contained in the kernel of $\operatorname{ad} t_i$, where

$$A_1 = \langle \mathfrak{D}_K(1), \mathfrak{D}_K(x_3), \mathfrak{D}_K(x_3^{(2)}) \rangle,$$
$$A_2 = A_5 = \langle \mathfrak{D}_K(x_1^{(2)}), \mathfrak{D}_K(x_1 x_2), \mathfrak{D}_K(x_2^{(2)}) \rangle,$$
$$A_3 = \langle \mathfrak{D}_K(x_1), \mathfrak{D}_K(x_1, x_2), \mathfrak{D}_K(x_1 x_2^{(2)}) \rangle,$$
$$A_4 = \langle \mathfrak{D}_K(x_2), \mathfrak{D}_K(x_1 x_2), \mathfrak{D}_K(x_1^{(2)} x_2) \rangle,$$

and

$$A_6 = \langle \mathfrak{D}_K(1), \mathfrak{D}_K(x_2 - x_1 x_2), \mathfrak{D}_K((x_3 - x_1 x_2)^{(2)}) \rangle.$$

Since each of these algebras is simple, the kernel of $\operatorname{ad} t$ for a toral element $t \in K(3:\mathbf{1})^{(1)}$ cannot be solvable. Thus, by Theorem 3.4, $R(2:(1,1))^{(1)}$ cannot be isomorphic to $K(3:\mathbf{1})^{(1)}$ and so $R(2:(n_1, n_2))^{(1)}$ is not isomorphic to any of the contact algebras $K(m:\mathbf{q})^{(1)}$. □

## References

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