BOUNDING A FREE ACTION OF A DIHEDRAL GROUP

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1. RESULTS

Let C be a cyclic group of order 2^{n+1} ($n \geq 1$), and let G be one of the non-abelian split extensions of C by Z_2. The dihedral group of order 2^{n+2} is one example; there are two others [5, p. 187].

This paper considers smooth actions of G preserving a unitary (that is, weakly complex) structure on a smooth manifold. Let $U_*(G)$ be the bordism of all such actions, and let $\hat{U}_*(G)$ be the corresponding bordism of free unitary G-actions. Full definitions can be found in [10]. By [10, Proposition 2.3], we know that $\hat{U}_*(G) \cong U_*(BG)$.

THEOREM. *The kernel of the forgetful homomorphism* $s: \hat{U}_*(G) \to U_*(G)$ *is precisely* $\tilde{U}_*(BG)$.

COROLLARY. *Let* $\phi : G \times M \to M$ *be a free unitary* G-*action on a closed manifold. Then* $[M, \phi] = 0$ *in* $U_*(G)$ *if and only if* $[M] = 0$ *in* U_*.

To derive the corollary, one uses the analogue of [4, (19.4)] for unitary actions; this shows that $[M] = 0$ if and only if $[M, \phi] \in \operatorname{im} \tilde{U}_*(BG)$.

It is worth noticing that to prove the theorem for any group, it suffices to establish it for the Sylow subgroups (see [7, Proposition 6]). In particular, our results imply the theorem and corollary for a dihedral group of any order.

2. A TRANSVERSALITY LEMMA

Suppose H is a finite group. Let M and N be smooth H-manifolds, and let $P \subseteq N$ be an invariant submanifold. One says that *transversality* holds for (M, N, P) if, given an equivariant $f: M \to N$ and a closed invariant $A \subseteq M$ such that f is transverse to P on A, one may deform f by an H-homotopy making it transverse to P on all of M and leaving f fixed in a neighborhood of A.

LEMMA 1. *Transversality holds for* (M, N, P) *if either*

(a) H *acts freely on* M *or*

(b) H *is nilpotent and the normal bundle* $\nu \to P$ *has the property that, if* $hp = p$ *for some* $h \in H$ *and* $p \in P$, *then* $hx = x$ *for all* $x \in \nu_p$.

Proof. The sufficiency of (a) is fairly well known; a proof is to appear in [8, Proposition 2.2]. The sufficiency of (b) is a generalization of [10, Lemma 4.2]. Since H is nilpotent, it contains a central cyclic subgroup T of prime order. By the argument of [10, Lemma 4.2], we may assume that the fixed set M^T of T is empty.

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Now consider $f \times id: M \to N \times M$, which is an equivariant map of H-manifolds. Since T acts freely on M, we may divide out its action. By induction on the order of the group, we may deform the map $h: M/T \to (N \times M)/T$ of orbit spaces to make it transverse to $(P \times M)/T$. To finish the argument of [10], we need to lift this H-homotopy to $N \times M$.

LEMMA 2. *Let* H *act smoothly on a manifold* X, *and let* T *be a central subgroup of* H *that acts freely on* X. *Then the projection* $\pi: X \to X/T$ *is an* H-*fiber map.*

Proof. By [1, Chapter III, p. 7], we have to verify that if $K \leq H$ is a subgroup, then

$$\pi \mid X^K: X^K \to (X/T)^K$$

is a fibration. Since T is central, X^K is T-invariant, and $X^K \to X^K/T$ is certainly a fibration. Thus it suffices to verify that X^K/T is a union of components in $(X/T)^K$.

If U is a tubular neighborhood of X^K in X, we can think of $U \to X^K$ as a vector bundle. If $u \in U$ and $k \in K$, then k(u) lies in the same fiber as u, while t(u) is in a different fiber whenever $1 \neq t \in T$. Thus, the equation $k(u) = t(u)$ can only occur if $t = 1$ and $k(u) = u$.

It follows that U/T is a neighborhood of X^K/T such that $(U/T)^K = X^K/T$. Therefore X^K/T is open in $(X/T)^K$; we know that it is closed, since π is a closed map. This completes the proof of Lemma 2 and thus the proof of Lemma 1(b).

3. SOME SMITH CONSTRUCTIONS

Recall that G is generated by elements a and b of orders 2^{n+1} and 2, respectively, such that $bab = a^j$ and $j = -1$ or $j = \pm 1 + 2^n$. The case $j = -1$ gives the dihedral group.

Let $\xi = \exp(\pi i/2^n)$. Then G admits irreducible complex representations ω and ρ defined by the equations

$$\omega(a) = 1, \qquad \omega(b) = -1, \qquad \rho(a) = \begin{bmatrix} \xi & 0 \\ 0 & \xi^j \end{bmatrix}, \qquad \rho(b) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

Let $\tilde{\omega}$ and $\tilde{\rho}$ represent the actions of G on $\mathbb{C}^k$ and $\mathbb{C}^{2k}$, respectively, by the k-fold direct sums of ω and ρ. The symbols $\tilde{\omega}$ and $\tilde{\rho}$ will also denote the induced actions on the unit spheres S^{2k-1} and S^{4k-1}, respectively.

Let (S^∞, σ) be the classifying space constructed by R. E. Stong [11, pp. 10-11] for G-actions in which C acts freely. The essential property of this object is that if (M, ϕ) is a free G-action, then there is an equivariant $f: M \to S^\infty$, unique up to G-homotopy. In addition, the image of f will lie in $S^{4K-1} \subset S^\infty$ for sufficiently large K, where S^{4K-1} is the unit sphere in some representation θ of G in $\mathbb{C}^{2K}$.

Let S^{4K+3} be the unit sphere in $(\mathbb{C}^{2K} \times \mathbb{C}^2, \theta \times \rho)$. By Lemma 1(a), we can deform f to be transverse to S^{4K-1} in S^{4K+3}. Assigning $f^{-1} S^{4K-1}$ to M then defines

$$\Delta_\rho: \hat{U}_m(G) \to \hat{U}_{m-4}(G).$$

The reader can verify that Δ_ρ is a well-defined homomorphism.

Similarly, if we replace S^{4K+3} by the unit sphere S^{4K+1} in $(\mathbb{C}^{2K} \times \mathbb{C}, \theta \times \omega)$, we obtain a homomorphism

$$\Delta_\omega: \hat{U}_m(G) \to \hat{U}_{m-2}(G).$$

Next we turn to an application of Lemma 1(b). Let H be nilpotent. Suppose (N, ψ) is a smooth H-action and $P \subseteq N$ is an invariant submanifold on which H acts freely. If P is of even codimension, we assume that its normal bundle is of the form $(\mathbb{C}^k \times P, \theta \times \psi)$. Here and subsequently, it is convenient to use the same symbol for a map and its restriction, so that $\theta \times \psi$ means $\theta \times (\psi \mid H \times P)$. If P is of odd codimension, we assume its normal bundle has the form $(\mathbb{C}^k \times \mathbb{R} \times P, \theta \times \varepsilon \times \psi)$, where $\varepsilon: G \times \mathbb{R} \to \mathbb{R}$ is the trivial representation.

Given $[M, \phi, f] \in U_*(H)(N, \psi)$, the equivariant bordism of (N, ψ) as defined in [10], we can assume that f is transverse to P, and put

$$\Delta_P(M, \phi, f) = (Q, \phi, f) \qquad (Q = f^{-1} P).$$

LEMMA 3. *If* P *is of even codimension, then*

$$\Delta_P: U_*(H)(N, \psi) \to U_*(H)(P, \psi)$$

is a well-defined homomorphism, and $\Delta_P[M, \phi, f] = 0$ *if and only if* $[M, \phi, f]$ *lies in the image of* $U_*(H)(N - P, \psi)$.

Proof. Most of the proof is obvious. For the "only if" part, suppose $(Q, \phi, f) = \partial(Q', \phi', f')$. We may assume that Q has a neighborhood $D^{2K} \times Q$ on which ϕ' looks like $\theta \times \phi$, and that

$$f = \mathrm{id} \times (f \mid Q): D^{2k} \times Q \to D^{2k} \times P.$$

Now paste

$$(D^{2k} \times Q', \theta \times \phi', \mathrm{id} \times f) \quad \text{and} \quad (M \times I, \phi \times \mathrm{id}, f \cdot \mathrm{proj.})$$

to the copy of $D^{2k} \times Q$ in one end of the latter; the result is a bordism from (M, ϕ, f) to some object in the image of $U_*(H)(N - P, \psi)$.

LEMMA 4. *Let* $T \subset S^1$ *be the cyclic group of* nth *roots of* 1, *and let* $\mu: T \times S^1 \to S^1$ *be multiplication. Then* $U_*(T)(S^1, \mu)$ *is* U_**-free of rank* 2, *and it is generated by the inclusion of* T *and the identity mapping on* S^1.

Proof. Clearly, $U_*(T)(T, \mu) \cong U_*$ with generator $[T, \mu, \mathrm{id}]$. We have the relation $\Delta_T[S^1, \mu, \mathrm{id}] = [T, \mu, \mathrm{id}]$, and $S^1 - T$ is equivariantly homotopy equivalent to T. The result now follows easily from Lemma 3.

4. EVEN DIMENSIONS

Let $D < G$ be the subgroup, isomorphic to $Z_2 \times Z_2$, generated by b and a^{2^n}. We use the extension and restriction homomorphisms

$$e: \hat{U}_*(D) \to \hat{U}_*(G) \quad \text{and} \quad r: \hat{U}_*(G) \to \hat{U}_*(D),$$

respectively, as defined (for example) by Stong [11, pp. 12-13].

Fix an integer $q \geq 1$. For $1 \leq k \leq q/2$, define

$$\theta_{k,q} = [S^{4k-1} \times S^{2q-4k+1}, \tilde{\rho} \times \tilde{\omega}] \in \hat{U}_{2q}(G).$$

Write $\theta'_{k,q} = r(\theta_{k,q}) \in \hat{U}_{2q}(D)$.

We must consider the bordism of free D-actions in some detail. By the Künneth theorem,

$$\tilde{H}_{2q}(BD) = \sum_{i=1}^{q} \tilde{H}_{2i-1}(BZ_2) \otimes \tilde{H}_{2(q-i)+1}(BZ_2).$$

By [6, Theorem B], the similar formula for unitary bordism also holds. The Thom homomorphism $\mu: \tilde{U}_{2q}(BD) \to \tilde{H}_{2q}(BD)$ is surjective. More precisely, let $\alpha_i \in \hat{H}_{2i-1}(BZ_2)$ be the generator. Then

$$\alpha_i \otimes \alpha_{q-i+1} = \mu(\lambda_{i,q}),$$

where $\lambda_{i,q} \in \hat{U}_{2q}(D)$ is represented by $S^{2i-1} \times S^{2(q-i)+1}$ with the following D-action: if a', b are generators of D, let a' act as $(-1) \times 1$ and b as $1 \times (-1)$.

LEMMA 5. *If* q *is even,* $\theta'_{q/2,q} = \lambda_{q,q} \in \hat{U}_{2q}(D)$.

Proof. Both $\theta'_{q/2,q}$ and $\lambda_{q,q}$ are represented by D-actions on $S^{2q-1} \times S^1$. We imitate [4, (35.1)]. Let a' act on $S^{2q-1} \times S^1$ as $(-1) \times 1$, and let b act as $t \times (-1)$, where t is any involution commuting with -1. This gives a class $x = x(t) \in \hat{U}_{2q}(D)$; it suffices to show that x does not depend on t.

Since $BD = BZ_2 \times BZ_2$, [10, Proposition 2.3] gives us an isomorphism $\hat{U}_*(D) \cong \hat{U}_*(Z_2)(BZ_2, \textit{trivial})$. To construct the image of x, note that the projection $S^{2q-1} \times S^1 \to S^1$ is equivariant. After dividing out the action of b, we have a mapping

$$h: (S^{2q-1} \times S^1)/\{1, b\} \to S^1 \to BZ_2,$$

which is Z_2-equivariant with respect to the trivial actions on S^1 and BZ_2.

Now $\hat{U}_*(Z_2)(S^1, \text{trivial})$ is a free $\hat{U}_*(Z_2)$-module on two generators, the inclusion of a point, and the identity map on S^1. Since $(S^{2q-1} \times S^1)/\{1, b\}$ is even-dimensional and bounds as a manifold, it follows that x corresponds to a multiple of $(S^1, \textit{trivial}, \text{id})$. The coefficient is $[h^{-1}(*)]$, where $* \in S^1$ is a regular point; clearly, this is S^{2q-1} with antipodal action, for any choice of t. This establishes the lemma.

Let $\Delta_H: \tilde{H}_{2q}(BD) \to \tilde{H}_{2q-2}(BD)$ by $\Delta_H(\alpha_i \otimes \alpha_{q-i+1}) = \alpha_i \otimes \alpha_{q-i}$. Then clearly

$$\mu \Delta_\omega = \Delta_H \mu: \tilde{U}_{2q}(BD) \to \tilde{H}_{2q-2}(BD).$$

In fact, one may show, much as in [3, (10.3)], that

$$\Delta_\omega(\lambda_{i,q}) = \lambda_{i,q-1} \quad \text{and} \quad \Delta_\omega(\theta'_{k,q}) = \theta'_{k,q-1}$$

for $i < q$ and $k < q/2$, while

$$\Delta_\omega(\lambda_{q,q}) = 0 = \Delta_\omega(\theta'_{q/2,q}).$$

PROPOSITION 1. *For each* q, *the classes* $\{\mu(\theta'_{k,q}): 1 \leq k \leq q/2\}$ *are* Z_2-*linearly independent in* $\tilde{H}_{2q}(BD)$.

Proof. For $q = 1, 2$, this follows from Lemma 5. Suppose $\sum a_k \mu(\theta'_{k,q}) = 0$, and use induction on q. Then

$$\sum a_k \mu(\theta'_{k,q-1}) = \sum a_k \mu \Delta_\omega(\theta'_{k,q}) = \Delta_H \left(\sum a_k \mu(\theta'_{k,q}) \right) = 0,$$

which shows that $a_k = 0$ for $k < q/2$. To see that $a_{q/2} = 0$ for even q, we again apply Lemma 5.

COROLLARY. *For each* q, *the classes* $\{\mu(\theta_{k,q}): 1 \leq k \leq q/2\}$ *are* Z_2-*linearly independent in* $\tilde{H}_{2q}(BG)$.

This is clear, since it is known that $\tilde{H}_{2q}(BG)$ is also a Z_2-vector space (C. T. C. Wall [12]).

PROPOSITION 2. *The Thom homomorphism* $\mu: \tilde{U}_{2q}(BG) \to \tilde{H}_{2q}(BG)$ *is surjective.* $\tilde{U}_{2*}(BG)$ *is generated as a* U_*-*module by the* $\theta_{k,q}$, *together with classes in the image of* $e: \tilde{U}_{2*}(BD) \to \tilde{U}_{2*}(BG)$ *in the case where* G *is dihedral.*

Proof. If G is not dihedral (that is, if $bab = a^j$ and $j = \pm 1 + 2^n$), then by [12] the rank of $\tilde{H}_{2q}(BG)$ is $[q/2]$, and Proposition 1 implies that μ is surjective.

If G is dihedral, then rank $\tilde{H}_{2q}(BG) = q$. The image of e can also be computed from [12]. Specifically, Wall writes $\tilde{H}_{2q}(BG) = \sum_{i=1}^{q} Z_{q_i}$, and in this case each $q_i = 2$. The cycle representing the generator of Z_{q_i} is Wall's rf_m/h_i, where $m = 2(q - i) + 1$. It is easy to see that this cycle falls in Im e if and only if $h_i = 2$, and that $h_i = 2$ if and only if i is odd.

Thus Im e has rank $[(q + 1)/2]$. In

$$(*) \qquad \tilde{H}_{2q}(BD) \xrightarrow{e} \tilde{H}_{2q}(BG) \xrightarrow{r} \tilde{H}_{2q}(BD),$$

the composition is multiplication by $[G: D] = 2^n$, hence is zero. Thus Im e $\subseteq$ Ker r. Since

$$\text{rank Coker } e = q - [(q + 1)/2] = [q/2],$$

Proposition 1 implies that (*) is exact; also, since μ is surjective for D, it is surjective for G.

The remaining assertions follow from the knowledge that μ is surjective if we apply the usual spectral-sequence arguments, as in [4, (15.1) and (18.1)].

This proposition proves our theorem in even dimensions, since the $\theta_{k,q}$ and the elements of Im e all map to zero in $U_*(G)$.

5. ODD DIMENSIONS

We prove the theorem in odd dimensions by induction on dimension. To start the induction, the reader should notice that Propositions 3 to 5 below remain true, and have simpler proofs, in dimensions 1 and 3. We now assume the theorem in dimensions less than m, for some odd m.

PROPOSITION 3. *If* $x \in \hat{U}_m(G)$, *then* $s(x) = [M, \phi]$ *for some action admitting an equivariant map* $f: (M, \phi) \to (S^3, \rho)$.

Proof. Let $x = [M', \phi']$. By induction, $s\Delta_\rho(x) = 0$; therefore we put

$$\Delta_\rho(M', \phi') = (N, \phi') = \partial(P, \psi).$$

Let $F \subset P$ be the fixed set of a^{2^n}. Then P gives us a bordism P' from (N, ϕ') to $(S\nu, \psi)$, where $S\nu$ is the boundary of a tubular neighborhood of F.

As in [9, Proposition 5], we compare $[M', \phi']$ and $[S(\nu \oplus \mathbb{C}^2), \psi']$, where ψ' is the obvious action derived from ψ on $D\nu$ and ρ on $\mathbb{C}^2$. Pasting $M' \times I$, $P' \times D^4$, and $D(\nu \oplus \mathbb{C}^2)$, one produces a bordism from (M', ϕ') to an action whose classifying map into (S^∞, σ) has image in some $S^{2K+3} - S^{2K-1}$. The latter retracts equivariantly onto (S^3, ρ), whence the result.

Next we need to know something about the equivariant bordism of (S^3, ρ). Let $X \subset S^3$ be the union of the circles

$$\{(w, 0): |w| = 1\} \quad \text{and} \quad \{(0, z): |z| = 1\}.$$

Then X is an invariant submanifold satisfying the hypotheses of Lemmas 1(b) and 3. Let $T^2 \subset S^3$ be the torus $\{(w, z): |w| = |z| = 1/\sqrt{2}\}$.

PROPOSITION 4. *If* $x \in U_m(G)(S^3, \rho)$ *and* m *is odd, then*

$$x = [P][S^3, \rho, \mathrm{id}] + [M', \phi', f']$$

for some action $[M', \phi']$ *admitting an equivariant map* $f': (M', \phi') \to (T^2, \rho)$.

Proof. Using [2, Proposition II. 3.2], we see that

$$U_{m-2}(G)(X, \rho) \cong U_{m-2}(C)(S^1, \psi),$$

where $\psi(a, z) = z \cdot \exp(\pi i/2^n)$ for some generator $a \in C$. Since $m - 2$ is odd, Lemma 3 implies that $\Delta_X x = [P][X, \rho, \mathrm{id}]$. Applying Lemma 3, we find that

$$x = [P][S^3, \rho, \mathrm{id}] + [M', \phi', f'],$$

where $\mathrm{Im}\, f'$ misses X. But T^2 is an equivariant deformation retract of $S^3 - X$.

Put $\eta = \exp(\pi i/2^{n+1})$ and consider the following subsets of T^2:

$$Y = \{(w, z): z = \eta^k w, \ k \text{ odd}\},$$

$$S = \{(w, z): z = \eta w\},$$

$$N = \{(w, z): z = w \cdot \exp i\theta;\ -\pi/2^{n+1} \le \theta \le \pi/2^{n+1}\}.$$

Y is invariant and G acts freely in a neighborhood of Y. Given $[M, \phi, f] \in U_*(G)(T^2, \rho)$, we can thus assume, by Lemma 1(b), that f is transverse to Y. The submanifold $f^{-1}Y$ has trivial normal bundle and is thus a unitary manifold, but ϕ may fail to preserve the induced unitary structure (note that $f^{-1}Y$ has codimension 1).

To dodge this difficulty, let $Z_2 < G$ be the subgroup generated by a^{2^n}. Since Z_2 acts trivially in the normal bundle of S, we see that $f^{-1} S$ is a unitary manifold with unitary Z_2-action. Assigning $f^{-1} S$ to M, we obtain the mapping

$$\Delta_S: U_m(G)(T^2, \rho) \to U_{m-1}(Z_2)(S, \theta),$$

where θ describes the antipodal involution $(w, z) \to (-w, -z)$.

We claim that $\Delta_S = 0$ if m is odd. Since $m - 1$ is then even, it suffices, by Lemma 4, to show that $f^{-1} S$ bounds as a unitary manifold. For this, consider $f^{-1} N$. This manifold is invariant under the action of b, which is a unitary involution, and its boundary consists of two copies of $f^{-1} S$ interchanged by b. Hence $2[f^{-1} S] = 0$, so that $[f^{-1} S] = 0$.

Since $\Delta_S = 0$, we see by the proof of Lemma 3 that we can assume Im f misses S, and in fact Y. But

$$W = \{(w, z): z = w \cdot \exp(k\pi i/2^n),\ k \in Z\}$$

is an equivariant deformation retract of $T^2 - Y$. We have proved the following result.

PROPOSITION 5. *If* $x \in \hat{U}_m(G)$ *and* m *is odd, then* $s(x) = [M, \phi]$ *for some* (M, ϕ) *admitting an equivariant map into* (W, ρ).

Consider the circle $S_1 = \{(w, w)\}$ contained in W, which is D-invariant, and the circle $S_2 = \{(w, \xi w)\}$ contained in W, which is invariant under the subgroup E generated by a^{2^n} and $a^{-1} b$. Observe that $W = G \times_D S_1 \cup G \times_E S_2$. Using again [2, Proposition II. 3.2], we obtain the isomorphism

$$U_*(G)(W, \rho) \cong U_*(D)(S_1, \rho') \oplus U_*(E)(S_2, \rho'),$$

where ρ' denotes the actions induced on the circles S_i by ρ. The following proposition will complete the proof of the theorem.

PROPOSITION 6. *If* m *is odd and* $[M, \phi, f] \in U_m(D)(S_1, \rho')$, *then* $[M, \phi] = 0 \in U_m(D)$.

Proof. Put $T = \{(w_0, w_0), (-w_0, -w_0)\} \subset S_1$ for some specific w_0. Then $U_*(D)(T, \rho') \cong U_*(Z_2)$, and, just as in the proof of Lemma 4, we see that $U_*(D)(S_1, \rho')$ is a $U_*(Z_2)$-module on two generators, the identity map on S_1 and the inclusion of T. Since m is odd, and since $U_*(Z_2)$ is U_*-free on even-dimensional generators [10], $[M, \phi, f]$ must be a multiple of $[S_1, \rho', id]$. This completes the proof.

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